



NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION - 2013

SOLUTIONS FOR CLASS: 12 (PCM)

Mathematics

- 1. (B) $\int_0^{\pi/2} \cos x \cdot e^{\sin x} dx = \left| e^{\sin x} \right|_0^{\pi/2}$ $= e^1 e^0 = e 1.$
- 2. (D) Let $\mathbf{\xi}$ P be the profit when x items are sold.

$$P = \left(5 - \frac{x}{100}\right)x - \left(\frac{x}{5} + 500\right)$$
$$= 5x - \frac{x}{5} - \frac{x^2}{100} - 500$$

$$\Rightarrow \frac{dP}{dx} = 5 - \frac{1}{5} - \frac{2x}{100} = \frac{24}{5} - \frac{x}{50}$$

and
$$\frac{d^2P}{dx^2} = -\frac{1}{5}$$
 for max. profit, let $\frac{dP}{dx} = 0$

$$\Rightarrow \frac{24}{5} - \frac{x}{50} = 0$$
$$\Rightarrow x = 240$$

Also
$$\frac{d^2P}{dx^2}$$
 x = 240 $-\frac{1}{5}$ < 0

 \Rightarrow P is max. when x = 240.

- 3. (D) adj (adj A) = $|A|^{n-2}$ A, whenever A is a non singular matrix of order n(>1).
- 4. (D) $f(x) = \frac{d}{dx} \left(\int x^{m-1} dx \right) = x^{m-1}$ $\Rightarrow f^{m+1}(x) = \frac{d^m}{dx^m} \left(f^1(x) \right) = \frac{d^m}{dx^m} \left(x^{m-1} \right) = 0$ only if 'm' is a positive integer.

5. (A)
$$\sin\left[\cot^{-1}(x+1)\right] = \cos\left(\tan^{-1}x\right)$$
$$\Rightarrow \frac{1}{\sqrt{(1+x)^2+1}} = \frac{1}{\sqrt{1+x^2}}$$
$$\Rightarrow x = -\frac{1}{2}$$

6. (D) If $(\overrightarrow{a} \perp \overrightarrow{b})$ then $\overrightarrow{a} \ltimes \overrightarrow{b} = 0$ and hence $(\overrightarrow{a} + \overrightarrow{b})^2 = |\overrightarrow{a} + \overrightarrow{b}|^2$ $= (\overrightarrow{a} + \overrightarrow{b}) \ltimes (\overrightarrow{a} + \overrightarrow{b})$ $= |\overrightarrow{a}|^2 + |\overrightarrow{b}|^2 + 2\overrightarrow{a}^2 - \overrightarrow{b} = (\overrightarrow{a})^2 + (\overrightarrow{b})^2$ Similarly, $(\overrightarrow{a} + \overrightarrow{b})^2 = (\overrightarrow{a} - \overrightarrow{b})^2$ and $(\overrightarrow{a} + \overrightarrow{b})^2 = (\overrightarrow{a})^2 + (\overrightarrow{b})^2 = |\overrightarrow{a} - \overrightarrow{b}|^2$

So, all the four statements are correct.

- 7. (B) $\frac{d^2y}{dx^2} = e^{-2x} \Rightarrow \frac{dy}{dx} = \frac{e^{-2x}}{-2} + c$ $\Rightarrow y = \frac{e^{-2x}}{4} + cx + d$
- 8. (D) $\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ therefore, inverse of $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$ is $\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$.

10. (B) Clearly, the relation is symmetric but it is neither reflexive nor transitive.

11. (D) Put
$$\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$$

$$\therefore \int \frac{\cos \sqrt{x}}{\sqrt{x}} dx = \int (\cos t) 2dt = 2 \sin t$$

$$= 2 \sin \sqrt{x}.$$

12. (B) The given differential equation can be written as
$$\left\{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right\}^5 = \left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)^3$$
.

Here order is 2 and the degree is 3.

- 13. (A) Expanding with R1, we obtain the value of the given determinant as $1 (\cos^2 x \sin^2 x) + 0 + 0 = \cos 2x.$
- 14. (B) For any $x_1, x_2 \in \mathbb{R}$, we have $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1 - \mathbf{m}}{x_1 - \mathbf{n}} = \frac{x_2 - \mathbf{m}}{x_2 - \mathbf{n}}$$

$$\Rightarrow x_1 = x_2$$

 \therefore f is one – one.

Now let $\alpha \in R f(x) = \alpha$

$$\Rightarrow \frac{x-m}{x-n} = \alpha$$

$$\Rightarrow x = \frac{m - n\alpha}{1 - \alpha}$$

Clearly, $x \notin R$ for $\alpha = 1$.

: f is not onto.

15. (C) Here, $D_f = R$ and $f^1(x) = 3kx^2 - 18x + 9$, $x \in R$.

Now f is increasing on R

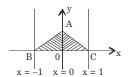
if $f^{1}(x) \ge 0$ for all $x \in R$

i.e., if 3k > 0 and $(-18)^2 - 4 \times (3k) \times 9 \le 0$

i.e., if k > 0 and $k \ge 3$ ($_{\square}$ $ax^2 + 6x + c \ge 0$ for

all $x \in R$: if a > 0 and $b^2 - 4ac \le 0$).

16. (D) The required area = Area of $\triangle ABC = \frac{1}{2} \times 1 \times 2 = 1$ sq. units



17. (D) Here, f(1) = a + 1,

LHS:
$$f'(1) = \frac{Lt}{h \to 0^{-}} \frac{f(1+h) - f(1)}{h}$$
$$= \frac{Lt}{h \to 0^{-}} \frac{a(1+h)^{2} + 1 - (1+a)}{h} - 2a$$

$$\begin{split} RHS: f^{'}(1) &= \frac{Lt}{h \to 0^{+}} \frac{f(1+h) - f(1)}{h} = \\ &= \frac{Lt}{h \to 0^{+}} \frac{1 + h + a - (1+a)}{h} = 1 \end{split}$$

: f is derivable at 1, only if 2a = 1, i.e., if $a = \frac{1}{2}$.

18. (B) We have, $2f(x) = f'(x) \implies \frac{f'(x)}{f(x)} = 2$.

Integrating, we get $\log f(x) = 2x + c$,

$$\Rightarrow$$
 f(x) = $e^{2x + c_1} = e^{c_1}$, $e^{2x} = ce^{2x}$

where $c = e^{c_1}$.

Putting x = 0, f(0) = 3, we get c = 3.

$$\therefore f(x) = 3e^{2x} \implies f(2) = 3e^4.$$

19. (C) Now, $\begin{vmatrix} 3 & 2 \\ a + b + c \end{vmatrix} = (3 + 3 + 2) \cdot (3 + 3 + 2)$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} - \vec{b} + \vec{b} + \vec{c} + \vec{c} \cdot \vec{a})$$

$$(\Box \stackrel{\Rightarrow}{a} - \stackrel{\Rightarrow}{b} = \stackrel{\Rightarrow}{b} + \stackrel{\Rightarrow}{c} + \stackrel{\Rightarrow}{c} \cdot \stackrel{\Rightarrow}{a} = 0)$$

$$=6^2 + 2^2 + 3^2 + 0 = 49$$

Hence $\begin{vmatrix} \Rightarrow & \Rightarrow \\ a + b + c \end{vmatrix} = 7$.

20. (D) Let $\phi(x) = \{f(x) + f(-x)\} \{g(x) - g(-x)\},\$

then $\phi(x)$ is an odd function as

$$\phi(-x) = \{f(-x) + f(x)\} \{g(-x) - g(x)\}\$$

$$= -\phi(x)$$
.

- 21. (D) Here, $u = \frac{\pi}{2} \operatorname{as} \cot^{-1} x + \tan^{-1} x = \frac{\pi}{2} \ \forall x \in \mathbb{R}$.
- 22. (A) $\int_{0}^{\frac{\pi}{2}} \sin^{2} x \cos^{2} x \, dx = \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} = \frac{\pi}{16}$
- 23. (C) The function $f(x) = a^x$, $x \in \mathbb{R}$ (a > 0) is strictly decreasing if 0 < a < 1 and strictly increasing if a > 1. When a = 1, $f(x) = 1^x = 1$ is a constant function.
- 24. (A) x + y = 2

$$2x + 2y = 3$$

i.e.,
$$x + y = 2$$

and
$$x + y = \frac{3}{2}$$

 $\Rightarrow 2 = \frac{3}{2}$, which is absurd.

Hence, the given system has no solution.

Alternatively,
$$|A| = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 0$$

$$\left| \right. A_{_{1}} \left| \right. \left| \left. \begin{array}{cc} 2 & 1 \\ 3 & 2 \end{array} \right| = 4 - 3 = 1 \, \neq 0.$$

.: Given system is inconsistent.

25. (A) If
$$y = \log_{10}^{x}$$
 and $z = \log_{x}^{10}$, then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}}{\mathrm{d}x} (\log_{10} x)}{\frac{\mathrm{d}}{\mathrm{d}x} (\log_{x} 10)} = \frac{\frac{\mathrm{d}}{\mathrm{d}x} \left(\frac{\log x}{\log 10}\right)}{\frac{\mathrm{d}}{\mathrm{d}x} \left(\frac{\log 10}{\log x}\right)}$$

$$\frac{\frac{1}{\log 10} \cdot \frac{1}{x}}{\log 10 \cdot \left\{ -\frac{1}{(\log x)^{2}} \cdot \frac{1}{x} \right\}} = -\frac{(\log x)^{2}}{(\log 10)^{2}}$$

26. (D) Range of
$$\tan^{-1} x$$
 is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

27. (D) Since
$$A \cap A = A \neq \phi$$
 for a non-empty set A, the relation R is not reflexive. If $A \cap B = \phi$ then $B \cap A = \phi$. So, the relation R is symmetric. If $A \cup B \cup C = \{1, 2, 3, 4\}$, $A = \{1\}$, $B = \{2, 3\}$ and $C = \{1, 4\}$ then $A \cap B = \phi$, $B \cap C = \phi$ and $A \cap C = \{1\}$. Thus ARB, BRC but A is not related to C. Hence R is not transitive.

28. (B)
$$A^2 \neq 0$$
 but $A^3 = 0$, therefore A^3 is a nilpotent matrix.

29. (A) Given,
$$\frac{d}{dx} \left(\frac{dy}{dx} \right) = e^{-2x}$$

Integrating,
$$\frac{d}{dx} = \frac{e^{-2x}}{-2} + a$$

Again integrating, we get

$$y = \frac{e^{-2x}}{(-2)(-2)} + ax + b$$

30. (C) Let
$$I = \int_{0}^{a} x f(x) dx$$
(1)

then
$$I = \int_{a}^{a} (a - x)f(a - x)dx$$

$$\left[\Box \int_{0}^{a} f(x) dx = \int_{0}^{a} f(a - x) dx \right]$$

or I =
$$\int_{0}^{a} (a - x)f(x)dx$$
(2)

Adding (1) and (2),

$$2\mathrm{I} = \int\limits_0^\mathrm{a} \mathrm{af}(x) \mathrm{d}x \, \Longrightarrow \, \mathrm{I} = \frac{\mathrm{a}}{2} \int\limits_0^\mathrm{a} \mathrm{f}(x) \mathrm{d}x \; .$$

31. (C) For unique solution
$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & \lambda \\ 1 & 2 & \lambda \end{vmatrix} \neq 0$$
$$\Rightarrow 1(2\lambda - 6) - 1(\lambda - 3) + 1(2 - 2) \neq 0$$
$$\Rightarrow \lambda - 3 \neq 0$$

32. (B)
$$|\hat{\mathbf{a}} - \hat{\mathbf{b}}|^2 = |(\hat{\mathbf{a}} - \hat{\mathbf{b}}) \cdot (\hat{\mathbf{a}} - \hat{\mathbf{b}})|$$

 $= |\hat{\mathbf{a}} \cdot \hat{\mathbf{a}} - 2 \hat{\mathbf{a}} \cdot \hat{\mathbf{b}} + \hat{\mathbf{b}} \cdot \hat{\mathbf{b}}|$
 $= |\hat{\mathbf{a}}|^2 - 2 |\hat{\mathbf{a}}| \hat{\mathbf{b}} | \cos \theta + |\hat{\mathbf{b}}|^2$
 $= 1 - 2\cos \theta + 1 = 2 (1 - \cos \theta)$
 $= 2 \times 2 \sin^2 \frac{\theta}{2}$

$$\Rightarrow |\hat{\mathbf{a}} - \hat{\mathbf{b}}| = |2\sin\left(\frac{\theta}{2}\right)|$$

$$=2\sin\left(\frac{\theta}{2}\right)\begin{bmatrix}\sin ce\\0\leq\theta\leq\pi\\0\leq\frac{\theta}{2}\leq\frac{\pi}{2}\end{bmatrix}$$

$$= \mathop{\rm Lt}_{x \to a} \frac{f(x) - 3^2}{\sqrt{f(x)} + 3} \cdot \frac{\sqrt{x} + 3}{x - 3^2}$$

$$= \underset{x \to a}{\operatorname{Lt}} \quad \frac{f(x) - 9}{x - 9} \cdot \frac{\sqrt{x} + 3}{\sqrt{f(x)} + 3}$$

$$= \underset{x \to a}{\operatorname{Lt}} \frac{f(x) - f(9)}{x - 9} \cdot \underset{x \to a}{\operatorname{Lt}} \frac{\sqrt{x + 3}}{\sqrt{f(x) + 3}}$$

= f'(a)
$$\frac{\sqrt{9}+3}{\sqrt{f(9)}+3}$$
 = $3\left(\frac{3+3}{\sqrt{9}+3}\right)$ = 3.

34. (A)
$$f(x) = x^2 + \frac{250}{x} \Rightarrow f'(x) = 2x - \frac{250}{x}$$
 and
$$f''(x) = 2 + \frac{500}{x^3}$$

Now,
$$f'(x) = 0 \implies x = 5$$

$$f''(5) = 2 + \frac{500}{5^3} = 2 + 4 = 6 > 0$$

 \therefore f has a local minimum at x = 5 whose value is f(5)

$$=5^2 + \frac{250}{5} = 25 + 50 = 75.$$

35. (D)
$$\overrightarrow{OP} \times \overrightarrow{OQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -6 & 3 \\ -2 & 1 & 2 \end{vmatrix}$$

$$= (-12 - 3) \hat{i} - (4 + 6) \hat{j} + (2 - 12) \hat{k}$$

$$= -15 \hat{i} - 10 \hat{j} - 10 \hat{k}$$

$$\Rightarrow \overrightarrow{OP} \times \overrightarrow{OQ} = \sqrt{(-15)^2 + (-10)^2 + (-10)^2}$$

$$= \sqrt{425} = 5\sqrt{7}$$

36. (B)
$$\sin\left(2\cos^{-1}\left(-\frac{4}{5}\right)\right)$$

$$= 2\sin\left(\cos^{-1}\left(-\frac{4}{5}\right)\right)\cos\left(\cos^{-1}\left(-\frac{4}{5}\right)\right)$$

$$= \sqrt[2]{1-\left(-\frac{4}{5}\right)^2\left(-\frac{4}{5}\right)}$$

$$= -\frac{8}{5}\sqrt{1-\frac{16}{25}} = -\frac{8}{5}\times\frac{3}{5} = -\frac{24}{25}$$

37. (B)
$$A^2 = -I$$

$$\Rightarrow A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Out of the given matrices, only for

$$\mathbf{A} = \begin{bmatrix} \mathbf{i} & \mathbf{0} \\ \mathbf{0} & -\mathbf{i} \end{bmatrix},$$

$$A^2 = -I \cdot$$

38. (D)
$$\int_{0}^{1} (a+x)^{2} f(x) dx = a^{2} \int_{0}^{1} f(x) dx + 2a$$
$$\int_{0}^{1} x f(x) dx + \int_{0}^{1} x^{2} f(x) dx$$
$$= a^{2} \cdot 1 + 2a \cdot a + a^{2} = 4a^{2}$$

39. (D)
$$\begin{vmatrix} R_2 \to R_3 - R_1 \\ R_3 \to R_3 - R_1 \end{vmatrix} \Rightarrow \begin{vmatrix} x & -6 & -1 \\ 2 - x & -3x + 6 & x - 2 \\ -3 - x & 2x + 6 & x + 3 \end{vmatrix} = 0$$

$$\Rightarrow (x-2)(x+3) \begin{vmatrix} x & -6 & -1 \\ -1 & -3 & 1 \\ -1 & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (x-2)(x+3) \Rightarrow (x-2)(x+3)$$

$$\begin{vmatrix} x-1 & -6 & -1 \\ 0 & -3 & 1 \\ 0 & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (x-2)(x+3)(x-3)(-1)^{1+1} \begin{vmatrix} -3 & 1 \\ 2 & 1 \end{vmatrix} = 0$$
[Expand C,]

⇒
$$(x-2)(x+3)(x-1)(-5) = 0$$

⇒ $x = 2, -3, 1$

40. (D)
$$f(x) = \frac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$$

$$\Rightarrow f(x) = \begin{cases} 0 & x < 0 \\ \frac{e^x - e^{-x}}{e^x + e^{-x}} & x \ge 0 \end{cases}$$

$$\Rightarrow f(x) > 0 \ \forall x \ge 0$$

f(x) is neither injective nor surjective $\forall x \in \mathbb{R}$.

Physics

- 41. (D) A potentiometer is preferred for measurement of the e.m.f of a cell. Potentiometer draws no current from cell while voltmeter draws some current. Therefore, e.m.f. measured by voltmeter is slightly less than the actual value of e.m.f. of a cell. Further, a potentiometer is used with a galvanometer which is set to null reading when the experiment is performed. This method of null reading avoids many errors.
- 42. (D) For total internal reflection to take place at AB i > θ_c , i.e., \sin i > \sin θ_c

$$i = 45^{\circ}$$
 and $\sin \theta_c = (1/\mu)$

$$\frac{1}{\sqrt{2}} > \frac{1}{\mu}$$
, i.e., $\mu > \sqrt{2}$

So,
$$(\mu)_{min} = \sqrt{2}$$

43. (D) In series (minimum value of capacitance):

$$C = \frac{C_{1}C_{2}C_{3}}{C_{1}C_{2} + C_{2}C_{3} + C_{3}C_{1}}$$

$${}^{3}C_{1} \times G_{1} \times G_{2}$$

$$=\frac{{}^{3}\cancel{9}\times\cancel{9}\times\cancel{9}}{\cancel{3}\times\cancel{8}\cancel{1}_{\cancel{9}}}=3\,\mu\mathrm{F}$$

 $In \ parallel \ (maximum \ value \ of \ capacitance)$

$$C = C_1 + C_2 + C_3$$

= 9 + 9 + 9 = 27 μ F

44. (B) $f_m = 20 \text{ kHz}$; $f_c = 1 \text{MHz} = 1,000 \text{ kHz}$ side bands are produced at,

$$f_{min} = f_c - f_m = 1,000 - 20 = 980 \text{ kHz}$$

 $f_{max} = f_c + f_m = 1,000 + 20 = 1,020 \text{ kHz}$

45. (B) When slit is wide (i.e. a>> λ), bending of light becomes so small that it cannot be detected upto a certain distance of screen from the slit. Hence, practically, no diffraction occurs.

In diffraction pattern, all the bright bands are not of the same intensity.

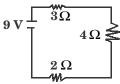
46. (C) Here,
$$\frac{dI}{dt}$$
 = 6A s⁻¹; e = 18 mV = 18×10⁻³ V

$$e = \frac{\angle dI}{dt}$$

$$L = \frac{e}{(dI/dt)} = \frac{18 \times 10^{-3}}{6} = 3 \times 10^{-3} \,H = 3 \,mH$$

During the positive half of the input a.c., one of the diodes is forward biased and the other diode is reverse biased. Hence, no current flows through the circuit.

Similarly, during the negative half of the input a.c., one of the diodes is reverse biased and the other diode is forward biased. Hence, no current flows through the circuit.



$$\Rightarrow I = \frac{9}{9} = 1A$$

So,
$$I_{AB} = 1A = I_{EH}$$

$$I_{BE} = \frac{1}{2} I_{AB} = 0.5A = I_{BC} = I_{EF}$$

$$I_{_{CF}} = \, \frac{1}{2} \, I_{_{BE}} = 0.25 A = I_{_{GH}} = I_{_{CD}} = I_{_{DG}}$$

: Current through 3Ω resistor in DG is $0.25 \, A$.

- 49. (A) Radio waves are electromagnetic waves whose wavelength ranges from 10^3 m to 10^5 m and has the longest wavelength compared to X rays, ultraviolet rays and visible light.
- 50. (C) $R = R_a A^{1/3}$

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{3}} = \left(\frac{1}{27}\right)^{\frac{1}{3}} = \left(\frac{1}{3^3}\right)^{\frac{1}{3}} = \frac{1}{3}$$

51. (C) Force on A due to B is F_1

$$=\frac{\left(1\times10^{-6}\right)\left(1\times10^{-6}\right)C^{2}}{4\pi\epsilon_{0}\left(8cm\right)^{2}}$$

$$= 9 \times 10^{9} \frac{Nm^{2}}{C^{2}} \times 10^{12} C^{2} \times \frac{1}{64 \times 10^{-4} m^{2}}$$
$$= 14.06 N$$

This force acts along BA. Similarly, the force on A due to C is \mathbf{F}_2

$$=\frac{\left(1\times10^{-6}\right)\left(1\times10^{-6}\right)C^{2}}{4\pi\,\epsilon_{o}\,\left(6cm\right)^{2}}$$

=
$$9 \times 10^{9} \frac{\text{Nm}^{2}}{\text{C}^{2}} \times 10^{-12} \text{C}^{2} \times \frac{1}{36 \times 10^{-4} \text{ m}^{2}}$$

= $0.25 \times 10 = 2.5 \text{ N}$

$$F = \frac{\sqrt{F_1^2 + F_2^2}}{\sqrt{(14.06)^2 + (2.5)^2}} = 14.3 \text{ N}$$

- 52. (D) (i) The first prism has split the white light incident on it into its component colours.
 - (ii) The component colours were recombined by the inverted prism to give out white light as emergent ray.
- 53. (B) Since, force on any section of wire will be outwards, the loop will have a tendency to expand.

54. (B) Frequency
$$v = \frac{w}{2\pi}$$

$$K_{max} = h \nu - \phi = \frac{h\omega}{2\pi} - \phi$$

$$= \left(\frac{h\omega - \phi(2\pi)}{2\pi}\right)$$

55. (C) If e is the magnitude of charge on an electron or a proton, then

$$F = 9 \times 10^9 \ \frac{e^2}{r^2} \ = 9 \times 10^9 \times \frac{\left(1.6 \times 10^{-19}\right)^2}{\left(0.53 \times 10^{-10}\right)^2}$$

$$= 8.2 \times 10^{-8} \text{N}$$

56. (B) The frequency of the electron is given by

$$v = \frac{v}{2\pi r} = \frac{mvr}{2\pi mr^2} = \frac{nh}{2\pi (2\pi mr^2)}$$
$$= \frac{nh}{4\pi^2 mr^2}$$

Hence
$$V_1 = \frac{h}{4\pi^2 mr^2}$$

57. (B) Given : q = 1 C, $E_0 = 8.854 \times 10^{-12} C^2 N^{-1} m^{-2}$ From Gauss' theorem, electric flux

$$\phi = \frac{\mathbf{q}}{\mathbf{E}_{a}}$$

Electric lines of force originating from a

charge of 1 C,
$$\phi = \frac{1}{8.854 \times 10^{-12}} = 1.129 \times 10^{11}$$
.

- 58. (D) According to the given information substance 'X' need to be ferromagnetic in nature which is gadolinium.
- 59. (B) All colours in the visible spectrum propagate with the same speed in a non-

dispersive medium. The colour of light can be differentiated either by their frequencies or their wavelengths.

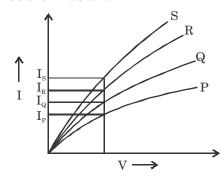
- 60. (A) In a series RLC circuit, resonance takes place when $X_L = X_C$ i.e., inductive reactance is equal to capacitive reactance. They are opposite as well as equal.
- 61. (A) A thermistor is a heat sensitive resistor, whose value varies appreciably with temperature more so than in standard resistors. Therefore, its temperature coefficient of resistivity should be high. Thermistors are usually made up of metal oxides.
- 62. (A) Given I = 20 A, n = 9×10^{30} m⁻³ ; A = 10^{-4} m² and e = 1.6×10^{-19} C

$$\begin{split} V_{_{d}} = \frac{I}{neA} = \frac{20}{9 \times 10^{30} \times 1.6 \times 10^{-19} \times 10^{-4}} \\ = 0.138 \times 10^{-6} \ m \ s^{-1} \end{split}$$

63. (A) Suppose that 4 wires P, Q, R, S possess resistances R_P , R_Q , R_R and R_S respectively and their resistivities be ρ_P , ρ_Q , ρ_R and ρ_S respectively. Secondly, all the four wires have same length l and area of cross section A,

$$\begin{split} R_{_{\mathrm{P}}} = \rho_{_{\mathrm{P}}} \frac{\mathit{l}}{A}, \; R_{_{\mathrm{Q}}} = \rho_{_{\mathrm{Q}}} \frac{\mathit{l}}{A}, \; R_{_{\mathrm{R}}} = \rho_{_{\mathrm{R}}} \frac{\mathit{l}}{A}, \\ R_{_{\mathrm{S}}} = \rho_{_{\mathrm{S}}} \frac{\mathit{l}}{A}, \end{split}$$

When a potential difference V is applied across each of the four wires P, Q, R and S as shown below.



$$\begin{split} R_{\mathrm{P}} &= \frac{\mathrm{V}}{\mathrm{I}_{\mathrm{P}}}; \ R_{\mathrm{Q}} = \frac{\mathrm{V}}{\mathrm{I}_{\mathrm{Q}}}; \ R_{\mathrm{R}} = \frac{\mathrm{V}}{\mathrm{I}_{\mathrm{R}}}; \ R_{\mathrm{S}} = \frac{\mathrm{V}}{\mathrm{I}_{\mathrm{S}}}; \\ I_{\mathrm{S}} &> I_{\mathrm{R}} > I_{\mathrm{Q}} > I_{\mathrm{P}} \Rightarrow R_{\mathrm{s}} < R_{\mathrm{R}} < R_{\mathrm{Q}} < R_{\mathrm{P}} \\ \rho_{\mathrm{S}} &\frac{l}{\mathrm{A}} < \rho_{\mathrm{R}} \frac{l}{\mathrm{A}} < \rho_{\mathrm{Q}} \frac{l}{\mathrm{A}} < \rho_{\mathrm{P}} \frac{l}{\mathrm{A}} \end{split}$$

This implies that P has greater resistivity.

64. (A) Flux density due to full circular line = $\frac{\mu_0 1}{2a}$

The arc AC is only $\frac{1}{3}$ of the circular line

$$B=\frac{1}{3}\times\frac{\mu_{o}i}{2a}=\frac{\mu_{o}i}{6a}$$

$$= \frac{4\pi \times 10^{-7} \times 6}{0.6} = 4 \pi \times 10^{-6}$$
$$= 3.14 \times 4 \times 10^{-6}$$

65. (B) Fringe width,

$$\omega = \frac{\lambda D}{d} = \frac{\left(6000 \times 10^{10}\right)(2)}{6 \times 10^{-3}}$$
$$= 0.2 \text{ mm}$$

Chemistry

66. (B) For reaction $3A \rightarrow 2B$.

 $= 1.26 \times 10^{-5} \,\mathrm{T}$

$$Rate = \frac{-1}{3} \frac{d(A)}{dt} = \frac{+1}{2} \frac{d(B)}{dt}$$

$$\therefore \frac{d[B]}{dt} = \frac{-2}{3} \frac{d[A]}{dt}$$

- 67. (D) $[Fe(CN)_g]^{3}$ is an octahedral complex ion and is paramagnetic in nature. Secondly it is an inner orbital complex ion with the presence of only one unpaired electron in it.
- 68. (D) Terylene is a polyester of ethylene glycol and terephthalic acid.
- 69. (B) The given cubic solid belongs to the body centred cubic lattice. Since, each corner atom Y is shared by 8 cubes, hence no. of Y atoms per cube = $8 \times \frac{1}{8} = 1$.

Atom X is at the body centre. So, there is only one X atom per cube. Therefore, formula of the compound is XY.

- 70. (A) The F- \sim -NH $_3^+$ is more acidic than \sim -NH $_3^+$ because of the presence of electron withdrawing (–I) F atom on the ring.
- 71. (C) Daniel cell is an electrochemical cell that converts chemical energy of a spontaneous redox reaction into electrical energy. But from the given redox reactions, the following can be concluded:
 - (i) A redox reaction is a combination of two half reactions whose addition gives the given overall reaction.
 - (ii) The reduction half reaction occurs on the copper electrode.
 - (iii) The oxidation half reaction occurs on the zinc electrode.
- 72. (B) On reduction, aldehydes give primary alcohols, while ketones give secondary alcohols. e.g.,
 - $\begin{array}{ccc} \text{(i) CH}_3 \text{ CHO} + \text{H}_2 & \xrightarrow{\text{Ni}} & \text{CH}_3 \text{CH}_2 \text{OH} \\ \text{acetaldehyde} & \text{(ethyl alcohol)} \end{array}$
 - (ii) CH_3 . $CO. CH_3 + H_2 \xrightarrow{Ni} CH_3 CH OH. CH_3$ acetone (propan - 2 - ol)

73.	(C)	Amorphous solids do not melt. They simply
15.	(\mathbf{C})	Amor phous somus do not mert. They simply
		soften on heating, and gradually begin to
		flow on further heating. These solids are,
		therefore considered as super cooled
		liquids.

$$\begin{array}{c} O \\ -C \\ -C \\ \longrightarrow \\ O \\ \longrightarrow$$

$$\begin{aligned} \mathbf{C_6H_{12}O_6} & \underline{\quad \mathbf{Zymase} \quad} \mathbf{2C_2H_5OH} + \mathbf{2CO_2} \\ \mathbf{Glucose} & \mathbf{Ethyl\ alcohol} \end{aligned}$$

- (ii) Conversion of milk into curd: It is an enzymatic reaction brought about by lacto bacilli enzyme present in curd.
- (iii) **Decomposition of urea into ammonia and CO₂:** The enzyme urease catalyses this decomposition.

$$NH_{2}CONH_{2} + H_{2}O \frac{Urease}{} 2NH_{3} + CO_{2}$$

- 78. (C) The elements belonging to 3d series are titanium, iron, because the valence electronic configuration of these elements are 3d² 4s², 3d⁶ 4s² respectively.
- 79. (B) Polystyrene is a thermoplastic polymer.
- 80. (A) Molality is a preferred unit for measuring concentration because it is temperature independent.

$$n_{solvent} = \frac{Mass \ of \ solute(w)}{Molar \ mass \ of \ solute(M)}$$

Molality (m) =
$$\frac{\text{w / M}}{\text{W}}$$

Molality (m) of a solution does not change with temperature.

81. (D) Xenon oxy tetra fluoride (XeOF₄) can be obtained by partial hydrolysis of XeF₆. Xe F₆ + H₂O \rightarrow XeOF₄ + 2HF

82. (B) Ethyl alcohol shows H-bonding and thus has higher boiling point. No H-bonding occurs between molecules of dimethyl ether, hence it exists as a gas.

The high boiling points of alcohols are mainly due to the presence of intermolecular hydrogen bonding in them which is lacking in ethers.

83. (B) Smelting is a process of adding a suitable fusible material (flux) to lower the melting point of the ore and reducing the ore into molten metal.

$$\mathrm{Fe_2O_3} + \mathrm{3C} \rightarrow \mathrm{2Fe} + \mathrm{3CO}$$

84. (D) In the complex ion, $[\text{Co(NH}_3)_3\text{C}l_3], \text{ both the NH}_3 \text{ molecule and } \\ \text{C}l^{\text{-}} \text{ ions are the ligands.}$

85. (D)
$$\Delta T_f = k_f m$$

$$k_f = \frac{\Delta T_f}{m} = \frac{4 \text{ deg}}{0.25 \text{ mol kg}^{-1}}$$

$$= 16 \text{ deg kg mol}^{-1}$$

86. (A) Soap coats the drops of an emulsion and checks them from coming together and the emulsion is thus stabilised.

87. (C) Phenol is manufactured from the hydrocarbon, cumene. Cumene (isopropyl benzene) is oxidised in the presence of air to cumene hydroperoxide. It is converted to phenol and acetone by treating it with dilute acid. Acetone, a by - product of this reaction is also obtained in large quantities by this method.

$$\begin{array}{cccc} CH_3 & CH_3 \\ CH_3-CH & & CH_3 \\ \hline & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & \\ & & & \\$$

88. (A) Fats and oils are glycerides as they are triesters of glycerol with higher fatty acids.

89. (B) All (copper, lead and chromium) have higher standard reduction potentials than Mn except Mg, which has lower standard reduction potential than Mn.

 \therefore Mg will displace Mn from its salt solution (MnSO₄).

90. (A) In aryl - alkyl ketones, the carbonyl group (>C=O) is attached to an alkyl and an aryl group. For example, acetophenone $(C_6H_5$. CO. CH_3) is an aryl - alkyl ketone.