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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION - 2015 (UPDATED)

Paper Code: UN412

Solutions for Class : 11 (PCM)

Mathematics

1. **(C)** We have, $\sec\theta + \tan^3\theta \ \mathrm{cosec}\,\theta$

$= \sec\theta\left(1+\tan^3\theta\dfrac{\mathrm{cosec}\,\theta}{\sec\theta}\right)$

$= \sec\theta\,(1+\tan^3\theta\ .\cot\theta) = \sec\theta\,(1+\tan^2\theta)$

$= \sec\theta\sec^2\theta = \sec^3\theta$

$= (\sec^2\theta)^{3/2} = (1+\tan^2\theta)^{3/2}$

$(1+1-e^2)^{3/2} = (2-e^2)^{3/2}$ [Since, $\tan^2\theta = 1-e^2$]

2. **(C)** Since a, b, c are in H.P.,

$\therefore \dfrac{1}{a}, \dfrac{1}{b}, \dfrac{1}{c}$ are in A.P.

$\Rightarrow \dfrac{1}{a}+\dfrac{1}{c}=\dfrac{2}{b}$

The given line is $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{1}{c}=0$

$\Rightarrow \dfrac{x}{a}+\dfrac{y}{b}+\left(\dfrac{2}{b}-\dfrac{1}{a}\right)=0$

$\Rightarrow \dfrac{1}{a}(x-1)+\dfrac{1}{b}(y+2)=0$

$\Rightarrow$ The given line passes through the point of intersection of $x-1=0$ and $y+2=0$ i.e., (1, –2) which is a fixed point.

3. **(D)** We know that

$n(A\cup B) = n(A)+n(B)-n(A\cap B)$

Also $n(A\cap B) \le \min\{n(A), n(B)\}$
$= \min\{3, 6\} = 3$

$\Rightarrow n(A\cap B)\le 3 \Rightarrow -n(A\cap B)\ge -3$

$\Rightarrow n(A\cup B) = n(A)+n(B)-n(A\cap B)$
$\ge n(A)+n(B)-3$

$\Rightarrow n(A\cup B)\ge 6$

$\Rightarrow$ Minimum number of elements in $A\cup B$ is 6.

4. **(C)** We have, $7^{103} = 7(49)^{51} = 7(50-1)^{51}$

$= 7(50^{51} - {}^{51}C_1\, 50^{50} + {}^{51}C_2\, 50^{49} - \ldots -1)$

$= 7(50^{51} - {}^{51}C_1\, 50^{50} + {}^{51}C_2\, 50^{49} - \ldots) - 25 + 18$

$= k + 18$ (say), since k is divisible by 25,

$\therefore$ Remainder is 18.

5. **(A)** Let $f(x) = x$ and $g(x) = x^3$. Then $f(x)\,g(x) = x^4$, even; option (A) is false.

Let $f(x)$ and $g(x)$ are two even functions and $F(x) = f(x) + g(x)$

$F(-x) = f(-x) + g(-x) = f(x) + g(x) = F(x)$

$\Rightarrow F(x)$ is even, option (B) is true.

Let $f(x)$ be an even function and $g(x)$ be an odd function and $F(x) = f(x)\,g(x)$

$F(-x) = f(-x)\,g(-x) = f(x)\,(-g(x))$
$\Rightarrow -F(x)$ is odd, option (C) is true

(D) $g(-x) = \dfrac{f(-x)+f(x)}{2} = \dfrac{f(x)+f(-x)}{2}$

$= g(x)$ is even, option (D) is ture.

6. **(C)** T_{20} = (20[th] term of 2, 4, 6, 8, ...) × (20[th] term of 4, 6, 8, ...)

$= [2+(19)(2)][4+(19)2] = (40)(42) = 1680$

7. **(B)** Given statement is not true for n = 1 as

$1 < \left(\dfrac{1+1}{2}\right)^1$ i.e., $1 < 1$ is not true.

For n = 2, the statement becomes

$1\times 2 < \left(\dfrac{2+1}{2}\right)^2$ i.e., $2 < \dfrac{9}{4}$, which is true.

8. **(C)** Since each question can be deal with in 3 ways

a) by selecting it

b) by selecting its alternative, or

c) by rejecting it.

Thus, the total number of ways of dealing with 10 given questions is 3^{10} including a way in which we reject all the questions.

Hence the number of all possible selections of one or more questions is $3^{10} - 1$.

9. **(A)** We have $\tan\alpha + \tan\beta + \tan\gamma = 0$

$\Rightarrow \tan\gamma = -h$, and since $\tan\gamma$ satisfies the given equation.

$\Rightarrow$ a $\tan^3\gamma + (2a - x)\tan\gamma + y = 0$

$\Rightarrow$ $a(-h)^3 + (2a - x)(-h) + y = 0$

$\therefore$ $ah^3 + (2a - x)h = y$

10. **(A)** Let Nisha secure x marks in the fifth test, then her average score

$$= \frac{87+92+94+95+x}{5} = \frac{368+x}{5}$$

According to given condition, we must have

$$\Rightarrow \frac{368+x}{5} \geq 90$$

$\Rightarrow$ $368 + x \geq 90 \times 5$

$\Rightarrow$ $x \geq 450 - 368$

$\Rightarrow$ $x \geq 82$

11. **(B)** $y = e^x$ and $y = x$

$\Rightarrow e^x = x \Rightarrow$ no $x \in R$

Hence, $A \cap B = \phi$.

12. **(C)** Solving the given equations in pairs, we get the coordinates of the vertices as

A = (–3, 3), B = (1, 1), C = (1, –1),

D = (–2, –2)

$$m_1 = \text{slope of AC} = \frac{-1-3}{1+3} = -\frac{4}{4} = -1$$

$$m_2 = \text{slope of BD} = \frac{-2-1}{-2-1} = -\frac{-3}{-3} = 1$$

$m_1 m_2 = (-1)(1) = -1.$

So, diagonals AC and BD are perpendicular.

$\therefore$ Angle between AC and BD is $\frac{\pi}{2}$.

13. **(A)** We have, $\cos(\alpha + \beta)\sin(\gamma + \beta)\sin(\gamma - \delta)$

$$\Rightarrow \frac{\sin(\gamma+\delta)}{\sin(\gamma-\delta)} = \frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)}$$

$$\Rightarrow \frac{\sin(\gamma+\delta)+\sin(\gamma-\delta)}{\sin(\gamma+\delta)-\sin(\gamma+\delta)}$$

$$= \frac{\cos(\alpha-\beta)+\cos(\alpha+\beta)}{\cos(\alpha-\beta)-\cos(\alpha+\beta)}$$

(applying components and dividendo)

$$\Rightarrow \frac{2\sin\gamma\cos\delta}{2\cos\gamma\sin\delta} = \frac{2\cos\alpha\cos\beta}{2\sin\alpha\sin\beta}$$

$$\Rightarrow \cot\delta = \cot\alpha\cot\beta\cot\gamma$$

14. **(D)** In case of R_1, $f(x) = 6x + 7$

Clearly, every element of A has a unique image.

Hence, R_1 represents a function.

Similarly, R_2 and R_3 also represent functions.

In case of R_4, $f(x) = \pm 4x$

$\therefore$ Every element of A has two unequal images.

e.g., $f(1) = \pm 4$, $f(2) = \pm 8$ etc.

$\therefore$ R_4 is not a function.

15. **(D)** $(1 + 4x)^{-5/4}.(1 + 2x)^{1/2}$

$$= \left(1 - \frac{5}{4}.4x\right)\left(1 + \frac{1}{2}(2x)\right)$$

$= (1 - 5x)(1 + x)$

$\therefore$ Coefficient of $x = 1 - 5 = -4$

16. **(A)** Number of triangles formed with 12 points $= {}^{12}C_3 = 220$

Number of triangles formed with 5 points $= {}^{5}C_3 = 10$

(Since, 5 points are collinear, so they cannot form a triangle)

Hence, total number of triangles $= 220 - 10 = 210$

17. **(B)** $n(A) = n(A - B) + n(A \cap B)$

$= 30 + x + 2x$

$= 30 + 3x$

$n(B) = n(B - A) + n(A \cap B) = x + 2x = 3x$

$n(A) = 2n(B)$

$\Rightarrow$ $30 + 3x = 2.3x$

$\Rightarrow$ $30 = 6x - 3x$

$\Rightarrow$ $30 = 3x$

$\Rightarrow$ $x = 10$

18. **(A)** We have,

$(1 + x)^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + ... + C_n x^n$ .... (1)

Differentiating w.r.t. x, we get

$n(1 + x)^{n-1} = C_1 + 2C_2 x + 3C_3 x^2 + 4C_4 x^3 + ... + nC_n x^{n-1}$ .... (2)

Putting $x = 1$ in (1) and (2), we get

$2^n = C_0 + C_1 + C_2 + C_3 + C_4 + ... C_n$ ..... (3)

and $n\, 2^{n-1} = C_1 + 2C_2 + 3C_3 + 4C_4 + ... + nC_n$ .... (4)

Adding (3) and (4), we get

$(n + 2)\, 2^{n-1} = C_0 + 2C_1 + 3C_2 + ... + (n + 1)\, C_n$.

19. **(C)** Since, a, b and c are in A.P.

$\Rightarrow$ 2b = a + c

Also, p is A.M. between a and b .... (1)

$\Rightarrow$ a, p, b are in A.P.

$\Rightarrow$ 2p = a + b .... (2)

Also, p' is G.M. between a and b

$\Rightarrow$ a, p', b are in G.P.

$\Rightarrow p' = \sqrt{ab}$ .... (3)

Similarly,

2q = b + c .... (4)

and $q' = \sqrt{bc}$

$$p^2 = \frac{(a + b)^2}{4} = \frac{1}{4}(a^2 + b^2 + 2ab); \quad p'^2 = ab$$

$$q^2 = \frac{(b + c)^2}{4} = \frac{1}{4}(b^2 + c^2 + 2bc); \quad q'^2 = bc$$

$$p^2 - q^2 = \frac{1}{4}(a^2 - c^2 - 2b(a - c)) = \frac{1}{4}(a^2 - c^2 + (a + c)(a - c))$$

[$\because$ a, b, c are in A.P, 2b = a + c]

$$= \frac{1}{2}(a^2 - c^2)$$

$$p'^2 - q'^2 = ab - bc = b(a - c)$$

$$\Rightarrow \frac{(a + c)}{2}(a - c) = \frac{1}{2}(a^2 - c^2)$$

$$\therefore p^2 - q^2 = p'^2 - q'^2$$

20. **(D)** The given expression

$$= \frac{\cos 6\theta + \cos 4\theta + 5\cos 4\theta + 5\cos 2\theta + 10\cos 2\theta + 10}{\cos 5\theta + 5\cos 3\theta + 10\cos\theta}$$

$$= \frac{2\cos 5\theta\cos\theta + 5.2\cos 3\theta\cos\theta + 10.2\cos^2\theta}{\cos 5\theta + 5\cos 3\theta + 10\cos\theta}$$

$$= \frac{2\cos\theta(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)}{(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)} = 2\cos\theta$$

21. **(D)** Since α is a root of the equation.

$a^2x^2 + bx + c = 0$

$\therefore a^2\alpha^2 + b\alpha + c = 0$ .... (1)

Since β is a root of the equation

$a^2x^2 - bx - c = 0$

$\therefore a^2\beta^2 - b\beta - c = 0$ .....(2)

Now let $f(x) = a^2x^2 + 2bx + 2c$

$\therefore f(\alpha) = a^2\alpha^2 + 2(b\alpha + c)$

$= a^2\alpha^2 + 2(-a^2\alpha^2)$ [from (1)]

$= -a^2\alpha^2 < 0$

$f(\beta) = -a^2\beta^2 + 2(b\beta + c)$

$= a^2\beta^2 + 2(a^2\beta^2)$ [from (2)]

$= 3a^2\beta^2 > 0$

Since $f(\alpha)$ and $f(\beta)$ are of opposite sign and γ is a root of the equation $f(x) = 0$,

$\therefore$ γ must lie between α and β.

Thus $\alpha < \gamma < \beta$.

22. **(B)** The given digits are 0, 1, 2, 3, 4 which are five in number.

Since we are to form the numbers that are greater than 20000 and as no digits is to be repeated, every such numbers contains five digits and it must have 2, 3 or 4 at extreme left.

Thus, the extreme left place can be filled up in 3 ways.

Now the remaining four places can be filled up with the remaining four digits in ${}^4P_4$ ways. By using the principle of association, the required number of numbers formed

$= 3 \times {}^4P_4 = 3 \times \underline{|4} = 3 \times 4 \times 3 \times 2 \times 1 = 72$

23. **(C)** Statement P, 'Cardinality of a singleton set is 1', is true.

Statement Q, 'Cardinality of a null set is 0' is true.

Hence (C) is the answer.

24. **(C)** Let h be the height of the triangle.

Since, the area of the triangle is a^2

$\therefore \quad \frac{1}{2} \times a \times h = a^2 \Rightarrow h = 2a$

Since the base lies along the line x = a, the vertex lies on the line parallel to the base at a distance 2a from it. So, the required lines are

x = a $\pm$ 2a i.e., x = –a or x = 3a

25. **(A)** The equation reduces to

$$E = (-1)(-2)(-2)^2 \ldots. (-2)^{15}$$

$$(x-1)\left(x-\frac{1}{2}\right)\left(x-\frac{1}{2^2}\right)\ldots\left(x-\frac{1}{2^{15}}\right)$$

$$\Rightarrow E = 2^{1+2+3+\ldots 15}$$

$$\left(x-\frac{1}{2}\right)\left(x-\frac{1}{2^2}\right)\ldots\left(x-\frac{1}{2^{15}}\right)$$

$\therefore$ Coefficient of x^{15} in E is

$$2^{1+2+3\ldots 15}.\left(-1-\frac{1}{2}-\frac{1}{2^2}-\ldots-\frac{1}{2^{15}}\right)$$

$$= 2^{120}\left\{(-1)\frac{1-\left(\frac{1}{2}\right)^{15}}{1-\left(\frac{1}{2}\right)}\right\}$$

$$= -2^{120}\left\{\left(\frac{2^{16}-1}{2-1}\right)\frac{2}{2^{16}}\right\}$$

$$= -2^{105}(2^{16}-1) = 2^{105} - 2^{121}$$

26. **(C)** We have, $(1 + i)^{2n} = (1 - i)^{2n}$

$$\Rightarrow \left(\frac{1+i}{1-i}\right)^{2n}$$

$$\Rightarrow \left[\frac{(1+i)^2}{2}\right]^{2n} = 1$$

$$\Rightarrow \left[\frac{1-1+2i^2}{2}\right]^{2n} = 1$$

$\Rightarrow i^{2n} = 1$

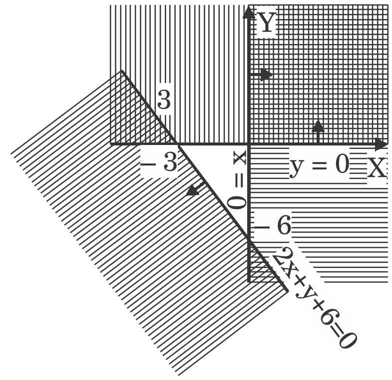
$\Rightarrow$ n = 2 is the required smallest positive integer.

27. **(C)** We have, $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$

$= \sin\theta\,(2\sin 2\theta \cos\theta)$

$= \sin^2 2\theta \geq 0$, for all real θ

28. **(C)**



29. **(C)** Let the last three numbers in A.P. be b, b + 6, b + 12 and the first number be a.

Hence the four numbers are a, b, b + 6, b + 12

Given, a = b + 12 .... (i)

and a, b, b + 6 are in G.P. i.e., $b^2 = a(b + 6)$

$\Rightarrow b^2 = (b + 12)(b + 6)$ [Since a = b + 12]

or 18b = –72

$\therefore$ b = – 4

From (i), a = –4 + 12 = 8

Hence the four numbers are 8, –4, 2 and 8.

30. **(A)** Since, $(a + ib)^5 = \alpha + i\beta$ (given)

$$\Rightarrow \left\{\iota\left(\frac{a}{\iota}+b\right)\right\}^5 = \alpha + \iota\beta$$

$$\Rightarrow \iota^5 (b - \iota a)^5 = \alpha + \iota\beta \;\; (\text{since } \iota = \frac{1}{\iota})$$

$$\Rightarrow \iota = (b - \iota a)^5 = \alpha + \iota\beta$$

$$\Rightarrow (b - \iota a)^5 = \frac{\alpha + \iota\beta}{\iota} = \iota(\alpha + \iota\beta)$$

$$\Rightarrow (b - \iota a)^5 = \iota\alpha + \iota\beta$$

Taking conjugate through out, we have

$$\left(\overline{b - \iota a}\right)^5 = \left(\overline{\beta + \iota\alpha}\right)$$

$$\therefore (b + \iota a)^5 = \beta + \iota\alpha$$

31. **(C)** Let n be the number of newspapers which are read.

$\Rightarrow 60n = (300)(5)$

$\therefore$ n = 25

32. **(A)** Let orthocentre be (α, β).

Then, $3 = \dfrac{2(6)+1\cdot\alpha}{2+1}$ and $3 = \dfrac{2(2)+1\cdot\beta}{2+1}$

$\Rightarrow \ \alpha = -3$ and $\beta = 5$

$\therefore$ Orthocentre is $(-3, 5)$.

33. **(B)** We have,

$$\frac{\cos A}{\sin B \sin C} + \frac{\cos B}{\sin A \sin C} + \frac{\cos C}{\sin A \sin B}$$

$$= \frac{\sin A \cos A + \sin B \cos B + \sin C \cos C}{\sin A \sin B \sin C}$$

$$= \frac{\sin 2A + \sin 2B + \sin 2C}{2 \sin A \sin B \sin C}$$

$$= \frac{4 \sin A \sin B \sin C}{2 \sin A \sin B \sin C} = 2$$

34. **(C)** $A_1 = \{2\}$, $A_2 = \{2, 4\}$, $A_3 = \{2, 4, 6\}$ and so on.

Therefore $\bigcap_{n=1}^{2007} A_n = \{2\}$

35. **(A)** $({}^7C_0 + {}^7C_1) + ({}^7C_1 + {}^7C_2) + \ldots + ({}^7C_6 + {}^7C_7)$

$= {}^8C_1 + {}^8C_2 + \ldots. + {}^8C_7$

$= ({}^8C_0 + {}^8C_1 + {}^8C_2 + \ldots + {}^8C_7 + {}^8C_8) - ({}^8C_0 + {}^8C_8)$

$= 2^8 - 1\,(1 + 1) = 2^8 - 2$

36. **(A)** Let $x^{18} = y^{21} = z^{28} = k$. Then,

$18 \log x = 21 \log y = 28 \log z = \log k$

$\Rightarrow \log_y x = \dfrac{21}{18}$, $\log_z y = \dfrac{28}{21}$, $\log_x z = \dfrac{18}{28}$

$\Rightarrow 3 \log_y x = \dfrac{7}{2}$, $3 \log_z y = 4$, $7 \log_x z = \dfrac{9}{2}$

$\Rightarrow 3, 3 \log_y z, 3 \log_z y = 4, 7 \log_x z$ are in A.P.

37. **(C)** We have $(x, y) \in R$ iff $x + y < 6$

Given the value $x = 1$, we get possible values of $y = 1, 2, 3, 4$.

Thus 1R1, 1R2, 1R3, 1R4. Similarly we may find other values. The set of such ordered pairs is R

$= \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1) (3, 2), (4, 1)\}$

$n(R) = 10$

38. **(C)** $\alpha + \beta + \gamma = p$, $\alpha\beta + \beta\gamma + \gamma\alpha = q$, $\alpha\beta\gamma = r$

$\therefore \ (\beta + \gamma - \alpha)(\gamma + \alpha - \beta)(\alpha + \beta - \gamma)$

$= (p - 2\alpha)(p - 2\beta)(p - 2\gamma)$

$= p^3 - 2(\alpha + \beta + \gamma)p^2 + 4(\alpha\beta + \beta\gamma + \gamma\alpha)$
$p - 8\alpha\beta\gamma$

$= p^3 + 4pq - 8r$

39. **(C)** We have, $\sin(A + B + C) = 1$,

$\tan(A - B) = \dfrac{1}{\sqrt{3}}$ and $\sec(A + C) = 2$

$\Rightarrow A + B + C = 90^o, A - B = 30^o$ and $A + C = 60^o$

$\Rightarrow B = 30^o, A = 60^o$ and $C = 0^o$

40. **(D)** We have $\dfrac{1}{n(n+1)} = \dfrac{1}{n} - \dfrac{1}{n+1}$

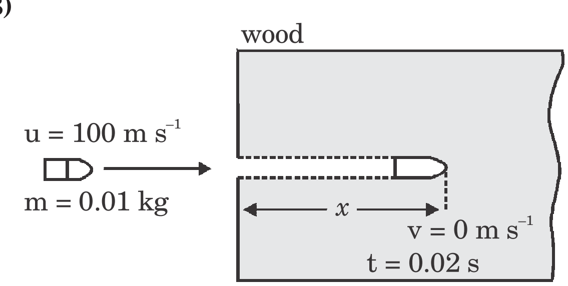
$$\therefore \ \frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \ldots\ldots. + \frac{1}{2007 \times 2008}$$

$$= 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \ldots.. + \frac{1}{2007} - \frac{1}{2008}$$

$$= 1 - \frac{1}{2008} = \frac{2007}{2008}$$

Physics

41. **(B)**



$u = 100$ m s^{-1} $\quad$ $v = 0$ m s^{-1}

$a = ?$ $\quad$ $t = 0.02$ s

$v = u + at$

$0 = 100 + a \times 0.02$

$0.02\, a = -100$

$a = -100 / 0.02$

$a = -5000$ m s^{-2}

A bullet penetrating a wooden block

$F = ?$ $\quad$ $m = 0.01$ kg $\quad$ $a = -5000$ m s^{-2}

$F = ma$

$F = 0.01 \times (-5000)$

The average retarding force exerted by the wood is – 50 N.

42. **(A)** $T_1 = 27 + 273 = 300$ K

$T_2 = -13 + 273 = 260$ K

Coefficient of performance =

$$\frac{T_2}{T_1 - T_2} = \frac{260}{300-260} = \frac{260}{40} = 6.5$$

43. **(B)** On the surface of the earth

$$g = \frac{GM}{R^2}\text{ ; Weight mg = 99 N}$$

At a height h above the earth

$$g' = \frac{GM}{(R+h)^2} \text{ , where } h = \frac{R}{2}$$

$$\frac{g'}{g} = \frac{R^2}{(R+h)^2} = \frac{R^2}{\left(R+\frac{R}{2}\right)^2} = \frac{R^2}{\frac{9}{4}R^2}$$

$$g' = \frac{4g}{9}$$

$$\text{Weight} = mg' = m\times\frac{4g}{9} = mg\times\frac{4}{9}$$

$$\text{Here mg = 99 N} = 99\times\frac{4}{9} = 44\text{ N}$$

44. **(B)** Mean diameter =

$$\frac{0.39+0.38+0.39+0.41+0.38+0.37+0.40+0.39}{8}$$

$\bar{d} = 0.38875$ mm

= 0.39 mm (rounded off to two significant figures)

Absolute error in the first reading =

0.39 – 0.39 = 0.00 mm

Similarly finding the absolute error in the other seven readings and taking the mean;

Mean absolute error = $\overline{\Delta d}$ =

$$\frac{0.00+0.01+0.00+0.02+0.01+0.02+0.01+0.00}{8}$$

= 0.00875 = 0.01 mm

$$\text{Relative error} = \frac{\overline{\Delta d}}{d} = \frac{0.01}{0.39} = 0.0256$$

45. **(B)** Energy stored per unit volume

$$U = \frac{1}{2}\text{stress}\times\text{strain}$$

$$= \frac{1}{2}\text{stress}\times\frac{\text{strain}}{Y}$$

$$= \frac{1}{2}S\times\frac{S}{Y} = \frac{1}{2}\frac{S^2}{Y}$$

46. **(B)** A raw egg behaves like a spherical shell and a half boiled egg behaves like a solid sphere.

$$\therefore \frac{I_r}{I_s} = \frac{2/3\ MR^2}{2/5\ MR^2} = \frac{5}{3} > 1$$

47. **(B)** $x_1 = 30\times\frac{1}{2}$ km, $x_2 = 50\times\frac{1}{2}$ km

$x = x_1 + x_2 = 40$ km

$$v = \frac{x}{t} = \frac{40\text{ km}}{1\text{ h}} = 40\text{ km h}^{-1}$$

48. **(C)** Mass m = 1500 kg, h = 50 m

t = 2 × 60 = 120 s

$$\text{Power} = \frac{W}{t} = \frac{mgh}{t} = \frac{1500\times9.8\times50}{2\times60}$$

= 6125 W

Power of the engine operating the lift

$$= \frac{100\times6125}{75} = 8166.67\text{ W}$$

49. **(C)** On the surface of the earth, the atmospheric pressure is quite high. The astronauts will feel great discomfort if they move on the earth immediately after coming back from the moon. To avoid it, they need to get used to normal air pressure gradually. That is why, they have to live for some days in a caravan with the air pressure lower than outside.

50. **(B)** Here $\frac{\Delta t}{t} = \frac{1}{10^{11}}$

$$\Delta t = \frac{1}{10^{11}}\times t = \frac{1}{10^{11}}\times10^{11} = 1$$

or $\Delta t = 1$ s

Hence, maximum difference in time between two such clocks = 2 s

One may be 1 s faster and the other may be 1 s slower.

51. **(D)** $\frac{4S}{r_1} - \frac{4S}{r_2} = \frac{4S}{r}$

or $\frac{1}{r} = \frac{1}{r_1} - \frac{1}{r_2} = \frac{1}{4} - \frac{1}{5} = \frac{1}{20}$ or r = 20 cm

52. **(B)** As no external torque acts on the system, the angular momentum L is conserved. As the beads slide down, the moment of inertia of the system shall change. As L = I ω = constant and I changes, therefore, ω would change. As no work is being done, total energy cannot change.

53. **(C)** $a = \frac{dm}{dt} = -10\text{ kg s}^{-1}$, $V_r = 5$ km s^{-1} (5000)

M = 1500 kg, t = 50 s

$$\frac{10\times5000}{1500-10\times50} = 50\text{ m s}^{-2}$$

54. **(A)** Here, m = 0.5 kg, v = 1.5 m s^{-1}

K = 50 N m^{-1}

x = ?

$$\frac{1}{2}Kx^2 = \frac{1}{2}mv^2$$

$$x = v\sqrt{\frac{m}{K}} = 1.5\sqrt{\frac{0.5}{50}} = 0.15\text{ m}$$

55. **(A)** Relative velocity of overtaking =

$40\ m\ s^{-1} - 30\ m\ s^{-1} = 10\ m\ s^{-1}$.

Total distance covered with this relative velocity during overtaking will be =

100 m + 200 m = 300 m.

Time taken t = 300 m/10 $m\ s^{-1}$ = 30 s

56. **(D)** Specific heat (in cal/g/°C) : Copper (0.09) Aluminium (0.21), Iron (0.11), Lead (0.03).

57. **(B)** Error in time period is

$\Delta T = (0.1/20)s = 0.005$ s.

Also T = (20 s / 20) = 1s

Hence $\frac{\Delta T}{T} = \frac{0.005}{1} = 0.005 \times 100\% = 0.5\%$

58. **(B)** $C_m = \frac{3}{2}R,\ C_{di} = \frac{5}{2}R$.

If change in temperature is ΔT, then

$1 \times \frac{3}{2} R\,\Delta T + 1 \times \frac{5}{2} R\,\Delta T = 2 \times C_v \times \Delta T$

This gives $C_v = 2R$

59. **(A)** $g = \frac{GM}{R^2} = \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \rho = \frac{4}{3}\pi G R \rho$,

i.e. $g \propto \rho$

$\therefore \frac{g'}{g} = \frac{2\rho}{\rho} = 2$

or $g' = 2g = 2 \times 9.8 = 19.6\ m\ s^{-2}$

60. **(C)** $\upsilon = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} = \frac{40 \times 4 + 60 \times 2}{40 + 60}$

$= 2.8\ m\ s^{-1}$

Loss in K.E. =

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}(m_1 + m_2)\upsilon^2$

$\frac{1}{2}\left[40 \times 16 + 60 \times 4 - 100 \times 2.8^2\right] = 48\ J$

61. **(A)** Temperature of source =

$T_1 = 100 + 273 = 373$ K

Temperature of sink =

$T_2 = 30 + 273 = 303$ K

Efficiency = $\eta = \frac{T_1 - T_2}{T_1}$

$= \frac{373 - 303}{373} = 0.188 = 18.8\%$

62. **(A)** The vertical displacement of the two stones and their initial velocities are the same. So, the final velocity acquired by them should also be equal.

For the stone thrown vertically upwards

$a = -g, \quad s = -h$

$v_1^2 = u^2 + 2as$

$v_1^2 = u^2 + 2(-g)(-h) = u^2 + 2gh$

For the stone thrown vertically downwards,

$a = +g,\ s = h$

$v_2^2 = u^2 + 2gh$

$\frac{v_1^2}{v_2^2} = \frac{u^2 + 2gh}{u^2 + 2gh} = 1$

$v_1 : v_2 = 1 : 1$

63. **(C)** Time taken in reaching bottom of incline is

$t = \sqrt{\frac{2l\left(1 + K^2/R^2\right)}{g \sin\theta}}$

For solid cylinder (SC), $K^2 = R^2/2$

For hollow cylinder (HC), $K^2 = R^2$

For solid sphere (S), $K^2 = \frac{2}{5}R^2$

$\therefore t_S < t_{SC} < t_{HC}$

64. **(B)** Here $dx_1 = dx_2$, $A_1 = A_2$, $\frac{K_1}{K_2} = \frac{2}{3}$

Let θ be the temp. of the junction.

As $\frac{dQ_1}{dt} = \frac{dQ_2}{dt}$

$\therefore K_1A_1\frac{dT_1}{dx_1} = K_2A_2\frac{dT_2}{dx_2}$

$K_1 = (100 - \theta) = K_2(\theta - 0)$

or $\frac{K_1}{K_2} = \frac{\theta}{100 - \theta} = \frac{2}{3}$

$3\theta = 200 - 2\theta$; $5\theta = 200$; $\theta = 40\ °C$

65. **(C)** m = 3 kg, $u = 10\ m\ s^{-1}$, v = 0

Impulse = F × t = ?

Impulse = Change in momentum

F × t = m(v – u)

= 3 [0 – 10] = – 30 N s

Chemistry

66. **(C)** The set of elements given in options (A), (B) and (D) have decreasing atomic radius.

Atomic radius in ($\mathring{A}$)

Oxygen 0.73

Sulphur 1.09

Selenium 1.16

Tellurium 1.35

As the atomic number increases within a group, the atomic size increases accordingly.

67. **(D)** All the alkali metals and their salts impart colour to bunsen flame. The colours imparted by different alkali metals are as follows.

Element	Li	Na	K	Rb	Cs
Colour	Crimson red	Golden yellow	Pale violet	Red violet	Bluish

When heat energy is supplied to alkali metal atom or ion in salt, the electronic excitation occurs in which electron jumps to higher energy level. When this excited electron de-excites to ground state, the energy is emitted in the form of electromagnetic radiation which lies in visible region thereby imparting colour to the flame. The colour of flame depends upon the wavelength of radiation emitted e.g., yellow D-line of Na-spectra arises from $3s^1 \rightarrow 3p^1$ transition.

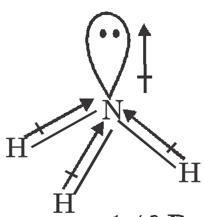
68. **(A)** $CO = N_2 = w\ g \therefore n_1(CO) = \frac{w}{28}$

$n_2(N_2) = \frac{w}{28}$. Hence, $P_{N_2} = P_{CO}$

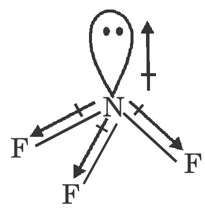
*69. **(C)**

70. **(D)** Only coloured salts will form coloured metal metaborates.

71. **(B)** BF_3 is triangular planar and B_2H_6 is a dimer of triangular planar molecule (BH_3), therefore, both of these have zero dipole moment. NH_3 and NF_3, on the other hand have pyramidal structures and thus have dipole moments.



$\mu = 1.46$ D



$\mu = 0.24$ D

In NH_3, the dipole moments of the three N–H bonds reinforce the dipole moment due to lone pair of electrons but in NF_3, the dipole moments of the three N–F bonds oppose the dipole moment due to lone pair of electrons. As a result, dipole moment of NH_3 ($\mu = 1.46$ D) is higher than that of NF_3 ($\mu = 0.24$ D).

72. **(B)** Rise in temperature,

$\Delta t = (300.78\ K - 294.05\ K) = 6.73\ K$

Heat capacity of the calorimeter = $8.93\ kJ\ K^{-1}$

Then,

Heat transferred to calorimeter =

Heat capacity of calorimeter $\times$ Rise in temperature

$= 8.93\ kJ\ K^{-1} \times 6.73\ K$

$= 60.1\ kJ$

73. **(B)** HNO_3 is added to decompose Na_2S and NaCN otherwise Na_2S will give black ppt. of Ag_2S and NaCN will give white ppt. of AgCN which would interfere with the test of halogens.

74. **(A)** 2-Ethylanthraquinol $\rightarrow$ 2-Ethylanthraquinone + $2H_2O_2$

75. **(C)** $K_c = \frac{[NO]^2}{[N_2O_4]} = \frac{(1.2 \times 10^{-2}\ mol\ L^{-1})^2}{4.8 \times 10^{-2}\ mol\ L^{-1}}$

$= 3 \times 10^{-3}\ mol\ L^{-1}$

76. **(A)** $C : H = \frac{12 \times 100}{13 \times 12} = \frac{1 \times 100}{13 \times 1} = 1 : 1$

$\therefore$ E.F. = CH

Since, P decolourises Br_2–H_2O, but Q does not, therefore, $P = C_2H_2$ (acetylene) and $Q = C_6H_6$ (benzene).

77. **(B)** According to Fajan's rule, the covalent bonding is maximum when W^+ is small and X^- is large.

78. **(A)** $TiH_{1.73}$ is a non-stoichiometric metallic or interstitial hydride.

79. **(A)** Except lime (50 – 60%), the major constituent of cement is silica (20 – 25%).

80. **(D)** (a) It is exact neutralisation. Hence, pH = 7.

(b) After neutralisation, $\frac{M}{10}$ HC*l* left = 10 m*l*.

Total volume = 100 m*l*

Dilution = 10 times.

$\therefore [H^+] = 10^{-2}$

or pH = 2

(c) After neutralisition, $\frac{M}{10}$ NaOH left = 80 m*l*.

Total volume = 100 m*l*. pH > 7.

(d) After neutralisation, $\frac{M}{5}$ HC*l* left = 50 m*l*.

Total volume = 100 m*l*

Dilution = 2 times

*69. **(C)** The sum of mass % is 99.8. Hence, there is no oxygen in the given compound.

Element	Mass %	Atomic mass	Atomic ratio	Simplest ratio	Simplest whole number ratio
C	64.4	12	64.4 / 12 = 5.37	5.37 / 0.53 = 10.1	10
H	5.5	1	5.5 / 1 = 5.5	5.5 / 0.53 = 10.4	10
Fe	29.9	56	29.9 / 56 = 0.53	0.53 / 0.53 = 1	1

Thus, the empirical formula of the compound is $C_{10}H_{10}Fe$.

$\therefore$ $[H^+] = \frac{1}{10} = 10^{-1}$ M or pH = 1

81. **(C)** $BaO_2 + H_2SO_4 \rightarrow BaSO_4 + H_2O_2$
In this reaction, none of the elements undergo, a change in oxidation number or valency.

82. **(A)** Structure of B_2H_6 contains four 2c - 2e bonds and two 3c – 2e bonds.

83. **(C)** Molar mass of acetylene (C_2H_2),
M = (2 × 12 + 2 × 1) g/mol = 26 g/mol
Mass of acetylene, m= 5.0 g
Temperature, T = (50 °C + 273) = 323 K

Pressure, P = 740 mm Hg = $\frac{740}{760}$ atm

= 0.9737 atm

Using the gas equation,

$$PV = nRT = \frac{m}{M} RT$$

$$V = \frac{m\,RT}{M\,P}$$

$$\frac{5.0\,g \times 0.082\text{ L atm K}^{-1}\text{mol}^{-1} \times 323\text{K}}{26\text{ g mol}^{-1} \times 0.9737\text{ atm}}$$

= 5.23 L

84. **(A)** $c = v\lambda$ or $\lambda = \frac{c}{v} = \frac{3\times10^{8}\text{ m s}^{-1}}{589\times10^{-9}\text{m}}$

$= 5.1 \times 10^{14}$ s^{-1} (or Hz)

85. **(B)** % of S = $\frac{32}{233} \times \frac{0.233}{0.32} \times 100 = 10$

86. **(D)** I has the tendency to lose as well as gain electrons. Oxidation states of
Cs = +1
F = –1, 0
Xe = Nil
I = –1, 0, +1, +3, +5, +7

87. **(A)** NH_4^+ is a conjugate acid of the base NH_3.

88. **(B)** Cs with low IE is used in photoelectric cells.

89. **(C)** For PCl_3,

$X = \frac{1}{2}$ = [VE + MA – c + a]

$\frac{1}{2}$ [5 + 3 – 0 + 0] = 4

$\therefore$ Hybridization of P in PCl_3 is sp^3 .
For PCl_5.

$X = \frac{1}{2}$ [5 + 5 – 0 + 0] = 5

$\therefore$ Hybridization of P in PCl_5 is sp^3d.

90. **(B)** Due to the poor shielding (screening) effect of d-electrons in case of Ga, the valence electrons are attracted more strongly and hence, the size is not increased.

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