UNIFIED COUNCIL

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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION

Solutions for sample questions

Class : 12 (PCM)

Mathematics

1. **(B)** $x = a\cos^3\theta$, $y = a\sin^3\theta$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin\theta}{\cos\theta}$$

Equation of the tangent is

$$y - a\sin^3\theta = \frac{-\sin\theta}{\cos\theta}(x - a\cos^3\theta)$$

$$\Rightarrow x\sin\theta + y\cos\theta$$

$$= a\cos\theta\sin\theta(\cos^2\theta + \sin^2\theta)$$

$$= a\cos\theta\sin\theta$$

Equation of the normal is

$$y - a\sin^2\theta = \frac{\cos\theta}{\sin\theta}(x - a\cos^2\theta)$$

$$\Rightarrow x\cos\theta + y\sin\theta$$

$$= a(\cos^4\theta + \sin^4\theta)$$

$$= a\,.\cos 2\theta$$

p = length of the perpendicular from origin to the tangent

$$= \frac{|a\cos\theta\sin\theta|}{\sqrt{\sin^2\theta+\cos^2\theta}} = \frac{1}{2}a.\sin 2\theta$$

q = length of the perpendicular from origin to the normal

$$= \frac{|a\cos 2\theta|}{\sqrt{\cos^2\theta+\sin^2\theta}} = a.\cos 2\theta$$

$$\therefore 4p^2 + q^2 = 4.\frac{1}{4}a^2\sin^2 2\theta + a^2\cos^2 2\theta$$

$$= a^2.$$

2. **(A)** $x^2 + y^2 + 2fy + c = 0$

$$\Rightarrow 2x + 2yy_1 + 2fy_1 = 0$$

$$\Rightarrow x + yy_1 + fy_1 = 0$$

$$\Rightarrow 1 + y_1^2 + yy_2 + fy_2 = 0$$

$$\Rightarrow 1 + y_1^2 + yy_2 - y_2\left[\frac{-(x+yy_1)}{y_1}\right] = 0$$

$$\Rightarrow y_1 + y_1^3 + yy_1y_2 + xy_2 - yy_1y_2 = 0$$

$$\Rightarrow y_1^3 - xy_2 + y_1 = 0$$

$$\Rightarrow xy'' - (y')^3 - y' = 0.$$

3. **(A)** $\int \frac{x\,e^x}{(x+1)^2}dx = \int \frac{(x+1-1)\,e^x}{(x+1)^2}$

$$= \int e^x\left[\frac{1}{x+1} - \frac{1}{(x+1)^2}\right]dx$$

$$= e^x\left[\frac{1}{x+1}\right] + c.$$

4. **(B)** $f(x) = \frac{1}{1+1/x} = \frac{x}{x+1}$

$$g(x) = \frac{1}{1+1/f(x)} = \frac{1}{1+(x+1)/x} = \frac{x}{2x+1}$$

$$g'(x) = \frac{(2x+1)1 - x(2)}{(2x+1)^2} = \frac{1}{(2x+1)^2}$$

$$\Rightarrow g'(2) = \frac{1}{25}.$$

5. **(B)** If a, b, c are the sides of the triangle respectively then

$$|a| = |2i + 3j - 6k|$$

$$= \sqrt{4+9+36} = \sqrt{49} = 7,$$

$|b| = |i + 2j - 3k|$

$= \sqrt{36+4+9} = 7$,

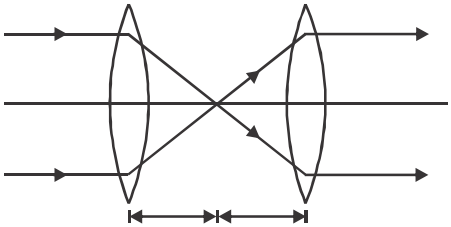
$|c| = |3i + 6j - 2k|$

$= \sqrt{9+36+4} = \sqrt{49} = 7.$

$\therefore$ Perimeter = 7 + 7 + 7 = 21

Physics

6. **(B)** As shown in the figure the distance between the lenses should be 30 cm.



f_1 = 20 cm f_2 = 10 cm

7. **(A)** $F = 6 \times 10^{-6}$ N

$$n = \frac{F}{3 \times 10^{-10}} = \frac{6 \times 10^{-6}}{3 \times 10^{-10}} = 2 \times 10^4$$

8. **(B)** Here, rate of production of energy at the atomic power house,

$P = 400$ MW $= 400 \times 10^6$ J s^{-1}

Therefore, total energy produced in a day i.e., $24 \times 60 \times 60$s,

$E = P \times 24 \times 60 \times 60 = 400 \times 10^6 \times 24 \times 60 \times 60 = 3.456 \times 10^{13}$ J

If mass of U^{235} consumed per day is m (in kg) so as to produce the required amount of energy, then

$E = m\,c^2$

or $3.456 \times 10^{13} = m\,c^2$

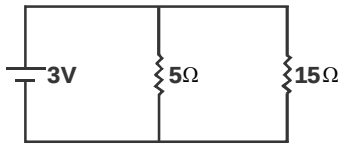
$$\text{or } m = \frac{3.456 \times 10^{13}}{c^2} = \frac{3.456 \times 10^{13}}{(3 \times 10^8)^2}$$

$= 0.384 \times 10^{-3}$ kg = 0.384 g

9. **(B)** Polarity of emf will be opposite in the two cases while entering and while leaving the coil. Only in option (B) polarity is changing.

10. **(B)** Equivalent resistance in series is sum of individual resistances.

In the given figure 3 resistors of 5Ω are connected in series.



$\therefore$ R' = 5 + 5 + 5 = 15Ω

This 15 Ω resistor is connected with the 5 Ω resistor in parallel hence, equivalent resistance now is

$$\frac{1}{R''} = \frac{1}{15} + \frac{1}{5}$$

$$\frac{1}{R''} = \frac{5+15}{5 \times 15}$$

$\Rightarrow$ R'' = 3.75Ω

From Ohm's law, V = IR

$$\therefore \quad I = \frac{V}{R} = \frac{3}{3.75} = 0.8A$$

Chemistry

11. **(C)** Ketones are less reactive than aldehydes.

The aromatic aldehydes and ketones are less reactive than the aliphatic aldehydes and ketones. So

$\underset{II}{CH_3CHO} > \underset{I}{CH_3COCH_3} > \underset{III}{PhCOPh}$

12. **(C)** Doping of Si with P gives extra electrons while doping with Al gives rise to holes.

13. **(C)** Vapour pressure of pure, P_A^o = 40 mm Hg

Vapour pressure of A in solution,

P_A = 32 mm Hg

According to the Raoult's law,

$P_A = P_A^o X_A$

$$\text{Then, } X_A = \frac{P_A}{P_A^o} = \frac{32 \text{ mm Hg}}{40 \text{ mm Hg}} = 0.8$$

14. **(C)** $PH_3(g) + 4Cl_2(g) \rightarrow PCl_5 + 3HCl(g)$
Phosphine

15. **(C)** Cell potential

$= E^o_{Red}$ (RHS) $- E^o_{Red}$ (LHS)

$= 0.34 - (-0.76) = +1.10$ V