NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION

Paper Code: UN423 (UPDATED)

Solutions for Class : 11 (PCM)

Mathematics

1. **(B)** The given equation is equivalent to

$2\sin^2((\pi/2)\cos^2 x) = 2\sin^2((\pi/2)\sin 2x)$

$\Rightarrow \cos^2 x = \sin 2x$

$\Rightarrow \cos x(\cos x - 2\sin x) = 0$

$\Rightarrow 1 - 2\tan x = 0$ as $\cos x \neq 0$,

$x \neq (2n+1)\pi/2$

$\Rightarrow \tan x = 1/2$

$\Rightarrow \cos 2x = \dfrac{1-\tan^2 x}{1+\tan^2 x} = \dfrac{3}{5}$

2. **(D)** The index of $(x-a)^8$ is 8 which are even and so the middle term in the expansion of $(x-a)^8$ is $\left(\frac{8}{2}+1\right)^{th}$ term i.e., 5^{th} term.

Now, the general term in the expansion of $(x-a)^8$ is

$t_{r+1} = {}^8C_r(-1)^r x^{8-r}a^r$

Putting r = 4 in (i), we get

$t_5 = {}^8C_4(-1)^4 x^4 a^4 = {}^8C_4 x^4 a^4$.

3. **(A)** $\log_{\sqrt{3}} x + \log_{\sqrt[4]{3}} x + \log_{\sqrt[6]{3}} x + \ldots + \log_{\sqrt[16]{3}} x = 36$

$$\Rightarrow \frac{\log x}{\log\sqrt{3}} + \frac{\log x}{\log\sqrt[4]{3}} + \frac{\log x}{\log\sqrt[6]{3}} + \ldots + \frac{\log x}{\log\sqrt[16]{3}} = 36$$

$$\Rightarrow \frac{\log x}{\log 3^{1/2}} + \frac{\log x}{\log 3^{1/4}} + \frac{\log x}{\log 3^{1/6}} + \ldots + \frac{\log x}{\log 3^{1/16}} = 36$$

$$\Rightarrow \frac{\log x}{\frac{1}{2}\log 3} + \frac{\log x}{\frac{1}{4}\log 3} + \frac{\log x}{\frac{1}{6}\log 3} + \ldots + \frac{\log x}{\frac{1}{16}\log 3} = 36$$

$$\Rightarrow \left(\frac{\log x}{\log 3}\right)\{2+4+6+\ldots+16\} = 36$$

$$\Rightarrow \left(\frac{\log x}{\log 3}\right)\left\{\frac{8}{2}(2+16)\right\} = 36$$

$$\Rightarrow \log x = \frac{1}{2}\log 3 \Rightarrow x = \sqrt{3}.$$

4. **(B)** Given limit

$$= \lim_{x\to 0} \frac{x^{1/2}\left(x + \frac{x^3}{3} + \frac{2}{15}x^5 + \ldots\ldots\right)}{\left\{\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\right) - 1\right\}^{3/2}}$$

$$= \lim_{x\to 0} \frac{x^{3/2}\left(1 + \frac{x^2}{3} + \frac{2}{15}x^4 + \ldots\ldots\right)}{x^{3/2}\left(1 + \frac{x}{2} + \frac{x^2}{6} + \ldots\ldots\right)^{3/2}}$$

$$= \lim_{x\to 0} \frac{\left(1 + \frac{x^2}{3} + \frac{2}{15}x^4 + \ldots\ldots\right)}{\left(1 + \frac{x}{2} + \frac{x^2}{6} + \ldots\ldots\right)^{3/2}} = \frac{1}{(1)^{3/2}} = 1$$

Hence, the correct answer is (b).

5. (C) **Condition of collinearity**

$$\Delta = \begin{vmatrix} p+1 & 1 & 1 \\ 2p+1 & 3 & 1 \\ 2p+2 & 2p & 1 \end{vmatrix} = 0$$

$\Rightarrow (p+1)(3-2p) - 1(\cancel{2p} + 1 - \cancel{2p} - 2)$

$+1(4p^2 + 2P - 6p - 6) = 0$

$\Rightarrow -2p^2 + p + 3 + 1 + 4p^2 - 4p - 6 = 0$

$\Rightarrow 2p^2 - 3p - 2 = 0$

$\Rightarrow 2p^2 - 4p + p - 2 = 0$

$\Rightarrow p = 2, \dfrac{-1}{2}$

6. **(B)** When x = 2, y = 4 we have 4x + 3y = 4(2) + 3(4) = 20.

$\therefore (2, 4) \in R$.

7. **(D)** $\frac{2x+3}{5} < \frac{4x-1}{2}$

$\Rightarrow$ $2(2x+3) < 5(4x-1)$

[Multiplying both sides by 10.

We have: $a > b \Rightarrow am > bm \ \forall\ m > 0$]

$\Rightarrow 4x + 6 < 20x - 5$

$\Rightarrow 0 < 16x - 11$

[Adding $(-4x - 6)$ on both sides.

We have: $a > b \Rightarrow a + m > b + m \ \forall\ m \in R$]

$\Rightarrow$ $11 < 16x$ [Adding 11 on both sides]

$\Rightarrow \frac{11}{16} < x$ [Multiplying $\frac{1}{16}$ on both sides]

$\therefore x \in \left[\frac{11}{16}, \infty\right)$

8. **(A)** $\tan(\pi \cos\theta)$

$= \cot(\pi \sin\theta) = \tan\left(\pm\frac{\pi}{2} - \pi\sin\theta\right)$

$\Rightarrow \pi\cos\theta = \pm\frac{\pi}{2} - \pi\sin\theta$

$\Rightarrow \cos\theta + \sin\theta = \pm\frac{1}{2}$

$\Rightarrow \sqrt{2}\cos\left(\theta - \frac{\pi}{4}\right) = \pm\frac{1}{2}$

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \pm\frac{1}{2\sqrt{2}}$

9. **(A)** $\sum_{k=1}^{4n+7} i^k = i^1 + i^2 + i^3 + i^4 + i^5 + i^6 + i^7 + i^8 + \ldots + i^{4n+1} + i^{4n+2} + i^{4n+3} + i^{4n+4} + i^{4n+5} + i^{4n+6} + i^{4n+7}$

$= 0 + 0 + \ldots + 0 + i - 1 - i = -1.$

10. **(B)** The desired point is the foot of the perpendicular from the origin on the line $3x - 4y = 25$.

The equation of a line passing through the origin and perpendicular to $3x - 4y = 25$ is $4x + 3y = 0$.

Solving these two equations we get $x = 3$, $y = -4$.

Hence, the nearest point on the line from the origin is $(3, -4)$.

11. **(B)** Let a, b, c be the three sides of the triangle ABC, such that $a < b < c$ and a, b, c are in A.P. with common difference d.

Then, $d > 0$ since a, b, c is an increasing A.P.

$\therefore$ We have : $a = b - d$ and $c = b + d$

Let A, B, C be the angles opposite to the sides a, b, c respectively.

Then, $A < B < C$.

[Side opposite the largest angle is the longest]

We have: $C = 2A$ (given)

And so, $B = \pi - (A + C) = \pi - (A + 2A) = \pi - 3A$.

Now, by sine Formula, we have:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{b-d}{\sin A} = \frac{b}{\sin(\pi - 3A)} = \frac{b+d}{\sin 2A}$$

$$\Rightarrow \frac{b-d}{\sin A} = \frac{b}{\sin 3A} = \frac{b+d}{\sin 2A}$$

$$\Rightarrow \frac{b}{b-d} = \frac{\sin 3A}{\sin A} \text{ and } \frac{b+d}{b-d} = \frac{\sin 2A}{\sin A}$$

$$\Rightarrow \frac{b}{b-d} = 3 - 4\sin^2 A \text{ and } \frac{b+d}{b-d} = 2\cos A$$

[Using $\sin 3A = 3\sin A - 4\sin^3 A$]

$$\Rightarrow \frac{b}{b-d} = 3 - 4\sin^2 A \text{ and } \frac{b+d}{b-d} = 2\cos A$$

[$3 - 4\sin^2 A = 3 - 4(1 - \cos^2 A) = 4\cos^2 A - 1$]

$$\Rightarrow \frac{b}{b-d} = \left(\frac{b+d}{b-d}\right)^2 - 1$$

$$\Rightarrow \frac{b}{b-d} = \frac{(b+d)^2 - (b-d)^2}{(b-d)^2} = \frac{4bd}{(b-d)^2}$$

$\Rightarrow 4d = b - d \Rightarrow b = 5b.$

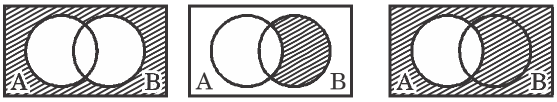
$\therefore$ The sides of the triangle are

$a = b - d = 5d - d = 4d$, $b = 5d$ and

$c = b + d = 5d + d = 6d$.

Hence, the sides are in the ratio 4 : 5 : 6

12. (C) $(A\cup B)' \cup (A'\cap B) = A'$ as shown in the following venn-diagrams:



$(A\cup B)'$ $(A'\cap B)$ $(A\cup B)'\cup(A'\cap B)=A'$

13. (D) Number of ways to enter the palace = 7.

Number of ways to come out of the palace = 12.

$\therefore$ By the fundamental principle of counting:

Total number of ways in which a person can enter and come out of the palace = 7 × 12 = 84.

14. (C) Putting $x = \cos\theta$, we get

$$\frac{d}{dx}\left\{\sin^2\left(\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)\right\}$$

$$=\frac{d}{dx}\left\{\sin^2\left[\cot^{-1}\cot\frac{\theta}{2}\right]\right\}=\frac{d}{dx}\left(\sin^2\frac{\theta}{2}\right)$$

$$=\frac{d}{dx}\left[\frac{1}{2}\left(2\sin^2\frac{\theta}{2}\right)\right]=\frac{d}{dx}\left[\frac{(1-\cos\theta)}{2}\right]$$

$$=\frac{d}{dx}\left\{\frac{1}{2}(1-x)\right\}=-\frac{1}{2}$$

$\therefore$ The correct answer is (c)

15. (B) The affixes of the points P, Q, R and S are $z_1 = 4 + i$, $z_2 = 1 + 6i$, $z_3 = -4 + 3i$ and $z_4 = -1 - 2i$ respectively.

$$PQ = |z_2 - z_1| = \sqrt{(1-4)^2+(6-1)^2} = \sqrt{34};$$

$$QR = |z_3 - z_2| = \sqrt{(-4-1)^2+(3-6)^2} = \sqrt{34};$$

$$RS = |z_4 - z_3| = \sqrt{(-1+4)^2+(-2-3)^2} = \sqrt{34};$$

$$SP = |z_1 - z_4| = \sqrt{(4+1)^2+(1+2)^2} = \sqrt{34};$$

$$\text{Also, } PR = |z_3 - z_1| = \sqrt{(-4-4)^2+(3-1)^2} = \sqrt{68}$$

$$\text{and } QS = |z_4 - z_2| = \sqrt{(-1-1)^2+(-2-6)^2} = \sqrt{68}.$$

Thus, all the four sides of the quadrilateral PQRS are equal i.e., PQ = QR = RS = SP = $\sqrt{34}$ and both the diagonals are equal i.e., PR = QS. Hence, PQRS is a square.

16. (A) Let x_1 and x_2 be the two roots of the equation

$x^2 - 2ax + a^2 = 0$

Then, $x_1 + x_2 = 2a$

$$\Rightarrow \frac{x_1+x_2}{2} = a$$

i.e., Arithmetic mean of the roots, A = a.

Also, $x_1x_2 = a_2$ [Product of the roots]

$$\Rightarrow \sqrt{x_1x_2} = a$$

i.e., Geometric Mean of the roots, G = a.

Clearly, A = G.

17. (B) By Binomial Expansion:

$(1 + x)^{20} = {}^{20}C_0 + {}^{20}C_1x + {}^{20}C_2x^2 + \dots + {}^{20}C_{20}x^{20}$

The general term in this expansion is given by

$t_{r+1} = {}^{20}C_rx^r$. Its coefficient is ${}^{20}C_r$.

$\therefore$ Coefficient of (m + 1)th term = coefficient of (m + 3)th term

$\Rightarrow {}^{20}C_m = {}^{20}C_{m+2} \Rightarrow {}^{20}C_{20-m} = {}^{20}C_{m+2}$ [$\because$ ${}^{20}C_m = {}^{20}C_{20-m}$]

$\Rightarrow 20 - m = m + 2 \Rightarrow 2m = 20 - 2 \Rightarrow m = 9.$

18. (D) X = Set of all positive divisors of 400

= {1, 2, 4, 5, 8, 10, 16, 20, 25, 40, 50, 80, 100, 200, 400}

Y = Set of all positive divisors of 1000

= {1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 125, 200, 250, 500, 1000}

We have : $X\cap Y$ = {1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200}

Hence, $n(X\cap Y) = 12$.

19. (B) We have tan A = a/b and tan B = b/a, so that $\tan A + \tan B = \frac{a}{b}+\frac{b}{a}$

$$=\frac{a^2+b^2}{ab}=\frac{c^2}{ab}$$

20. (B) The condition satisfies 4abc for a,b,c in A.P.

21. (C) $\lim_{x\to 2}\frac{xf(2)-2f(x)}{x-2}\left[\text{Form }\frac{0}{0}\right]$

$=\lim_{x\to 2}\frac{xf(2)-2f'(x)}{1}$

$=\{f(2)-2f'(2)\}=(4-2\times 4)=-4$

$\therefore$ The correct answer is (c).

22. **(C)** $|(1-i)(1+2i)(2-3i)|$

$=|1-i|.|1+2i|.|2-3i|$

[$\because$ $|z_1.z_2...z_n|=|z_1|.|z_2|...|z_n|$]

$\left\{\sqrt{1^2+(-1)^2}\right\}.\left\{\sqrt{1^2+2^2}\right\}.\left\{\sqrt{2^2+(-3)^2}\right\}$

$=\sqrt{2}.\sqrt{5}.\sqrt{13}=\sqrt{130}$

23. **(B)** Total number of ways of selecting 6 out of 10 friends $= {}^{10}C_6 = 210$.

Number of ways of selecting 6 out of 10 friends when 2 particular friends are to be always included (together) in each selection $= {}^{10-2}C_{6-2} = {}^8C_4 = 70$.

$\therefore$ Number of ways of selecting 6 out of 10 friends when 2 particular friends are not together in any selection

$= 210 - 70 = 140$.

24. **(C)** Here $m_1 = 2-\sqrt{3}$ and $m_2 = 2+\sqrt{3}$

If θ is the angle between lines, then

$\tan\theta = \left[\frac{m_1-m_2}{1+m_1m_2}\right]$

$\Rightarrow \tan\theta = \left[\frac{2-\sqrt{3}-2-\sqrt{3}}{1+\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)}\right]$

$\Rightarrow \tan\theta = \frac{2\sqrt{3}}{1+(4-3)}$

$\Rightarrow \tan\theta=\sqrt{3}$

$\therefore \theta=60^\circ$

25. **(C)** Let d be the common difference of the A.P. a,b,c.

Then, $b-a=d$ and $c-b=d$.

$\therefore a=b-d$ and $c=b+d$.

Now, abc = 4 (given)

$\Rightarrow (b-d)\,b\,(b+d)=4 \Rightarrow b(b^2-d^2)=4$ .... (i)

But $b^2-d^2<b^2$ [Since both b^2 and d^2 are positive]

$\Rightarrow b.(b^2-d^2)<b.b^2$ [$\because$ be is positive]

$\Rightarrow 4<b^3 \Rightarrow 2^2<b^3 \Rightarrow b>2^{2/3}$ [Using (i)]

Thus, the minimum possible value of b is $2^{2/3}$.

26. **(C)** The sum of the coefficients in the expansion of $(x+y)^n = 4096$

$\Rightarrow {}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 4096$

$\Rightarrow 2^n = 4096 = 2^{12} \Rightarrow n = 12.$

Since n is even, the greatest coefficient is the coefficient of the middle term i.e., of the $\left(\frac{n}{2}+1\right)$th term i.e., of the 7^{th} term.

Thus, the greatest coefficient in the expansion is

${}^nC_{n/2} = {}^{12}C_6 = \frac{12\times 11\times 10\times 9\times 8\times 7}{6\times 5\times 4\times 3\times 2\times 1} = 924.$

27. **(C)** The number of proper subsets of a set having n elements is 2^{n-1}.

$\therefore$ The number of proper subsets of $\{a, b, c\}$ i.e. of a set containing 3 elements is 2^3-1 *i.e.* 7.

28. **(A)** Ler P (h, 0) divides the join of A(2,–3) and B(5, 6) in the ratio of λ : 1, such that the co-ordinates of point P are

$\left(\frac{5\lambda+2}{\lambda+1}, \frac{6\lambda-3}{\lambda+1}\right)$

Since, the point P lies on the x-axis, therefore, the y- part(ordinate) must be zero.

$\Rightarrow \frac{6\lambda-3}{\lambda+1}=0$

$\Rightarrow \lambda=\frac{1}{2}$

The ratio in which P on x-axis divided by join of AB is 1 : 2

29. **(C)** When n = 1, we have :

$p^{n+1} + (p+1)^{2n-1} = p^2 + (p+1)^1 = P^2 + p + 1$, which is divisible by $p^2 + p + 1$.

[**Note :** p2 + p + 1 is not divisible by any of the remaining alternatives $p-1, p^2+1$ or p^3-1.]

We shall show that $p^{n+1} + (p+1)^{2n-1}$ is divisible by p^2+p+1 for all natural numbers n.

Let P (n) : $p^{n+1} + (p+1)^{2n-1}$ is divisible by $p^2 + p + 1$.

Then, P (1) is true. [Proved above]

Let P (m) be true.

Then, $p^{m+1} + (p+1)^{2m-1}$ is divisible by $p^2 + p + 1$.

Let $p^{m+1} + (p+1)^{2m-1} = k(p^2 + p + 1)$ where $k \in N$.

$\Rightarrow p^{m+1} = k(p^2 + p + 1) - (p+1)^{2m-1}$

Now, $p^{(m+1)+1} + (p+1)^{2(m+1)-1}$

$= p^{m+2} + (p+1)^{2m+1} = p.p^{m+1} + (p+1)^{2m+1}$

$= p\{k(p^2 + p + 1) - (p+1)^{2m-1}\} + (p+1)^{2m+1}$

$= kp(p^2 + p + 1) - p(p+1)^{2m-1} + (p+1)^{2m+1}$

$= kp(p^2 + p + 1) + (p+1)^{2m-1}\{-p + (p+1)^2\}$

$= kp(p^2 + p + 1) + (p+1)^{2m-1}(p^2 + p + 1)$

$= (p^2+p+1)\{kp + (p+1)^{2m-1}\}$, which is divisible by p^2+p+1.

$\therefore$ P (m+1) is true whenever P (m) is true.

Thus, P (n) is true for all $n \in N$.

Hence, $p^{n+1} + (p+1)^{2n-1}$ is divisible by $p^2 + p + 1$, for all $n \in N$.

30. (C) $\frac{{}^2C_2}{{}^2C_4} = \frac{2}{1} \Rightarrow \frac{n!}{2!(n-2)!} \times \frac{4!(n-4)!}{n!} = \frac{2}{1}$

$\Rightarrow \frac{4 \times 3}{(n-2)(n-3)} = \frac{2}{1}$

$\Rightarrow n^2 - 5n + 6 = 6$

$\Rightarrow n^2 - 5n = 0$

$\Rightarrow n(n-5) = 0$

$\Rightarrow n = 0$ or $n = 5$

$\Rightarrow n = 5$ [$\because$ ${}^nC_2$ and ${}^nC_4$ are not defined for n = 0]

31. (C) $y = \log_e(1 + x)$

$\Rightarrow \frac{dy}{dx} = \frac{1}{(1+x)}$

$\Rightarrow \left.\frac{dy}{dx}\right]_{(x=1)} = \frac{1}{2}$

$\therefore$ The correct answer is (C)

32. (B) $f(x) = \frac{x-1}{x+1} \Rightarrow (x+1)f(x) = x - 1$

$\Rightarrow x\{f(x) - 1\} = -\{1 + f(x)\} \Rightarrow x = \frac{1 + f(x)}{1 - f(x)}$

Now, f(2x) =

$$\frac{2x-1}{2x+1} = \frac{2\left\{\frac{1+f(x)}{1-f(x)}\right\}-1}{2\left\{\frac{1+f(x)}{1-f(x)}\right\}+1} = \frac{1+3f(x)}{3+f(x)} = \frac{3f(x)+1}{f(x)+3}$$

33. (C) The given equation can be written as

$$\frac{1+\sin x}{\cos x} = 2\cos x$$

$\Rightarrow 1 + \sin x = 2\cos^2 x = 2(1 - \sin^2 x)$

$\Rightarrow 2\sin^2 x + \sin x - 1 = 0$

$\Rightarrow (1 + \sin x)(2\sin x - 1) = 0$

$\Rightarrow \sin x = -1$ or $1/2$

Now $\sin x = -1 \Rightarrow x = 3\pi/2$ for which the given equation is not meaningful and $\sin x = 1/2 \Rightarrow x = \pi/6$ or $5\pi/6$.

The required number of solutions is 2.

34. (B) Let $y = \lim_{x\to 0}\left(\frac{a^x + b^x}{2}\right)^{1/x}$, Then,

$\log y = \lim_{x\to 0}\frac{1}{x}\log\left(\frac{a^x + b^x}{2}\right)$

$= \lim_{x\to 0}\frac{\log(a^x + b^x) - \log 2}{x}$ $\left[\text{form } \frac{0}{0}\right]$

$= \lim_{x\to 0}\frac{1}{(a^x + b^x)}(a^x \log a + b^x \log b)$

$= \frac{1}{2}(\log a + \log b) = \log\sqrt{ab}$

$\therefore y = e^{\log\sqrt{ab}} = \sqrt{ab}$

Hence, the correct answer is (b).

35. (C) The two digit natural numbers which leave a remainder 5 when divided by 7 are 12, 19, 26, ..., 89, 96

Let S = 12+ 19 + 26 + ... + 89 + 96

Let there be n terms in the A.P. 12, 19, 26, ..., 89, 96.

The first term of this A.P. is a = 12 and its common difference d = 19 – 12 = 7.

Then the nth term $t_n = 96 \Rightarrow a + (n-1)d = 96$

$\Rightarrow 12 + (n-1)(7) = 96 \Rightarrow 7n - 7 = 84 \Rightarrow n = 13$.

Thus, there are 13 terms in the A.P. 12,19,26,...,89,96.

$\therefore S = \frac{n}{2}(a + l) = \frac{1}{2}(12 + 96) = 702$.

36. (D) The given expression is equal to

$(\sin 47° + \sin 61°) - (\sin 11° - \sin 25°)$

$= 2\sin 54°\cos 7° - 2\sin 18°\cos 7°$

$= 2\cos 7°(\sin 54° - \sin 18°)$

$= 2\cos 7°\left[\frac{\sqrt{5}+1}{4} - \frac{\sqrt{5}-1}{4}\right] = \cos 7°$

37. (B) $\frac{2}{5}+\frac{3}{5^2}+\frac{2}{5^3}+\frac{3}{5^4}+\ldots\infty$

$$\left(\frac{2}{5}+\frac{2}{5^3}+\ldots\infty\right)+\left(\frac{3}{5^2}+\frac{3}{5^5}+\ldots\infty\right)$$

$$=\frac{\left(\frac{2}{5}\right)}{\left(1-\frac{1}{5^2}\right)}+\frac{\left(\frac{3}{5^2}\right)}{\left(1-\frac{1}{5^2}\right)}=\left(\frac{2}{5}\cdot\frac{25}{24}\right)+\left(\frac{3}{25}\cdot\frac{25}{24}\right)$$

$$=\frac{10}{24}+\frac{3}{24}=\frac{13}{24}$$

38. (C) $A = \{1, 2\}, C = \{\{1\}, \{2\}\}$

Clearly, $A \in C$.

39. (C) $3 \le 3t - 18 \le 18$........ (i) (given)

$\Rightarrow\ 21 \le 3t \le 36$ .... (ii)

$[a > b \Rightarrow a + m > b + m\ \forall\ m \in R]$

$\therefore$ Alternative (B) is not true.

$\Rightarrow 7 \le t \le 12$ .... (iii)

$\left[\ a > b \Rightarrow \frac{a}{m} > \frac{b}{m}\ \forall m > 0\right]$

$\therefore$ Alternative (A) is not true.

$\Rightarrow 8 \le t + 1 \le 13$ .... (iv)

[Adding 1 on both sides]

$\therefore$ Alternative (C) is true.

Again, from (iii) we have: $7 \le t \le 12$

$\Rightarrow 14 \le 2t \le 24$ [Multiplying both sides by 2]

$\Rightarrow 15 \le 2t + 1 \le 25$ [Adding 1 on both sides]

$\therefore$ Alternative (D) is not true.

40. (C) We have: $\alpha - i\beta = \left(\frac{3-4i\,x}{3+4i\,x}\right)$

$$\Rightarrow |\alpha - i\beta| = \left|\frac{3-4i\,x}{3+4i\,x}\right| = \frac{|3-4i\,x|}{|3+4i\,x|}\quad\left[\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$$

$$\Rightarrow |\alpha - i\beta|^2 = \frac{|3-4i\,x|^2}{|3+4i\,x|^2}$$

$$\Rightarrow \alpha^2+\beta^2 = \frac{3^2+(-4x)^2}{3^2+(4x)^2} = \frac{9+16x^2}{9+16x^2} = 1.$$

Physics

41. (B) Here, $m_1 = 10$ kg, $m_2 = 5$ kg, $v_1 = ?$, $v_2 = 0$, $v = 4$ m/s

Applying the principle of conservation of linear momentum,

$m_1v_1 + m_2v_2 = (m_1 + m_2)v$

$10v_1 + 5 \times 0 = (10 + 5)4 = 60$

$$v_1 = \frac{60}{10} = 6\ \text{m s}^{-1}$$

42. (C) $d = 0.005$ mm $= 0.005 \times 10^{-3}$ m,

$R = 10/2 = 5$ cm $= 0.05$ m,

$\sigma = 0.072\ \text{Nm}^{-1}$

Force required

$$= \frac{2\sigma}{d}\times\pi R^2 = \frac{2\times 0.072\times 3.14\times(0.05)^2}{0.005\times 10^{-3}}$$

$= 226$ N

43. (C) Here $\alpha = (10/2)$ rad s^{-2}

$I = MR^2 = \frac{1}{2} \times (0.2)^2 = 0.02$ kg ms^2.

Here $\tau = I\alpha = 5 \times 0.02$ Nm $= 0.10$ Nm.

44. (D) $\eta = \frac{W}{Q_1}$ $\quad W = \eta Q_1 = \frac{1}{3}\times 1000$ cals.

$$= \frac{1000}{3} \times 4.2\ \text{joule} = 1400\ \text{J}.$$

45. (B) Mean value of refractive index.

$$n_m = \frac{1.49+1.50+1.52+1.54+1.48}{5}$$

$= 1.506 = 1.51$

46. (C) $K = \frac{4\times 9.8\ \text{N}}{2\times 10^{-2}\ \text{m}}$ or $K = 19.6\times 10^2\ \text{N m}^{-1}$

$$\text{Workdone} = \frac{1}{2}\times 19.6\times 10^2\times\left(5\times 10^{-2}\right)^2$$

J = 2.45 J

47. (A) $\rho = 0.09$ kg m^{-3}, $v_{rms} = 1.84 \times 10^3$ m s^{-1}

$$\text{Pressure} = P = \frac{1}{3}\rho v_{rms}^2$$

$$P = \frac{1}{3}\times 0.09\times(1.84\times 10^3)^2 = 1.015\times 10^5\ \text{N/m}^2$$

48. **(D)** At height h, $\frac{g'}{g} = 1 - \frac{2h}{R} = \frac{90}{100}$

or $\frac{2h}{R} = 1 - \frac{90}{100} = \frac{10}{100} = \frac{1}{10}$

or R = 20 h = 20 × 320 = 6400 km

At depth d,

$\frac{g'}{g} = 1 - \frac{d}{R} = \frac{95}{100}$

or $\frac{d}{R} = 1 - \frac{95}{100} = \frac{5}{100} = \frac{1}{20}$

or $d = \frac{R}{20} = \frac{6400}{20} = 320$ km

49. **(C)** Hydrogen atom is assumed to be at the origin

$x_1 = 0$, $m_1 = m$, $x_2 = 1.27$ Å

$= 1.27 \times 10^{-10}$ m

$m_2 = 35.5$ m

The position of centre of mass

$x = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$

$= \frac{m \times 0 + 35.5\ m \times 1.27 \times 10^{-10}}{m + 35.5\ m}$

$= \frac{45.08 \times 10^{-10} m}{36.5}$

$= 1.24 \times 10^{-10}$ m

50. **(C)** Power = $\vec{F}.\vec{v}$ = Fv

$F = v\left(\frac{dm}{dt}\right)$

$= v\left\{\frac{d(\rho \times \text{volume})}{dt}\right\}$ $(\rho = \text{density})$

$= \rho v\left\{\frac{d(\text{volume})}{dt}\right\} = \rho v(Av) = \rho A v^2$

$\therefore$ power P = $\rho A v^3$

or $P \propto v^3$

51. **(D)** The temperature increases due to compression. There is flow of heat away from the system. The temperature falls. Consequently, the temperature decreases.

52. **(D)** A + B = 17 .... (i)

A – B = 7 .... (ii)

On solving A = 12 units and B = 5 units

$R = \sqrt{A^2 + B^2} = \sqrt{12^2 + 5^2} = 13$ units.

53. **(B)** Impulse = change in linear momentum

= m v – m u = 0.1 × 40 – 0.1 × (– 30)

54. **(B)** Sum of P.E. and K.E. of the ball at A = Sum of P.E. and K.E. of the ball at B. At the point A, the ball is at rest, so K.E. = 0.

At B the speed of the ball is v. At B, h = 0 So, P.E. = 0

(P.E. + K.E.) at A = (P.E. + K.E.) at B

mgh + 0 = 0 + $\frac{1}{2}$ mv^2

$v\sqrt{2gh}$

$= \sqrt{2 \times 9.8 \times 10}$

= 14 m/s

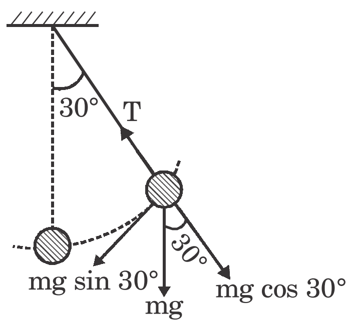
55. **(B)** Rate of change of speed

$\frac{dv}{dt}$ = tangential acceleration

$= \frac{\text{tangential force}}{\text{mass}}$

$= \frac{mg\sin 30°}{m} = g\sin 30°$

$= 10\left(\frac{1}{2}\right) m/s^2 = 5\ m/s^2$



56. **(B)** Workdone = $\frac{1}{2}$ × Y × (strain)^2 × volume

For a given strain, workdone $\propto$ Y.

As Y for copper is less than that of steel, so less work is done on copper spring than that on steel spring.

57. **(A)** As on the surface of planet g = (GM/R^2)

$$\frac{g_M}{g_E} = \frac{M_M}{M_E} \times \left(\frac{R_E}{R_M}\right)^2 = \frac{1}{80} \times (4)^2 = \frac{1}{5}$$

$$\therefore g_M = \frac{g}{5} = \frac{9.8}{5} = 1.96 \text{ m/s}^2$$

Further more as escape velocity

$$v_e = \sqrt{(2GM/R)}$$

$$\frac{v_M}{v_E} = \sqrt{\frac{M_M}{M_E} \times \frac{R_E}{R_M}} = \sqrt{\frac{1}{80} \times 4} = \frac{1}{\sqrt{20}}$$

i.e., $v_M = \left(v_E / \sqrt{20}\right)$ = (11.2/4.47) = 2.5 km/s.

58. **(A)** Mass = m = 700 gm = 0.7 kg

Radius of gyration = K = 20 cm = 0.2 m

Moment of inertia = I = mK^2 = 0.7 × 0.2 × 0.2 = 0.028 kg m^2.

59. **(C)** Let a be the acceleration of sprinter for time t = 4 s.

Distance travelled in 4 s = $\frac{1}{2}at^2 = \frac{1}{2} \times a \times 4^2 = 8a$

Velocity of sprinter at time 4 sec is

v = at = a × 4 = 4 a

Distance covered by sprinter with uniform velocity in time 4 sec to 9 sec (= 5 s) = 4 a × 5 = 20 a

As per question, 8 a + 20 a = 140

or a = 5 m s^{-2}

60. **(D)** $\frac{P_1}{P} = \frac{T_1}{T}$

$$\text{or } T_1 = P_1 \frac{T}{P} = \frac{2\,P \times 293}{P} = 586 \text{ K} = 313° \text{ C}$$

61. **(B)** Here, m = 120 g = 0.12 kg

$$\vec{v} = \left(2\hat{i} + 5\hat{j}\right)\text{m/s}$$

$$v = |\vec{v}| = \sqrt{2^2 + 5^2} = \sqrt{29} \; m/s$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.12 \times 29 = 1.74 \text{ joule.}$$

62. **(A)** Work done in a cyclic process = area enclosed by cycle

Work done = AB × BC

or W = (2P – P) × (2V – V) or

W = PV

63. **(A)** Given, $\frac{mg'}{mg} = \frac{30}{90}$ or $\frac{g'}{g} = \frac{1}{3}$

Now $g' = g\frac{R^2}{(R+h)^2}$ or $g' = g\frac{R^2}{(R+h)^2} = \frac{1}{3}$

or $\frac{R}{R+h} = \frac{1}{\sqrt{3}}$ or $(R+h) = \sqrt{3}\,R$

or h = $\left(\sqrt{3}-1\right)$R = 0.73 R

64. **(B)** $\frac{4}{3}\pi R^3 = 10^6\left(\frac{4}{3}\pi r^3\right)$

or R = (10^2r)

$U_i = 10^6\,(4\pi r^2)T$

$U_f = (4\pi R^2)T$

$$\therefore \frac{U_f}{U_i} = \frac{1}{10^2}$$

65. **(B)** Thrust = normal force

Mass ejected per sec = 250 × 10^{-3} kg s^{-1}

Change of velocity = 8000 m s^{-1}

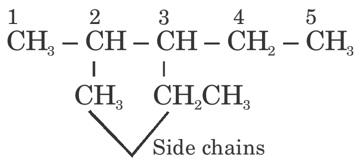
$$F = \frac{m(v-u)}{t}$$

= Mass per sec × Change of velocity

F = 250 × 10^{-3} × 8 × 1000 = 2000 N

Chemistry

66. **(B)** As per the nomenclature of branched chain and complex alkanes as shown below.



If two chains contain the same number of carbon atoms, then the one containing a larger number of side chains is considered as the parent chain. Hence, the parent chain contains two alkyl substituents. So, the correct IUPAC name is 3-Ethyl-2-methyl-pentane

67. **(D)** O^{2-} (8 + 2 = 10) and Mg^{2+} (12 – 2 = 10) are isoelectronic with F^- (9 + 1 = 10)

68. **(A)** $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O,\ \Delta H_1 = -x$ kJ .... (i)

$CH_3OH + \frac{3}{2}O_2 \rightarrow CO_2 + 2H_2O,\ \Delta H_2 = -y$ kJ .... (ii)

Subtracting (ii) from (i), we get

$$CH_4 + \frac{1}{2}O_2 \rightarrow CH_3OH,\ \Delta H_3 = -ve$$

i.e., – x – (–y) = –ve

y – x = –ve.

Hence, x > y

69. **(B)**

$2BCl_3$	+	$3H_2(g)$	$\rightarrow$	2 B	+	6 HCl
		3 mol		2 mol		
		3×22.4 L (STP)		2×10.8 g = 21.6 g		
		= 67.2 L				

70. **(C)**

We can write,	2 HI(g)	$\rightleftharpoons$ $H_2(g)$	+ $I_2(g)$	
Initial pressure,	0.2 atm,	–	–	
At equilibrium partial presssure,	(0.2-P)atm	P/2atm	P/2atm	
	(0.04)atm	0.08	0.08	

Then, $K_p = \dfrac{[PH_2][PI_2]}{[PHI]^2} = \dfrac{0.08 \times 0.08}{(0.04)^2} = 4$

71. **(D)** Na_2CO_3 is completely dissociated to Na^+ and CO_3^{2-} ions thus giving high concentration of CO_3^{2-} ions which would precipitate Mg^{2+} as $MgCO_3$.

72. **(C)** From the given data,

$$n(H_2) = \frac{0.5\text{ g}}{2\text{ g / mol}} = 0.25\text{ mol}$$

$$n(O_2) = \frac{10\text{ g}}{32\text{ g / mol}} = 0.31\text{ mol}$$

$$n(H_2) = \frac{15\text{ L}}{22.4\text{ L / mol}} = 0.67\text{ mol}$$

$$n(N_2) = \frac{5\text{ L}}{22.4\text{ L / mol}} = 0.22\text{ mol}$$

Number of molecules $\propto$ No. of moles of the gas

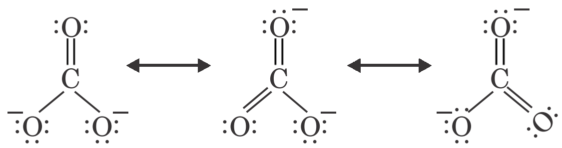
So, 15 L of H_2 gas at STP will contain the maximum number of molecules.

73. **(A)** $\left(\frac{1}{1^2}-\frac{1}{2^2}\right)=\frac{3}{4},\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=\frac{5}{36},\left(\frac{1}{3^2}-\frac{1}{4^2}\right)=\frac{7}{144}$ and so on.

Thus, $1/\lambda$ decreases from transition n = 2 to n = 1, to n = 3 to n = 2 and so on or λ increases in the reverse order, i.e., λ is maximum for n = 5 to n = 4. As by de Broglie equation, $\lambda \propto 1/p$, therefore, momentum *p* will be maximum for the transition n = 2 to n = 1.

74. **(D)** $2K_2CrO_4 + 2HCl \longrightarrow K_2Cr_2O_7 + 2KCl + H_2O$

75. **(B)** According to the experimental findings, all carbon to oxygen bonds in CO_3^{2-} are equivalent. Therefore, the carbonate ion is best described as a resonance hybrid of the canonical forms I, II and III shown below.



(I) (II) (III)

76. **(B)** The equation is $BaO_2 + H_2SO_4 \longrightarrow BaSO_4(s) + H_2O_2$

The most electronegative atom in the products is oxygen. The oxidation state of O in H_2O_2 is –1, whereas that is $BaSO_4$ is –2.

77. **(D)** $\%P = \dfrac{62}{222} \times \dfrac{0.444}{0.31} \times 100 = 40.$

78. **(D)**

2 Mg	+	O_2	$\rightarrow$	2MgO
2 × 24		32 g		2 (24 + 16)
= 48 g				= 80 g

30 g Mg will react completely with $O_2 = \dfrac{32}{48} \times 30 = 20$ g

$\therefore$ O_2 left = 30 – 20 = 10 g

MgO formed = $\dfrac{80}{48} \times 30 = 50$ g

79. **(D)** CCl_4 has tetrahedral, symmetrical structure, hence does not show dipole moment.

80. **(C)** Boron halides, BX_3 have only six electrons (three from B atom and three from three halogen atoms) around central B atom and thus B octet is not complete. Hence, boron halides behave as electron deficient compounds or Lewis acids.

81. **(A)**

	A	+	2 B	$\rightleftharpoons$	C
Initial moles	1		0.5		
Moles at eqm.	0.9		0.3		0.1
Molar concs.	0.3		0.1		0.1/3

$$K = \frac{0.1/3}{0.3 \times (0.1)^2} = 11.11$$

82. **(B)** Here, IE_2 involves the removal of an electron from the inert gas configuration ($1s^2\ 2s^2\ p^6$)

83. **(C)** $PbS + 4\,H_2O_2 \rightarrow PbSO_4 \downarrow + 4H_2O\,(l)$

84. **(C)** The boiling points of benzene (353 K) and chloroform (334 K) differ by 19° and hence can be separated by distillation.

85. **(B)** Given

Amount of the gas, n = 4.0 mol

Volume of the gas, V = 5 dm^3

Pressure of the gas, P = 3.32 bar

R = 0.083 bar $dm^3\,K^{-1}\,mol^{-1}$

From the gas equation, PV = nRT

$$T = \frac{PV}{nR} = \frac{3.32\text{ bar} \times 5\text{ dm}^3}{4.0\text{ mol} \times 0.083\text{ bar dm}^3\text{K}^{-1}\text{mol}^{-1}} = 50\text{ K}$$

86. **(B)** Eqm. const. is constant at constant temperature.

87. **(A)** $MgSO_4 \cdot 7\,H_2O$. Because of smaller size, Mg^{2+} ions are extensively hydrated.

88. **(C)** The process (C) cannot go by itself after initiation.

89. **(B)** Graphite is thermodynamically, the most stable allotrope of carbon. That is why $\Delta_f H^\circ$ (graphite) is taken as zero.

$\Delta_f H^\circ$ (diamond) = + 1.90 kJ mol^{-1}

$\Delta_f H^\circ$ (fullerene) = + 38.1 kJ mol^{-1}

90. **(B)** Heterocyclic and aromatic, i.e., furan.

91. **(B)**

92. **(C)**

93. **(D)**

94. **(B)**

95. **(C)**

96. **(C)**

97. **(B)**

98. **(C)**

99. **(C)**

100. **(D)**