

# **NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION - 2013**

## **SOLUTIONS FOR CLASS: 11 (PCM)**

#### **Mathematics**

1. (B)  $\tan \theta \tan 2\theta = 1$ 

$$\Rightarrow \tan\theta \left( \frac{2\tan\theta}{1-\tan^2\theta} \right) = 1$$

 $\Rightarrow 2 \tan^2 \theta = 1 - \tan^2 \theta$ ,  $\tan^2 \theta \neq 1$ 

 $\Rightarrow$  3 tan<sup>2</sup>  $\theta$  = 1

$$\Rightarrow \tan^2 \theta = \frac{1}{3} \Rightarrow \tan \theta = \tan \frac{\pi}{6}$$

$$\Rightarrow \theta = n \pi \pm \frac{\pi}{6}, n \in I$$

2. (C) If z = x + y i;  $x, y \in R$ ,

then 
$$z \overline{z} = (x + yi) (x - yi) = x^2 + y^2 = |z|^2$$
,

$$\left| \mathbf{Z}_{2}^{2} \right| = \left| \mathbf{x}^{2} - \mathbf{y}^{2} + 2i\mathbf{x}\mathbf{y} \right| = \sqrt{\left( \mathbf{x}^{2} - \mathbf{y}^{2} \right)^{2} + \left( 2\mathbf{x}\mathbf{y} \right)^{2}}$$

$$= \mathbf{x}^2 + \mathbf{y}^2 = \ \left| \mathbf{z} \right|^2$$

and z = (Re z) + (Im z)i.

So, (A), (B) and (D) are true but (C) is not true as  $\sqrt{z^2} \neq |z|$  when z is a complex number.

3. (A) To obtain the elements of RoR, we proceed as follows. Since (1, 4) and  $\in R$  we have  $(1, 5) \in \text{RoR}$ . Again, since  $(1, 4) \in R$  and  $(4, 6) \in R$  we have  $(1, 6) \in \text{RoR}$ . Similarly,  $(3, 6) \in \text{RoR}$  since  $(3, 7) \in R$  and

Similarly,  $(3, 6) \in \text{Nort since } (3, 7) \in \text{N and } (7, 6) \in \text{R}.$ 

Hence,  $RoR = \{(1, 5), (1, 6), (3, 6)\}$ 

- 4. (D) Solution set of  $x \ge 2$ ,  $x \le -3$  is  $(-\infty, -3] \cap [2, \infty) = \{ \}$ .
- 5. (C) If A is the single A.M. between the two numbers, then the sum of 40 A.M's = 40 A

$$\Rightarrow 120 = 40A$$

$$\Rightarrow$$
 A = 3.

Hence, the sum of 50 A.M's between the two numbers =  $50 \times 3 = 150$ .

6. (D) For y to be defined, we must have  $x^2 - 1 > 1 > 0$ 

 $\Rightarrow |x| \ge 1$  and  $x < 1 \Rightarrow x > 1$ .

 $\therefore$  Domain of y is  $(1, \infty)$ .

- 7. (D) The result (D) is incorrect as  ${}^{n}C_{r} + {}^{n}C_{r-1} \neq {}^{n}C_{r+1}$
- 8. (A) Here,  $|F_1F_2| = \sqrt{(3-0)^2 + (4-0)^2} = 5$  $\Rightarrow |PF_1| + |PF_2| = 10 > 5$   $\Rightarrow |PF_1| + |PF_2| = a \text{ constant greater}$

than  $|F_1F_2|$ , therefore, locus of P is an ellipse with foci at  $F_1$  and  $F_2$ .

9. (D)  $P(41) = 41^2 - 41 + 41$ 

i.e., 41<sup>2</sup> is prime, is certainly false.

10. (B) Equation of the line joining (3, 4) and (5, 6) is

$$y - 4 = \frac{6 - 4}{5 - 3}(x - 3)$$

or v - 4 = x - 3 or x - y + 1 = 0

When y = -1, we get x - (-1) + 1 = 0

$$\Rightarrow x = -2.$$

11. (C)



 $(A-B)\cup (B-C)\cup (C-A)$  is represented by the shaded portion in the figure. The unshaded portion is  $A\cap B\cap C$ .

$$\{(A-B)\cup (B-C)\cup (C-A)\}^1=A\cap B\cap C.$$

 $\Rightarrow \alpha = ar (1+r^2+r^4+....+r^{198}) .....(1)$ 

12. (A)  $\alpha = \sum_{n=1}^{100} a_{2n} \implies \alpha = a_2 + a_4 + \dots + a_{200}$  $\Rightarrow \alpha = ar + ar^3 + \dots + ar^{199}$ 

and

$$\beta = \sum_{n=1}^{100} a_{2n-1} \implies \beta = a_1 + a_3 + \dots + a_{199}$$

$$\implies \beta = a + ar^2 + \dots + a^{198}$$

$$\implies \beta = a(1+r^2 + \dots + ar^{198} \dots (2))$$

From (1) and (2), we get  $\frac{\alpha}{\beta} = r$ .

13. (B) In the expression of  $(1 + x)^{m+n}$ , general term  $T_{r+1} = {}^{m+n}C_r$   $x^r \Rightarrow coeff.$  of  $x^r = {}^{m+n}C_r$  Taking r = m, n we obtain

$$\begin{aligned} &\operatorname{coeff} \operatorname{of} x^{\operatorname{m}} = {}^{\operatorname{m+n}} C_{\operatorname{m}} = \frac{\left( \operatorname{m} + \operatorname{n} \right)!}{\left[ \left( \operatorname{m} + \operatorname{n} - \operatorname{m} \right)! \right] \operatorname{m}!} \ \operatorname{and} \\ &\operatorname{coeff} \operatorname{of} x^{\operatorname{n}} = {}^{\operatorname{m+n}} C_{\operatorname{n}} \ \overline{\left[ \left( \operatorname{m} + \operatorname{n} - \operatorname{n} \right)! \right] \operatorname{n}!} \ , \end{aligned}$$

which are certainly equal.

14. (C) The centre of the circle is the point of outersection of the diameters 2x - 3y = 5 and 3x - 4y = 7, i.e., the point (1, -1).

If r is the radius of the circle, then its area  $\pi r^2 = 154$  (Given)

$$\Rightarrow \frac{22}{7} \times r^2 = 154 \Rightarrow r = 7.$$

: Equation of the circle is

$$(x-1)^2 + (y+1)^2 = 7^2$$

or 
$$x^2 + y^2 - 2x + 2y = 47$$
.

15. (A) If the given points A, B, C and D are taken in order and |AC| and |BD| bisect each other then A B C D is a parallelogram.

Note that  $|AB| \neq |BC|$  and  $|AC| \neq |BD|$ .

16. (C) For any two integers a, & b, if |a-b| < 1

then 
$$|a-b|=0$$
.

So, 
$$a = b$$
.

Hence,  $R = \{(a,a); a \in I\}$ 

Thus R is reflexive, symmetric and transitive.

17. (D)  $a \le b, c < 0$ 

 $\Rightarrow$  b – a is either +ve or 0 and c is –ve.

 $\Rightarrow$  (b – a) c is either –ve or 0.

$$\Rightarrow$$
 bc – ac  $\leq 0$ 

$$\Rightarrow$$
 bc  $\leq$  ac

$$\Rightarrow$$
 ac  $\geq$  bc

18. (A) Given  $C = 90^{\circ}$ .

Also, 
$$\sin C = \frac{c}{2R}$$
;

$$Sin 90^{\circ} = \frac{c}{2R} \implies 1 = \frac{c}{2R} \implies 2R = c$$

Also 
$$r = (S - c) \tan \frac{C}{2} = (S - c) \tan \frac{90^{\circ}}{2}$$
  
= 2 S -

Here 2 (R + r) = 2R + 2r = c + 2(S - c)= 2S - c = a + b + c - c = a + b

19. (B) a, b, c are in G.P.,

$$\Rightarrow \frac{b}{a} = \frac{c}{b} \Rightarrow \frac{b}{a} - 1 = \frac{c}{b} - 1$$

$$\Rightarrow \frac{b - a}{a} = \frac{c - b}{b}$$

$$\Rightarrow \frac{b - a}{c - b} = \frac{a}{b}$$

$$\Rightarrow \frac{a - b}{b - c} = \frac{a}{b}.$$

- 20. (C) Clearly, the lengths of perpendiculars from (0, 0) on the given lines are each equal to 2.
- 21. (A)  $(\cos \theta + i \sin \theta)^2 = \cos^2 \theta + (i \sin \theta)^2 +$

 $2i \sin \theta \cos \theta$ 

$$= \cos^2 \theta - \sin^2 \theta + 9 (2 \sin \theta \cos \theta) (\Box i^2 = -1)$$
$$= \cos 2\theta + i \sin 2\theta$$

- 22. (C) Thousands place can be filled in only 9 ways as 0 cannot be placed there. Corresponding to each way of doing so, the remaining three places can be filled in  ${}^9P_3$  ways.
- 23. (A)  $\|PF_1| |PF_2\|$  is a fixed constant = length of transverse axis = 2a.

24. (C) 
$$S = \bigcup_{i=1}^{30} A_i$$

Now, elements of union S belongs to exactly 10 of the A's.

So, 
$$n(S) = \frac{1}{10} (5 \times 30) = 15$$
.

Again, 
$$S = \bigcup_{i=1}^{n} B_i$$
,  $So n(S) = \frac{1}{9}(3 \times n) = \frac{n}{3}$ .

Thus, 
$$\frac{n}{3} = 15$$
 and  $n = 45$ .

25. (B) The points representing the given complex numbers are (-4, 3); (2, -3) and (0, P). These are collinear,

i.e., if 
$$|12-6+2P-0+4P-0|=0$$

i. e., 
$$|6+6P|=0 \implies P=-1$$

26. (D) P(4): "2<sup>4</sup> < 1 × 2× 3× 4" is true.

27. (B)  $\sin^4 \theta + \cos^4 \theta = (\sin^2 \theta + \cos^2 \theta)^2 - 2\sin^2 \theta$  $\cos^2 \theta$ 

$$=1 - \frac{1}{2} (4\sin^2\theta \cos^2\theta)$$
$$= 1 - \frac{1}{2} (\sin^2\theta)^2$$

As  $0 \le \sin^2 2\theta \le 1$ ,

$$\begin{split} 0 &\geq -\,\frac{1}{2}\,\sin^22\theta \geq -\,\frac{1}{2} \\ \\ &\Rightarrow 1+0 \,\geq\, 1-\frac{1}{2}\sin^2\,2\,\theta \geq\, 1-\frac{1}{2} \\ \\ &\Rightarrow 1 \geq\, \sin^4+\cos^4\theta \geq \frac{1}{2}\,. \end{split}$$

28. (A) Eq. of the normal at point  $(b t_1^2, 2bt_1)$  on the parabola is

$$y = t_1 x + 2bt_1 + bt_1^3$$

It also passes through (b t<sub>2</sub> , 2bt<sub>2</sub>)

$$\begin{aligned} 2bt_2 &= -t_1 \cdot b \ t_2^2 + 2bt_1 + b \ t_1^3 \\ \Rightarrow 2t_2 - 2t_1 &= -t_1 \left( \ t_2^2 - t_1^2 \right) = t_1 \left( t_2 + t_1 \right) \left( t_2 - t_1 \right) \\ \Rightarrow 2 &= -t_1 \left( t_2 + t_1 \right) \\ \Rightarrow t_2 + t_1 &= \frac{-2}{t_1} \Rightarrow t_2 = -t_1 - \frac{2}{t_1} \end{aligned}$$

29. (B) Since  $R = \{(1, 2), (2, 3)\}$ 

i.e., 
$$A = \{1, 2\}$$
 and  $B = \{2, 3\}$ 

Now, if R, is the reflexive relation, such that

 $R_1 = \{(1, 2), (2, 3), (1, 1), (2, 2), (3, 3)\}$  has 5 elements

Now if R<sup>1</sup> is both symmetric and reflexive relation, then,

 $\begin{array}{l} R_2 = \{(1,\,2),\,(2,\,3),\,(1,\,1),\,(2,\,2),\,(2,\,1)\,(3,\,2),\\ (3,\,3)\} \; has \; 7 \; elements. \end{array}$ 

Again, if  $R_3$  is reflexive, symmetric and transitive all together, then  $R_3 = \{(1, 2), (2, 3), (1, 1), (2, 2), (2, 1), (3, 2), (3, 3), (1, 3), (3, 1)\}$  has 9 elements starting from 2 elements.

 $\therefore$  The minimum number of elements to be added is 7.

30. (C) Since  $(x + a)^n + (x - a)^n$ 

$$= 2 \left\{ {}^{n}C_{o} x^{n} + {}^{n}C_{2}x^{n-2}a^{2} + \dots \right\},\,$$

$$\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5$$

$$= 2\{{}^{5}\mathrm{C}_{0}x^{5} + {}^{5}\mathrm{C}_{2}x^{3}\left(\sqrt{x^{3}-1}\right)^{2} + {}^{5}\mathrm{C}_{4}x\left(\sqrt{x^{3}-1}\right)^{4}$$

= 
$$2\{x^5 + {}^5 c_2 x^3 (x^3 - 1) + {}^5 c_4 x (x^3 - 1)^2\}$$

which is a polynomial of degree 7.

31. (D) 
$$f[f(x)] = \frac{\alpha f(x)}{f(x)+1} = \frac{\alpha^2 x}{\alpha x + x + 1}$$

$$\therefore x = \frac{\alpha^2 x}{(\alpha + 1)x + 1} \text{ or } x (\alpha + 1)x + 1 - \alpha^2 \} = 0$$

$$\Rightarrow (\alpha+1)x^2 + (1-\alpha^2)x = 0$$

This should hold for all x.

$$\Rightarrow \alpha+1=0, 1-\alpha^2=0 : \alpha=-1.$$

32. (C) Given  $\sin \theta + \sin \phi = a$  .....(1)

and 
$$\cos \theta + \cos \phi = b$$
 .....(2)

$$\Rightarrow 2 \sin \frac{\theta + \phi}{2} \cos \frac{\theta - \phi}{2} = a \dots (3)$$

and 
$$2\cos\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2}=b$$
 .....(4)

Dividing (3) and (4), we get

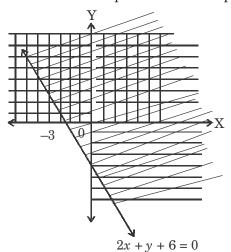
$$\tan \frac{\theta + \phi}{2} = \frac{a}{b}.$$

Hence  $\sin (\theta + \phi) = \sin \left( 2 \left( \frac{\theta + \phi}{2} \right) \right)$ 

$$=\frac{2\tan\left(\frac{\theta+\phi}{2}\right)}{1+\tan^2\left(\frac{\theta+\phi}{2}\right)}$$

$$=\frac{2^{a}/b}{1+a^{2}/b^{2}}=\frac{2ab}{a^{2}+b^{2}}$$

33. (A) In this case the graph of the given system is the set of all points in the first quadrant.



34. (D) Let z = x + yi; where  $x, y \in R$ , then

$$|3z - 1| = 3|z - 2|$$

$$\Rightarrow |3(x + yi) - 1| = 3|x + yi - 2|$$

$$\Rightarrow |3x - 1 + (3y)i| = 3|x - 2 + yi|$$

$$\Rightarrow \sqrt{(3x - 1)^2 + (3y)^2} = 3\sqrt{(x - 2)^2 + y^2}$$

Squaring, we get

$$9x^2 - 6x + 1 + 9y^2 = 9(x^2 - 4x + 4 + y^2)$$

$$\Rightarrow$$
 -6x + 1 = -36x + 36

$$\Rightarrow 30x = 35$$

$$\Rightarrow$$
 x =  $\frac{7}{6}$ , which means z is always at a

constant distance  $\frac{7}{6}$  from Y – axis.

35. (D) If the co-ordinates of A are  $(\alpha, \beta, \gamma)$ , then centroid of  $\triangle$  ABC is

$$\left(\frac{\alpha+0+0}{3},\frac{\beta+6+0}{3},\frac{\gamma+0+6}{3}\right)$$

But it is given to be (0, 0, 0), therefore,

$$\frac{\alpha}{3} = 0, \ \frac{\beta + 6}{3} = 0, \ \frac{\gamma + 6}{3} = 0$$
$$\Rightarrow (0, -6, -6).$$

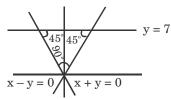
36. (B)



Since  $T \cap C \neq \emptyset$  and  $S \cap T \cap C = T \cap C$ . So, option (A) is true. Also  $T \subset S$  and  $C \subset S$ , So,  $S \cup T \cup C = S$ .

Also 
$$S \cup T = S = S \cup C$$
.

- 37. (D) Each selection of three points gives us a triangle. Hence, the required number of triangles =  ${}^{n}C_{a}$
- 38. (A) Lines x + y = 0 and x y = 0 are the bisectors of the angles between the axes and the line y = 7 is a line parallel to X axis.



- 39. (D)  $(1+x^2)^5 \times (1+x)^4 = (1+5x^2+10x^4+10x^6+5x^8+x^{10}) \times (1+4x+6x^2+4x^3+x^4)$   $= \dots + (5x^2) (4x^3) + (10x^4) (4x) + \dots$ Hence, the coefficient of  $x^5 = 20 + 40 = 60$ .
- 40. (A) For  $a \neq 0$ , the eq., can be written as

$$x^2 + y^2 + \frac{2g}{a}x + \frac{2fy}{a} + \frac{c}{a} = a$$
.

It will represent a circle if

$$\left(\frac{g}{a}\right)^2 + \left(\frac{f}{a}\right)^2 - \frac{c}{a} > 0$$
 ,i.e., if  $g^2 + f^2 - ac > 0$ .

### **Physics**

41. (A) Here,  $v = 7\sqrt{3}$  m s<sup>-1</sup>;  $r = 5\sqrt{3}$  m.

Let  $\theta$  be the inclination of the cyclist with the vertical.

Then, 
$$\tan \theta = \frac{v^2}{rg} = \frac{\left(7\sqrt{3}\right)^2}{5\sqrt{3} \times 9.8} = \sqrt{3}$$

$$\Rightarrow \theta = 60^{\circ}$$

- 42. (A) Dimensions of :
  - (i) Torque x angular displacement

$$= M^{1}L^{2}T^{-2} \times M^{0}L^{0}T^{0} = ML^{2}T^{-2}$$

 $\begin{array}{l} \mbox{(ii) Rotational inertia x (angular frequency)^2} \\ = M^{\scriptscriptstyle 1}L^{\scriptscriptstyle 2}T^{\scriptscriptstyle 0} \times (M^{\scriptscriptstyle 0}L^{\scriptscriptstyle 0}T^{\scriptscriptstyle 1})^2 \end{array}$ 

$$M^1L^2T^{-2}$$

(iii) Displacement × momentum

$$= M^{o}L^{1}T^{o} \times M^{1}L^{1}T^{-1} = ML^{2}T^{-1}$$

Dimensions of P.E. =  $mgh = ML^2T^{-2}$ 

Both (i) and (ii) have same dimensions of P.E.

43. (C) Volume of raft  $V = \frac{M}{\rho} = \left(\frac{30}{750}\right) m^3 = 0.04 m^3$ 

Maximum water the raft can displace

$$= V \times 1000 \text{ kg} = 0.04 \times 1000 = 40 \text{ kg}$$

So, we can place 40 - 30 = 10 kg mass on

- 44. (A) When the brakes are on, the wheels of the bike cannot rotate. So, it has to slide. The sliding friction is comparatively larger than the rolling friction.
- 45. (C) A couple produces a pure rotational motion as it is a combination of two equal, unlike, parallel forces acting on a body producing a turning movement.
- 46. (C) Maximum possible strain =  $\frac{0.4}{100}$

$$\therefore A = \frac{F}{Y \times strain} = \frac{2 \times 10^4 \times 100}{7 \times 10^9 \times 0.4}$$
$$= 7.15 \times 10^4 \text{ m}^2$$
$$\approx 7.1 \times 10^4 \text{ m}^2$$

- 47. (A) For a liquid solid interface, if the angle of contact is acute, then
  - (i) the liquid will wet the solid.
  - (ii) the liquid will rise in the capillary tube made of such a solid and
  - (iii) Meniscus of the liquid will be concave.
- 48. (D) Let the angle between vectors P and Q

he A

$$\begin{split} (P+Q)^2 + (P-Q)^2 + 2 \, (P+Q) \, (P-Q) \times \cos\theta \\ &= P^2 + Q^2 \end{split}$$

which gives  $\cos \theta = (P^2 + Q^2)/2(Q^2 - P^2)$ 

$$(or)\,\theta\ = cos^{\text{-}1} \Biggl[ \frac{\left(P^2 + Q^2\right)}{2\left(Q^2 - P^2\right)} \Biggr] \label{eq:theta}$$

49. (C)  $T^2 \alpha R^3$ 

$$\frac{T_{\!_{A}}^{\phantom{A}2}}{T_{\!_{B}}^{\phantom{B}2}} = \frac{R_{\!_{A}}^{\phantom{A}3}}{R_{\!_{B}}^{\phantom{B}3}}$$

or 
$$\frac{R_A^3}{R_B^3} = 64$$

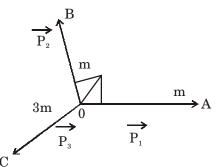
Or 
$$R_{A} = 4R_{B}$$

50. (B) Let the mass of the three fragments be m kg, m kg and 3m kg

Total mass = 1 kg

$$\Rightarrow m + m + 3m = 1kg$$
$$\Rightarrow 5m = 1 kg$$

$$\Rightarrow$$
 m =  $\frac{1}{5}$  kg



Momentum along OA is P<sub>1</sub>

= mass 
$$\times$$
 velocity =  $\frac{1}{5} \times 30$ 

$$P_2 = 6 \text{ kg m s}^{-1} = 6 \text{ kg m s}^{-1}$$

$$P_3 = 3 \text{ m} \times V$$

Total momentum after collision

= Total momentum before collision.

$$\overrightarrow{P}_1 + \overrightarrow{P}_2 + \overrightarrow{P}_3 = 0$$

$$\overrightarrow{P_3} = (\overrightarrow{P_1} + \overrightarrow{P_2}); |\overrightarrow{P_3}| = \sqrt{P_1^2 + P_2^2}$$

$$(\overrightarrow{P_1} + \overrightarrow{P_2}); |\overrightarrow{P_3}| = \sqrt{P_1^2 + P_2^2}$$

$$\sqrt{6^2 + 6^2} = \sqrt{72} = 6\sqrt{2}$$
 kg m s<sup>-1</sup>

$$3 \text{ m V} = 6\sqrt{2}$$

$$V = \frac{\sqrt[2]{6}\sqrt{2}}{\cancel{3} \times \frac{1}{5}} = 10\sqrt{2} = 14.14 \ m \ s^{-1}$$

51. (D) As oxygen and hydrogen are diatomic gases, their specific heat is the same.

$$\therefore 1 \times C \times (100 - \theta) = 1 \times C \times (\theta - 10)$$

$$2\theta = 110^{\circ} \Rightarrow \theta = 55^{\circ}$$
.

52. (C) The angle of contact depends upon the nature of liquid taken in the container, temperature and on the nature of material of the container.

53. (A) For a perfectly elastic collision e = 1 and for a perfectly inelastic collision, e = 0. Therefore 0 < e < 1. While in actual practice, collision between all real objects are neither perfectly elastic nor perfectly inelastic.

54. (B) Velocity at the top is  $\sqrt{gr}$  and that at the bottom is  $\sqrt{5}$  gr.

Difference in kinetic energy

$$= \frac{1}{2} M (5gr - gr)$$
= 2 M gr  
= 2 × 1 × 10 × 1  
= 20 J

55. (C) 
$$\frac{\Delta R}{R} = \frac{\Delta R1}{R1} + \frac{\Delta R2}{R2} + \frac{\Delta R1 + \Delta R2}{R1 + R2}$$
$$= \frac{0.2}{5} + \frac{0.1}{8} + \frac{0.2 + 0.1}{5 + 8}$$
$$= 0.04 + 0.0125 + 0.02 = 0.0725$$
Hence % of error = 7.25%

56. (B) 
$$\frac{gR^{2}}{(R+h)^{2}} = g\left(1 - \frac{h}{R}\right)$$
or 
$$\left(1 - \frac{h}{R}\right)\left(1 + \frac{h^{2}}{R^{2}} + \frac{2h}{R}\right) = 1$$
or 
$$\frac{h^{3}}{R^{3}} + \frac{h^{2}}{R^{2}} - \frac{h}{R} = 0$$
or 
$$\frac{h}{R}\left(\frac{h^{2}}{R^{2}} + \frac{h}{R} - 1\right) = 0$$
or 
$$\frac{h}{R} = \frac{-1 \pm \sqrt{1 + 4}}{2}$$

$$= \frac{\sqrt{5} - 1}{2}$$

or 
$$h = \frac{\sqrt{5R - R}}{2}$$

57. (D) 
$$V_0 = \sqrt{\frac{GM}{r}} \implies V_0 \alpha \frac{1}{\sqrt{r}}$$

So,  $\frac{V_1}{V_2} = \sqrt{\frac{r_2}{r_1}}$  (orbital velocity of a satellite is independent of mass of satellite).

$$=\sqrt{\frac{2r}{r}}=\sqrt{2}:1$$

58. (D) 
$$W = Fd = 1 \text{ gf} \times 1 \text{ cm}$$
  

$$= \frac{1}{1000} \text{kgf} \times \frac{1}{100} \text{ m}$$

$$= \frac{1}{1000} \times 10 \text{ N} \times \frac{1}{100} \text{ m} = 10^{-4} \text{ J}$$

$$= 0.0001 \text{ J}.$$

$$\begin{split} 59. \quad & (D) \quad \text{ Total kinetic energy } (U_{_k}) = \frac{1}{2} I \omega^2 + \frac{1}{2} M v^2 \\ & = \frac{1}{2} \times \frac{2}{5} M R^2 \omega^2 + \frac{1}{2} M v^2 \\ & = \frac{1}{5} M v^2 + \frac{1}{2} M v^2 \\ & = \frac{7}{10} M v^2 = \frac{7}{10} \times \frac{1}{2} \times \left(0.2\right)^2 \end{split}$$

- 60. (B) Impulse = change in momentum = mv - mu = m(v - u)= 0.1[30 - (-20)] = 5 N s
- $Dn = u + \frac{a}{2}(2n 1)$ (D) 61.  $2 = 0 + \frac{a}{2}(2 \times 1 - 1)$  $\therefore$  a = 4 cm s<sup>-2</sup>. Again from  $v = u + at = 0 + (4 \times 4) = 16 \text{ cm s}^{-1}$ .
- (C) 1 Poiseuille =  $N s m^{-2}$ 62.  $= 10^5 \, \text{dyne s} \times (100 \, \text{cm}^{-2})$  $= 10 \text{ dyne cm}^{-2} \text{ s} = 10 \text{ poise}$

1 poise = 
$$\frac{1}{10}$$
 N m<sup>2</sup> s  
= 0.1 N m<sup>-2</sup> s

- 63. (C) Damping is the periodic decrease in the amplitude, thereby frequency and time period also changes. But phase of the S.H.M. wave does not change.
- (B) The moment of inertia of a spherical shell 64. with axis along the diameter is  $\frac{2}{9}$  MR<sup>2</sup>
- The centre of mass of system of particles 65. (D) depends upon the:
  - (i) masses of the particles
  - (ii) position of the particles
  - (iii) relative separation between the particles.

#### Chemistry

66. (C) Dipole moment is defined as the product of magnitude of charge and distance of separation between the charges.

> Alkanes are non-polar in nature. Nonpolar molecules have zero dipole moment. Hence, pentane's dipole moment is zero.

 $1 \text{ m}^3 = 1000 \text{ litres}$ 67. (B) At S.T.P.,

> 1 mole of any gas occupies 22.4 l ? mole of any gas occupies 1000 l

Number of moles =  $\frac{1000}{22.4}$  = 44.6 moles.

68. (A) If S is the solubility product of AX,

Then,  $K_{SD} = [A^{+2}][X^{-}]^{2} = S \times (2S^{2}) = 4S^{3}$  $=4\times(1.0\times10^{-5}\ mol\ L^{-1})^3$  $= 4 \times 10^{-15} \text{ mol}^3 \text{ L}^{-3}$ 

- 69. (C) Half filled or completely filled orbitals are found to be more stable. Therefore, the ionisation enthalpy is higher when an electron is to be removed from a fully filled or half filled orbitals.
- 70. (D) Hydrogen is given off when palladium is heated in vacuum. This phenomenon is known as occlusion.
- 71. (C) When the boiling points of the two, four or more miscible liquids in any mixture do not differ very much and hence boil within a narrow range of temperature, then a fractionating column helps in condensing these liquids at different levels of the column. Thus, separating the liquid components by this process is known as fractional distillation.
- 72. Lithium is the only alkali metal which (A) reacts with nitrogen to give Lithium nitride, Li<sub>2</sub>N (ruby red solid).

$$6 \text{ Li} + \text{N}_2 \rightarrow 2 \text{Li}_3 \text{N}$$

- The state of hybridisation of nitrogen in 73. (B) NH<sub>4</sub><sup>+</sup> is sp<sup>3</sup> and its molecular shape is tetrahedral.
- 74. (B) BeH<sub>a</sub> cannot be prepared by direct action of H<sub>o</sub> on Be. BeH<sub>o</sub> is prepared by the action of LiAlH<sub>4</sub> on BeC $l_2$

 $2 \operatorname{BeC} l_2 + \operatorname{LiA} l_4 \rightarrow 2 \operatorname{BeH}_2 + \operatorname{LiC} l + \operatorname{A} l \operatorname{C} l_3$ .

(C) The reaction is 75.

$$\begin{array}{ccccc} N_{_2} & + & 3H_{_2} & \rightarrow & 2NH_{_3} \\ 1 \ mol & & 3 \ mol & & 2 \ mol \\ (2\times 14)g & & & 2\times (1\times 14 + 3\times 1)g \\ = 28 \ g & & = 34 \ g \end{array}$$

Thus, to produce 34 g of ammonia (NH<sub>2</sub>), 28g of nitrogen is needed.

76. (B) Na<sub>2</sub>ZnO<sub>3</sub> and H<sub>3</sub> are produced on dissolving metallic zinc in excess of NaOH.

 $2 \text{ NaOH} + \text{Zn} \rightarrow \text{Na}_{2} \text{ZnO}_{2} + \text{H}_{2}$ 

In the shell, n = 4 a maximum of 77.  $2n^2 = 2 \times 4^2 = 32$  electrons can be

accommodated. In this shell  $m_s = -\frac{1}{2}$  will be present for half of these 32 electrons i.e., 16 electrons.

- 78. (A) In potassium dichromate titrations the most commonly employed indicators are diphenylamine or N phenylanthranilic acid.
- 79. (C) The maximum limit of nitrate in drinking water is 50 ppm. Excess nitrate in drinking water can cause a disease known as methemoglobinemia or 'blue baby syndrome'.
- 80. (A) According to de Broglie equation  $\lambda = \frac{h}{mv}$   $= 6.63 \times 10^{-27} \text{ erg sec}$   $= 200 \text{ g} \times 3 \times 10^{3} \text{ cm s}^{-1}$   $= 1.1 \times 10^{-32} \text{ cm}$
- 81. (C)  $\begin{aligned} P_1 V_1 &= P_2 \ V_2 &= P_3 \ V_3 &= P_4 \ V_4 \\ P_2 &= 125 \ Torr \ ; \ P_3 &= 200 \ Torr \ ; \\ V_2 &= 64 \ ml \ ; \qquad V_3 &= ? \\ P_2 V_2 &= P_3 V_3 \\ or \ V_3 &= \frac{P_2 V_2}{P_2} = \frac{125 \times 64}{200} = 40 \ ml \end{aligned}$
- 82. (D) Quartz, cristobalite and tridymite are some of the crystalline forms of silica and they are interconvertible at suitable temperature.
- 83. (A) The first law of thermodynamics can be mathematically represented as:

$$\Delta U = q + w$$

where  $\Delta \, U$  - change in internal energy,

q - heat given to the system

w - work done on the system.

84. (D) Total no. of moles of  $CO_2 = \frac{\text{wt.in g}}{\text{mol. wt}}$ 

$$=\frac{0.2}{44}=0.00454$$

No. of moles removed

$$= \frac{10^{21}}{6.022 \times 10^{23}} = 0.00166$$

No. of moles of  $CO_2$  left = 0.00454 - 0.00166= 0.00288.

85. (C) Graphite is thermodynamically, the most stable allotrope of carbon and therefore,  $\Delta_i H^o$  of graphite is taken as zero.  $\Delta_i H^o$  value of diamond and fullerene are 1.90 and 38.1 kJ mol<sup>-1</sup> respectively.

86. (B) 
$$\begin{array}{c} & & & & & & \\ & & & & & \\ 2H_2^{-2}S(g) + SO_2(g) \rightarrow 3S(s) + 2H_2O(g) \\ & & & & \\ L & & & & \\ L & & & & \\ \end{array}$$

In 
$$\mathbf{H_2S}: \mathbf{H_2S}^{-2} \to \mathbf{S}^0$$

The oxidation number of S (in  $H_2S$ ) is -2 and it changes to 0 in the reaction. Thus,  $H_9S$  gets oxidised to S.

In 
$$SO_2: SO_2^{+4} \rightarrow \overset{0}{S}$$

The oxidation number of  $S(in SO_2)$  is + 4 and it changes to 0 in the reaction. Thus,  $SO_2$  gets reduced to S.

87. (B) 780 mm of Hg = 
$$\frac{780}{760}$$
 atm  
w = 22 g; M = 44 g mol<sup>1</sup>

$$T = 27^{\circ}C = 27 + 273 = 300 \text{ K}$$

Volume occupied

$$= \frac{w}{M}, \frac{RT}{P} = \frac{22}{44} \times \frac{0.0821 \times 300 \times 760}{780}$$
$$= 12 L$$

- 88. (A) When KCl is dissolved in water, heat is absorbed. Thus, the enthalpy of solution of KCl is positive. For a dilution of 200, the enthalpy of KCl is + 18.6 kJ mol<sup>-1</sup>
- 89. (A) Element Position

  Cesium 6<sup>th</sup> period, I A group

  Magnesium 3<sup>rd</sup> period, II A group

  Barium 6<sup>th</sup> period, II A group

  Lead 6<sup>th</sup> period, IV A group

For an element to have largest atomic size, it should be in greatest period and least group. So, in the given elements cesium has the largest atomic radius.

90. (C) For the reaction

 $Cu(s) + 2 Ag^{+}(aq) \square Cu^{+2}(aq) + 2Ag(s)$ the reaction quotient (Q) is,

$$Q = \frac{\left[Cu^{^{+2}}(aq)\right]}{\left[Ag + (aq)\right]^2} = \frac{1.8 \times 10^{^{-2}} mol \, L^{^{-1}}}{\left(3 \times 10^{^{-9}} mol \, L^{^{-1}}\right)^2}$$

under the given conditions, value of Q = K i.e., reaction quotient = equilibrium constant. Therefore, the system is at equilibrium.