UNIFIED COUNCIL

An ISO 9001:2008 Certified Organisation

nstse

Test • Assess • Achieve

NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION - 2015 (UPDATED)

Solutions for Class : 12 (PCM)

Paper Code: UN412

Mathematics

1. **(A)** $f(1) = \underset{x\to 1}{\text{Lt}}\, f(x) = \underset{x\to 1}{\text{Lt}}\, f(1-x)\tan\left(\frac{\pi}{2}x\right)$

$= \underset{h\to 0}{\text{Lt}}\,(1-(1+h))\tan\left(\frac{\pi}{2}(1+h)\right)$

$= \underset{h\to 0}{\text{Lt}}\, f(-h)\tan\left(\frac{\pi}{2}+\frac{\pi}{2}h\right)$

$= \underset{h\to 0}{\text{Lt}} -h\left(-\cot\left(\frac{\pi}{2}h\right)\right)$

$= \underset{h\to 0}{\text{Lt}}\, \frac{\frac{\pi}{2}h}{\tan\left(\frac{\pi}{2}h\right)}.\frac{2}{\pi} = \frac{2}{\pi}$

2. **(A)** As domain of $\cos^{-1} x$ is $[-1, 1]$, therefore, domain of $f(x)$ is given by

$-1 \le \frac{2-|x|}{4} \le 1 \;\Leftrightarrow\; -4 \le 2-|x| \le 4$

$\Leftrightarrow -6 \le -|x| \le 2 \;\Leftrightarrow\; 6 \ge |x| \ge 2$

$\therefore$ $f(x)$ is defined only for $|x| \le 6$.

3. **(D)** $\Delta = \begin{vmatrix} \cos\alpha & \sin\alpha & 0 \\ \cos\beta & \sin\beta & 0 \\ \cos\gamma & \sin\gamma & 0 \end{vmatrix} \times \begin{vmatrix} \cos\alpha & \sin\alpha & 0 \\ \cos\beta & \sin\beta & 0 \\ \cos\gamma & \sin\gamma & 0 \end{vmatrix}$

$= 0 \times 0 = 0$

4. **(B)** Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

then $\vec{a}\times\hat{i} = \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)\times\hat{i}$

$= a_1\vec{0} + a_2\left(-\hat{k}\right) + a_3\hat{j}$

$= -a_2\hat{k} + a_3\hat{j}$

$\Rightarrow |\vec{a}\times\hat{i}|^2 = (-a_2)^2 + a_3^2 = a_2^2 + a_3^2$

Similarly, $|\vec{a}\times\hat{j}|^2 = a_1^2 + a_3^2$

and $|\vec{a}\times\hat{k}|^2 = a_1^2 + a_2^2$

$\therefore |\vec{a}\times\hat{i}|^2 + |\vec{a}\times\hat{j}|^2 + |\vec{a}\times\hat{k}|^2$

$= 2\left(a_2^1 + a_2^2 + a_3^2\right) = 2|\vec{a}|^2 \; 2\times 1^2 = 2$

5. **(C)** Let $I = \int 5^{5^{5^x}}.5^{5^x}.5^x\, dx$

Put $5^{5^{5^x}} = t \Rightarrow 5^{5^{5^x}}\; 5^{5^x}\; 5^x (\log 5)^3\, dx = dt$

$\therefore\; I = \frac{1}{(\log 5)^3}\int 1\, dt = \frac{t}{(\log 5)^3} + c$

$\Rightarrow I = \frac{5^{5^{5^x}}}{(\log 5)^3} + C$

6. **(D)** Since, $\cos^{-1}\cos x = x$ if $x \in [0, \pi]$

$\therefore\; y = \cos^{-1}(\cos x)$

$\Rightarrow\; y = \cos^{-1}(-\cos(\pi + x))$

$\Rightarrow\; y = \pi - \cos^{-1}(\cos(\pi + x))$

$\Rightarrow\; y = \pi - (\pi + x)$

Since, $x = \frac{\pi}{4}$ and $\pi + x$ will lie between 0 and π.

Since, $\cos^{-1}\cos x = x$ if $x \in [0, \pi]$

$\Rightarrow\; y = -x$

$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{4}} = -1$

7. **(C)** By property, $\text{adj}\, A^T - (\text{adj}\, A)^T = O$ (null matrix)

8. **(B)** Since, $f(-x) = f(x)$

$\Rightarrow$ $f(x)$ is an even function and differential of an even function is an odd function.

9. **(C)** Let $f(x) = 18x^2$

$\Rightarrow \int_{-1}^{7} 18x^2\, dx = 18\left[\frac{x^3}{3}\right]^7$

$= 6((7)^3 - (-1)^3)$

$= 6(7)^3 - 6(-1)^3$

$\therefore$ Of the given options the value of $f(x)$ is $18x^2$.

10. **(B)** Given $\frac{dy}{dx} = y + \frac{y}{x}$ or $\frac{dy}{y} = \left(1 + \frac{1}{x}\right)dx$

On integration, we get

$\log|y| = x + \log|x| + c$ or

$\log|y| - \log|x| = x + c$

$\Rightarrow \log\left|\frac{y}{x}\right| = x + c \Rightarrow \left|\frac{y}{x}\right| = e^{x+C}$

$\Rightarrow \frac{y}{x} = \pm e^c e^x$ or $\frac{y}{x} = A\ e^x$

Where $A = \pm e^c$ is a constant.

$\therefore\ y = A\,x\,e^x$

$\therefore$ The solution $y = x\,e^x$ corresponds to $A = 1$.

11. **(C)** $\cos x + \sin x = \sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$

So, $f(x) = \left[\sqrt{2}\cos\left(x - \frac{\pi}{4}\right)\right]$

Since, $[x]$ is discontinuous at integral values of x.

Now, $\sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$ is an integer at

$$x = \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \frac{7\pi}{4}$$

12. **(A)**

13. **(B)** $\int x\log x(\log x - 1)dx$

$= \int \log x(x\log x - x)dx$

Note that

$\frac{d}{dx}(x\log x - x) = x\frac{1}{x} + \log x - 1 = \log x$

$\int (x\log x - x)^1(\log x)dx \quad |(f(x)^n f'(x)$ type

$= \frac{(x\log x - x)^2}{2} + C$

14. **(C)** Since, $(\vec{a}\times\vec{b})^2 = |\vec{a}\times\vec{b}|^2$

$\Rightarrow (\vec{a}\times\vec{b})^2 = \left||\vec{a}||\vec{b}|\sin\frac{\pi}{6}\right|^2$

$\therefore (\vec{a}\times\vec{b})^2 = 16\times4\times\frac{1}{4} = 16$

15. **(A)** $\bar{z} = \begin{vmatrix} 3-2i & 5+i & 7+3i \\ -i & -2i & 3i \\ 3+2i & 5-i & 7-3i \end{vmatrix}$

Applying $(R_1 \leftrightarrow R_3)$

$= \begin{vmatrix} 3+2i & 5-i & 7-3i \\ i & 2i & -3i \\ 3-2i & 5+i & 7+3i \end{vmatrix} = z$

$\therefore\ z = \bar{z}$, z is purely real.

16. **(D)** Let the two positive numbers be x and y

$\therefore\quad x + y = k$

Let $E = x^2 + y^2$

$\Rightarrow\quad E = x^2 + (k - x)^2$

for E to be minimum, we must have

$\Rightarrow \frac{dE}{dx} = 2x - 2(k - x) = 0$

$\Rightarrow x = \frac{k}{2}$

Now, $\frac{d^2E}{dx^2} = 4 > 0\ \forall x \in R$ throughout

$\therefore$ E attains its minimum at $x = \frac{k}{2}$

$\Rightarrow y = \frac{k}{2}$

17. **(C)** Given equation is

$e^x\,dy + (y\,e^x + 2x)\,dx = 0$

or $e^x\frac{dy}{dx} + ye^x + 2x = 0$

or $\frac{dy}{dx} + y = -2xe^{-x}$, which is linear in y.

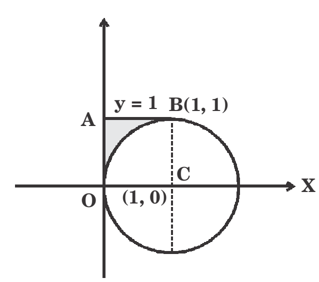
Here, I.F. $= e^{\int 1dx} = e^x$

and solution is given by

$y\,e^x = \int(-2xe^{-x})e^x dx + C$

or $y\,e^x = -x^2 + C$

18. (C)



Given curve is

$x^2 + y^2 - 2x = 0$

or $x^2 - 2x + 1 + y^2 = 1$

or $(x-1)^2 + y^2 = 1$

which is a circle with centre at (1, 0) and radius = 1

Required area = $\int_0^1 \left(1-\sqrt{1-(x-1)^2}\right)dx$

For the circle,

$y = \sqrt{1-(x-1)^2}$ in the first quadrant.

Required area = area of OABC $-\frac{1}{4}$ (area of the circle)

$= 1 \times 1 - \frac{1}{4}(\pi)$

$= \left(1 - \frac{\pi}{4}\right)$ sq.units

19. **(A)** If A and B are orthogonal

$\Rightarrow AA' = I, BB' = I$

Now, $(AB)(AB)' = AB(B'A')$

$\Rightarrow (AB)(AB)' = A(BB')A'$

$(AB)(AB)' = AI.A' = AA' = I$

$\therefore$ AB is also orthogonal

20. **(A)** $\sin[\cot^{-1}\{\tan(\cos^{-1}x)\}]$

$= \sin\left[\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right]$

$\left[\text{Since } \tan\left(\cos^{-1}x\right) = \frac{\sin\left(\cos^{-1}x\right)}{\cos\left(\cos^{-1}x\right)}\right.$

$\left. = \frac{\sqrt{1-x^2}}{x} = \frac{\sqrt{1-x^2}}{x}\right]$

$= \sin\theta$, where $\theta = \cot^{-1}\frac{\sqrt{1-x^2}}{x} \in (0, \pi)$

$= \frac{1}{\operatorname{cosec}\theta} = \frac{1}{\sqrt{1+\cot^2\theta}}$

$= \frac{1}{\sqrt{1+\frac{1-x^2}{x^2}}} = \sqrt{x^2} = |x|$

21. **(B)** Since, $f(x) = \sin x - bx + c$ is decreasing on $(-\infty, \infty)$, therefore,

$f'(x) = \cos x - b \le 0$ for all $x \in R$

$\Rightarrow \cos x \le b$ for all $x \in R$

$\therefore b \ge 1 \quad (\because |\cos x| \le 1)$

22. **(C)** $|\vec{a} + \vec{b}| < 1 \quad \Rightarrow |\vec{a} + \vec{b}|^2 < 1$

$\Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) < 1$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} < 1$

$\Rightarrow 1 + 1 + 2\vec{a} \cdot \vec{b} < 1$

$\Rightarrow \vec{a} \cdot \vec{b} < -\frac{1}{2}$

$\Rightarrow |\vec{a}|\,|\vec{b}| \cos\theta < -\frac{1}{2}$

$\Rightarrow 1 \times 1 \times \cos\theta < -\frac{1}{2}$

$\Rightarrow \cos\theta < -\frac{1}{2}$

$\Rightarrow -1 \le \cos\theta < -\frac{1}{2}$

$\Rightarrow \pi \ge \theta > \frac{2\pi}{3}$

23. **(D)** $\text{adj}(\text{adj } A) = |A|^{n-2}A$

24. **(B)** $f(2) = 4, f'(2) = 1$

Let $E = \lim_{x\to 2} \frac{xf(2) - 2f(x)}{x-2} \quad \left(\frac{0}{0} \text{ form}\right)$

By La Hospital Rule

$\Rightarrow f'(2) = \lim_{x\to 2} \frac{f(2) - 2f'(x)}{1}$

$\therefore f'(2) = \frac{f(2) - 2f'(2)}{1} = 4 - 2(1) = 2$

25. **(D)** Let $I = \int \frac{1}{1+\sin x}dx$

$\Rightarrow I = \int \frac{1}{1+\cos\left(\frac{\pi}{2} - x\right)}dx$

$= \int \frac{dx}{2\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}$

$\Rightarrow I = \int \frac{1}{2}\sec^2\left(\frac{\pi}{4} - \frac{x}{2}\right)dx$

$\Rightarrow I = -\tan\left(\frac{\pi}{4} - \frac{x}{2}\right) + c$

$\Rightarrow I = \tan\left(\frac{x}{2} - \frac{\pi}{4}\right) + c$

$\therefore a = -\frac{\pi}{4}; b = c$

26. **(C)** Here,

$$\underset{x\to -1^+}{\text{Lt}} f(x) = \underset{x\to -1^+}{\text{Lt}} \frac{|x+1|}{\tan^{-1}(x+1)}$$

$$= \underset{x\to -1^+}{\text{Lt}} \frac{x+1}{\tan^{-1}(x+1)} = 1$$

$\left[\text{Since } x \to -1^+ \Rightarrow x + 1 \to 0^+ \text{ and } \underset{t\to 0}{\text{Lt}} \frac{\tan t}{t} = 1 = \underset{s\to 0}{\text{Lt}} \frac{s}{\tan^{-1} s}\right]$

and $\underset{x\to -1^-}{\text{Lt}} f(x) = \underset{x\to -1^-}{\text{Lt}} \frac{|x+1|}{\tan^{-1}(x+1)}$

$$= \underset{x\to -1^-}{\text{Lt}} \frac{-(x+1)}{\tan^{-1}(x+1)} = -1$$

27. **(B)** $g(x) = \sin\left(\frac{\pi x}{2}\right)$ is a bijection.

28. **(C)** $\frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right) = \frac{1}{1+\frac{1}{x^2}}\left(-\frac{1}{x^2}\right) = -\frac{1}{1+x^2}$

$$\Rightarrow I = \int \frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)dx = -\int_{-1}^{1} \frac{dx}{1+x^2}$$

$$\Rightarrow I = -\tan^{-1} x\Big|_{-1}^{1}$$

$$\Rightarrow I = -\frac{\pi}{4} + \left(-\frac{\pi}{4}\right) = -\frac{\pi}{2}$$

Note:

$I = \tan^{-1}\left(\frac{1}{x}\right)\Bigg|_{-1}^{1} = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$ is incorrect.

since, the function $\tan^{-1}\left(\frac{1}{x}\right)$ is not an antiderivative of $\frac{d}{dx}\tan^{-1}\left(\frac{1}{x}\right)$ on the interval [–1, 1].

29. **(C)** Since, $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ (given)

$$\Rightarrow \cos^{-1}\frac{1}{\sqrt{(x^2+x)^2+1}} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1}\frac{1}{\sqrt{(x^2+x)^2+1}} = \frac{\pi}{2} - \sin^{-1}\sqrt{x^2+x+1}$$

$$\Rightarrow \cos^{-1}\frac{1}{\sqrt{(x^2+x)^2+1}} = \cos^{-1}\sqrt{x^2+x+1}$$

$$\Rightarrow \frac{1}{\sqrt{(x^2+x)^2+1}} = \sqrt{x^2+x+1}$$

$\Rightarrow 1 = (x^2 + x^2 + 1)\{(x^2 + x^2) + 1\}$

$\Rightarrow (x^2 + x)^3 + (x^2 + x)^2 + (x^2 + x) + 1 = 1$

$\Rightarrow (x^2 + x)\{(x^2 + x)^2 + (x^2 + x) + 1\} = 0$

$\Rightarrow x^2 + x = 0$

$\Rightarrow x = 0, -1$

30. **(D)** If e is the identity element w.r.t. 'o' then aoe = eoa = a for all a $\neq -1$

$\Rightarrow$ a + e + ae = a

$\Rightarrow$ e(1 + a) = 0

$\Rightarrow$ e = 0 (since a $\neq -1$, $\therefore$ a + 1 $\neq$ 0)

31. **(A)** Let at any instant of time t min, the depth of the wheat in the cylinder be h ft., then the volume V ft^3 of the wheat at that instant is given by

$$V = \pi(10)^2 h = 100\pi h$$

$$\Rightarrow \frac{dV}{dt} = 100\pi\frac{dh}{dt} \Rightarrow 314 = 100\pi\frac{dh}{dt}$$

(Since $\frac{dV}{dt} = 314$)

$$\Rightarrow \frac{dh}{dt} = \frac{314}{100(3.14)} \text{ (Taking } \pi = 3.14)$$

$\therefore$ The depth of the wheat in the cylinder is increasing at the rate of 1 cubic ft, per minute.

32. **(A)** The resultant force is given by

$$\vec{F} = 6\frac{(\hat{i} - 2\hat{j} + 2\hat{k})}{\sqrt{1+4+4}} + 7\frac{(2\hat{i} - 3\hat{j} - 6\hat{k})}{\sqrt{4+9+36}}$$

$\vec{F} = 4\hat{i} - 7\hat{j} - 2\hat{k}$

Now $\vec{d}$ = displacement

$\Rightarrow \vec{d} = \vec{PQ} = (5\hat{i} - \hat{j} + \hat{k}) - (2\hat{i} - \hat{j} - 3\hat{k})$

$\Rightarrow \vec{d} = (3\hat{i} + 4\hat{k})$

$\therefore$ Workdone = $\vec{F}\cdot\vec{d}$ = 12 + 0 – 8 = 4 unit

33. **(B)** If 'a' is the radius of the circle, then its centre is (–a, a).

Hence, the equation of the circle is

$(x + a)^2 + (y - a)^2 = a^2$ ..... (1)

or $x^2 + y^2 + 2ax - 2ay + a^2 = 0$ ..... (2)

Differentiating w.r.t. x, we get

$2x + 2yy_1 + 2a - 2ay_1 = 0$

$$\Rightarrow \quad \frac{x + yy_1}{y_1 - 1} = a$$

Substituting this value of a in (1), we get .... (2)

$$\left(x + \frac{x + yy_1}{y_1 - 1}\right)^2 + \left(y - \frac{x + yy_1}{y_1 - 1}\right)^2 = \left(\frac{x + yy_1}{y_1 - 1}\right)^2$$

or $(xy_1 + yy_1)^2 + (-y - x)^2 = (x + yy_1)^2$

or $(x + y)^2 (y_1^2 + 1) = (x + yy_1)^2$

34. (A) i) (a, b)∈ R ⇔ aRb ⇔ $bR^{-1}a$ ⇔ (b, a)∈ R^{-1}

∴ R^{-1} = {(b, a) : (a, b)∈ R}

ii) Let R = R^{-1}

Let (a, b)∈ R

∴ (a, b)∈ R^{-1}

(b, a)∈ R

∴ (a, b)∈ R ⇒ (b, a)∈ R

∴ R is symmetric

Conversely, let (a, b)∈ R

⇒ (b, a)∈ R

⇒ (a, b)∈ R^{-1} Since R ⊆ R^{-1}

Let (a, b)∈ R^{-1} Since (b, a)∈ R

⇒ (a, b)∈ R

R^{-1} ⊆ R

Combining, we get R = R^{-1}

35. (B) Required area = $2\int_0^1 \left(|x| - x^2\right)dx$

$$= 2\int_0^1 \left(x - x^2\right)dx$$

$$= 2\left[\left(\frac{x^2}{2}\right)_0^1 - \left(\frac{x^3}{3}\right)_0^1\right] = 2\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$= 2\left(\frac{1}{6}\right) = \frac{1}{3}$$

36. **(A)** The given system of equations has a unique solution if

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$$

⇒ k ≠ 0

37. **(A)** Let $\Delta = \begin{vmatrix} a_{21} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$= \begin{vmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{vmatrix}$$

= 0 + 1(0 + 1) – 1 (1) = 0

38. **(C)** Since, $\tan^{-1}(1 + x) + \tan^{-1}(1 - x) = \frac{\pi}{2}$

$$\Rightarrow \tan^{-1}(1+x) = \frac{\pi}{2} - \tan^{-1}(1-x)$$

$$\Rightarrow \tan^{-1}(1+x) = \cot^{-1}(1-x)$$

$$\Rightarrow \tan^{-1}(1+x) = \tan^{-1}\left(\frac{1}{1-x}\right)$$

$$\Rightarrow 1 + x = \frac{1}{1-x}$$

$$\Rightarrow 1 + x^2 = 1$$

∴ $x = 0$

39. **(D)** Here, $(fog)(x) = f(g(x))$

$= f(x - [x])$ (∵ $0 \le x - [x] < 1$)

$= [x - [x]] = 0$

40. **(B)** $\overrightarrow{AB} + \overrightarrow{AC}$ = P.V. of B = P.V. of C (Taking A as origin)

= 2(P.V. of mid-point of ([BC])

= 2 P.V of D

= $2\overrightarrow{AD}$

Physics

41. **(A)** $B = \mu_0\, n\, I = \mu_0 \frac{N}{l} I = 4\pi \times 10^{-7} \times \frac{120}{0.2} \times 2.5$

$= 1.885 \times 10^{-3}$ T

42. **(B)** As the image formed is erect and hence virtual, the magnification produced by the lens is positive i.e. m = + 4.

Also, f = + 20 cm

Now, $m = \frac{f}{u + f}$

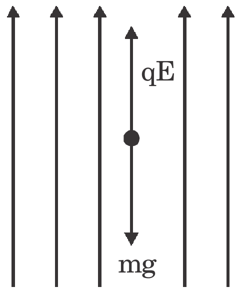
∴ $4 = \frac{20}{u + 20}$ or u + 20 = 5

or u = –15 cm

Again, $m = \frac{f - v}{f}$ ∴ $4 = \frac{20 - v}{20}$

or v = 20 – 80 = – 60 cm

43. **(B)** For the equilibrium of the proton as per the figure shown below.



qE = mg

$$E = \frac{mg}{q}$$

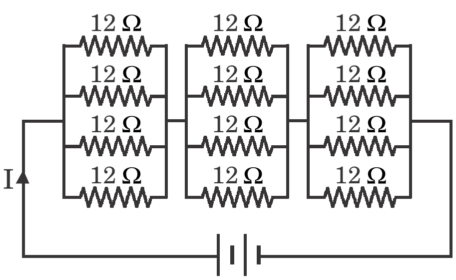
$$= \frac{1.67 \times 10^{-27} \times 9.8}{1.6 \times 10^{-19}}$$

$= 1.02 \times 10^{-7}$ N C^{-1}

The electric field is directed vertically upwards.

44. **(A)** The network of resistors connected to the battery of e.m.f. 9 V is shown below. Let I be the total current in the circuit. If R' is the effective resistance of the four resistors of 12 Ω each connected in parallel, then

$$\frac{1}{R'} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{4}{12} \quad \text{or} \quad R' = 3\ \Omega$$



Therefore, the effective resistance of the network of resistors,

$R = R' + R' + R' = 3 + 3 + 3 = 9\ \Omega$

The current in the circuit

$$I = \frac{E}{R} = \frac{9}{9} = 1\ A$$

Since all the four resistors are of the same resistance, same current will pass through the each resistor, Therefore,

the current through each resistor =

$$\frac{1}{4} I = \frac{1}{4} \times 1 = 0.25\ A$$

45. **(C)** $e\ B \times \pi\ r^2 \times v = \frac{1}{2}\ B\ r^2\ \omega$

Here, r = 200 cm = 2 m; B = 0.05 Wb m^{-2}

and ω = 60 rad s^{-1}

$$\therefore \quad e = \frac{1}{2} \times 0.05 \times (2)^2 \times 60 = 6\ V$$

46. **(A)** The average atomic mass of neon,

$$A = \frac{90.51 \times 19.99 + 0.27 \times 20.99 + 9.22 \times 21.99}{100}$$

= 20.18 a.m.u.

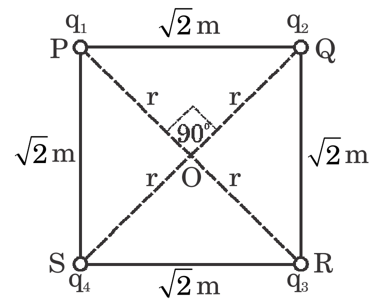
47. **(B)** A 0.1 m long bird will require frequency,

$$v = \frac{c}{\lambda} = \frac{3 \times 10^8}{0.1} = 3000\ \text{MHz}$$

It is very close to the popular radar frequency. Indeed, radars occasionally detect birds.

48. **(A)** Four charges q_1, q_2, q_3 and q_4 are placed at the four corners of the square PQRS as shown below.

Here,



$q_1 = 2\ \mu C = 2 \times 10^{-6}$ C;

$q_2 = -2\ \mu C = -2 \times 10^{-6}$ C;

$q_3 = -3\ \mu C = -3 \times 10^{-6}$ C;

$q_4 = 6\ \mu C = 6 \times 10^{-6}$ C;

and PQ = QR = RS = PS = $\sqrt{2}$ m

Let r be the distance of each charge from the centre O of the square.

Then, $\sqrt{r^2 + r^2} = \sqrt{2}$ or r = 1 m

Potential at point O due to charges at the four corners,

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right)$$

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{1}{r}(q_1 + q_2 + q_3 + q_4)$$

$$\frac{9 \times 10^9}{1}\left(2 \times 10^{-6} + \left(-2 \times 10^{-6}\right) + \left(-3 \times 10^{-6}\right) + 6 \times 10^{-6}\right)$$

$= 2.7 \times 10^4$ V

49. **(B)** Here, I = 10 A ; n= 100;

A = 40 cm × 20 cm = 800 cm^2

$= 8 \times 10^{-2}\ m^2$; B = 5 T ;

$\alpha = 60^o$ (angle between field and plane of coil)

Now, $\tau = n\ B\ I\ A \cos\alpha$

$= 100 \times 5 \times 10 \times 8 \times 10^{-2} \times \cos 60^o$

= 200 N m

50. **(C)** Statements (A), (B) and (D) are true of photons. Photons are electrically neutral as they are not deflected by electric and magnetic fields.

51. **(B)** When cells are joined in series,

net e.m.f. of cells,

E = 2.0 + 1.8 + 1.5 = 5.3 V

The total internal resistance of the cells in series,

r = 0.5 + 0.7 + 1 = 2.2 Ω

$$\therefore I = \frac{E}{R+r} = \frac{5.3}{4+2.2} = \frac{5.3}{6.2} = 0.854 \text{ A}$$

52. **(C)** Here, d = 0.02 cm ; D = 80 cm ;

$\lambda = 6000 \times 10^{-8}$ cm $= 6 \times 10^{-5}$ cm ;

For nth bright fringe, $y_n = \frac{nD\lambda}{d}$

For fifth maximum, n = 5

$$y_5 = \frac{5 \times 80 \times 6 \times 10^{-5}}{0.02} = 1.2 \text{ cm}$$

53. **(A)** Half-life = T = 12.5 years

Let the initial amount be N_0. This will be reduced to $\frac{N_0}{2}$ after 12.5 years. After 25 years it will be reduced to $\frac{N_0}{4}$. So, the fraction left undecayed is $\frac{1}{4}$.

54. **(B)** In descending order of wavelengths, the sequence of electromagnetic waves is; radio waves, microwaves, infrared radiation, visible light, ultraviolet rays, X-rays, gamma rays. Ultraviolet waves come after visible light.

55. **(D)** Here, $h = 6.62 \times 10^{-34}$ J s

de-Broglie wavelength of electron :

Here, $m_e = 9.1 \times 10^{-31}$ kg ; $v_e = 10^5$ m s^{-1}

$$\therefore \lambda_e = \frac{h}{m_e v_e} = \frac{6.62 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^5}$$

$= 7.27 \times 10^{-9}$ m

56. **(B)** Here, L = 1.0 H ; E_v = 110 V ; f = 70 Hz

Now $X_L = \omega L = 2\pi f L$

$$= 2 \times \frac{22}{7} \times 70 \times 1.0 = 440 \ \Omega$$

57. **(A)** De Broglie proposed that the wave length λ associated with a particle of momentum p is given as

$$\lambda = \frac{h}{p} = \frac{h}{m\upsilon}$$

where m is the mass of the particle and v its speed is known as the de Broglie relation and the wavelength λ of the matter wave is called de Broglie wave length.

58. **(C)** When the amplifiers are connected in series, the net voltage gain (A_v) is equal to the product of the gain of the individual amplifiers i.e.

$$A_v = A_v' \times A_v'' \quad \text{...... (i)}$$

$$\text{Also, } A_v = \frac{V_{out}}{V_{in}} \quad \text{...... (ii)}$$

From the equations (i) and (ii), we have

$$\frac{V_{out}}{V_{in}} = A_v' \times A_v''$$

or $V_{out} = V_{in} \times A_v' \times A_v''$

$= 0.01 \times 10 \times 30 = 3$ V

59. **(C)** R = 0.4 m, f = 10 MHz, K.E. = ? m = 10 $\times 10^6$ kg

Maximum K.E. = $2\pi^2 f^2 R^2 m$ =

$2 \times (3.14)^2 \times (10^7)^2 \times (0.4)^2 \times 1.67 \times 10^{-27}$

$= 5.263 \times 10^{-13}$ J

60. **(B)** $\frac{1}{v} + \frac{1}{9} = \frac{1}{10}$

i.e., v = – 90 cm

Magnitude of magnification =

$\frac{90}{9}$ = 10 cm

61. **(C)** Here, A = 90 cm^2 = 90×10^{-4} m^2 ;

d = 2.5 mm = 2.5×10^{-3} m ; V = 400 volt

$$\text{Now, } C = \frac{\varepsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 90 \times 10^{-4}}{2.5 \times 10^{-3}}$$

$= 3.187 \times 10^{-11}$ F

Now,

$$W = \frac{1}{2} C V^2 = \frac{1}{2} \times 3.187 \times 10^{-11} \times (400)^2$$

$= 2.55 \times 10^{-6}$ J

62. **(C)** $\lambda_{vacuum} = \frac{c}{v} = \frac{3 \times 10^8}{5 \times 10^{14}} = 6 \times 10^{-7}$ m

63. **(D)** K.E. $= \frac{e^4 m}{8\varepsilon_0^2 n^2 h^2} = 13.6$ eV,

$$P.E. = \frac{-Ze^2}{8\pi\varepsilon_0 r} = -2 \times K.E. = -27.2 \text{ eV}$$

64. **(C)** Here, $R = 10\ \Omega$; $L = 2$ H ; $C = 25\ \mu F$

$= 25 \times 10^{-6}$ F;

$E_v = 200$ V

When the frequency of the a.c. supply equals the natural frequency of the circuit,

$X_L = X_C$ i.e. $Z = R$

$\therefore P_{av} = E_v I_v$

$$= E_v \times \frac{E_v}{R} = \frac{200 \times 200}{10} = 4000 \text{ W}$$

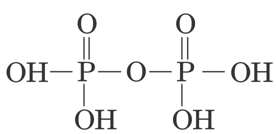
65. **(C)** Let E be the e.m.f of the battery. The charging current I is opposed by the e.m.f. of the battery. According to Ohm's law.

$V - E = I r$

or $E = V - I r = 12.5 - 0.5 \times 1 = 12$ V

Chemistry

66. **(D)** Pyrophosphoric acid ($H_4P_2O_7$)



contains four P–OH groups and hence, it is a tetrabasic acid.

67. **(C)** $Al^{3+} + 3e^- \rightarrow Al$

$Cu^{2+} + 2e^- \rightarrow Cu$

$Na^+ + e^- \rightarrow Na$

Thus, 1 F will deposit $\frac{1}{3}$ mol, Al, $\frac{1}{2}$ mol, Cu and 1 mol Na, i.e., moles deposited are in the ratio

$\frac{1}{3} : \frac{1}{2} : 1$ i.e., 2 : 3 : 6 or 1 : 1.5 : 3.

68. **(A)** It is a disubstituted complex and will show geometrical isomerism.

69. **(D)** When o- or p-phenolsulphonic acid is treated with HNO_3 ,nitration occurs at o, p-positions with simultaneous replacement of SO_3H group by NO_2 group to give ultimately 2, 4 6-trinitrophenol.

70. **(D)** Aldehydes which do not have an α-hydrogen atom (e.g., HCHO and C_6H_5CHO) undergo self-oxidation and reduction reaction on treatment with concentrated alkali solution. One of the molecules gets reduced to alcohol and the other gets oxidised to the acid.

$$2C_6H_5CHO \xrightarrow{NaOH} C_6H_5CH_2OH + C_6H_5COO^-Na^+$$

71. **(D)** The reagent reacts with H_2O

$RMgX + H_2O \rightarrow RH + Mg(OH)X$

72. **(B)** 100 mol NaCl is doped with $SrCl_2$ $= 2 \times 10^{-3}$ mol

$\therefore$ 1 mol NaCl is doped with $SrCl_2$ $= 2 \times 10^{-5}$ mol

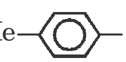
As each Sr^{2+} ion creates one cation vacancy, therefore cation vacancies = 2×10^{-5} mol/mol of NaCl

$= 2 \times 10^{-5} \times 6.02 \times 10^{23}\ mol^{-1}$

$= 12.04 \times 10^{18}\ mol^{-1}$

73. **(D)** Order may or may not be equal to molecularity.

74. **(A)** ClF_3 where Cl is sp^3d hybridised has a T-shape structure with two lone pairs of electrons on Cl atom.

75. **(B)** Only 1° amines (i.e., Me—  —NH_2 in the present case) gives positive carbylamine reaction.

76. **(C)** Since, the compound is optically active and does not rotate the plane of polarized light, therefore, the compound must be a racemic mixture.

77. **(B)** Xanthoproteic test involves heating of a protein with conc. HNO_3 when yellow colour is obtained.

78. **(C)** Aldehydes are easily oxidised to the corresponding acids by Tollens' reagent while all others are strong oxidising agents and hence, cleave the molecule at the site of the double bond yielding a mixture of products.

$$CH_3CH = CHCHO \xrightarrow{\text{Tollens' reagent}} CH_3CH = CHCOOH$$

79. **(B)** In the smelting process, the ore copper pyrites is converted into FeO which then combines with silica to give $FeSiO_3$ as a slag.

$FeO + SiO_2 \rightarrow FeSiO_3$

80. **(A)** Reactions I and II give 2-propanol, i.e.,

I. $CH_3CH = CH_2 + H_2O \xrightarrow[\text{Mark.add.}]{H^+} CH_3CHOHCH_3$
2-Propanol

II. $CH_3CHO \xrightarrow[\text{(ii) } H^+/H_2O]{\text{(i) } CH_3MgI} CH_3\text{–CHOH–}CH_3$
2-Propanol

In contrast, reaction III gives 1-propanol and IV gives 1, 2-propanediol.

III. $CH_2O \xrightarrow[\text{(ii) } H^+/H_2O]{\text{(i) } C_2H_5MgI} C_2H_5CH_2OH$
1-Propanol

IV. $CH_2 = CHCH_3 \xrightarrow[KMnO_4]{\text{Neutral}} CH_2(OH) - CH(OH) - CH_3$
1, 2 -Propanediol

81. **(D)** Bredig's arc method cannot be used for the preparation of colloidal sol of Na as it reacts with water vigorously.

82. **(A)** $He–O_2$ (80% : 20%) mixture is used by deep sea divers for artificial respiration. Because of low intermolecular forces in He, it is much less soluble in aqueous solutions (as compared to N_2) such as blood and does not cause "caisson sickness" or "bends" by bubbling out of blood when the worker moves from high pressure (while in deep sea) to atmospheric pressure.

83. **(A)** For a bcc unit cell

$$r = \frac{\sqrt{3}}{4}a = \frac{1.732}{4} \times 4.28\,\mathring{A} = 1.86\,\mathring{A}$$

84. **(A)** Van-Arkel method involves converting the metal to a volatile stable compound.

$$\underset{\text{Impure}}{Ti} + 2\,I_2 \xrightarrow{500\text{ K}} TiI_4 \text{ (volatile stable) ;}$$

$$TiI_4 \xrightarrow{1700\text{ K}} \underset{\text{Pure}}{Ti} + 2\,I_2$$

85. **(B)** Magnetic moment, $\mu_{eff} = 3.87$ B.M. corresponds to the number of unpaired electrons n = 3 by applying the formula

$$\mu_{eff} = \sqrt{n(n+2)} \text{ B.M.}$$

For n = 1, μ = 1.73 B.M.; for n = 2, μ = 2.83 B.M. ; for n = 3, μ = 3.87 B.M. and so on.

86. **(D)** Copper ferrocyanide ppt. acts as a semipermeable membrane.

87. **(C)** Reducing character of hydrides increases down the group.

88. **(C)** $C_6H_5N_2Cl + KCN \xrightarrow{\Delta} C_6H_5CN + N_2 + KCl$

89. **(C)** Wilkinson's catalyst is used for the hydrogenation of alkenes.

90. **(A)** $HCOOH \xrightarrow{P_2O_5} CO + H_2O$

❖❖❖