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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION

Paper Code: UN421

Solutions for Class : 12 (PCM)

Mathematics

1. **(D)** Let 'r' be the radius and θ be the angle of a sector.

$$\therefore\ 2r + r\theta = k \Rightarrow r = \frac{k}{\theta+2}$$

$$\frac{dA}{d\theta} = \frac{k^2}{2}\left[\frac{(\theta+2)^2\ 1-\theta,\ 2(\theta+2)}{(\theta+2)^4}\right]$$

$$= \frac{k^2}{2}\frac{(2-\theta)}{(\theta+2)^3}$$

$$\frac{dA}{d\theta} = 0$$

$$\Rightarrow\ 2 - \theta = 0$$

$$\therefore\ \boldsymbol{\theta = 2}$$

2. **(A)** $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{bmatrix}$ is singular

$$\Rightarrow \begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 8(7\lambda - 16) + 6(-6\lambda + 8) + 2(24 - 14) = 0$$

$$\Rightarrow 56\lambda - 128 - 36\lambda + 48 + 20 = 0$$

$$\Rightarrow 20\lambda - 60 = 0 \Rightarrow \boldsymbol{\lambda = 3}.$$

3. **(A)** $\displaystyle\int \frac{1}{x}\frac{\sqrt{x-1}}{x+1}dx = \int \frac{1}{x}\frac{x-1}{\sqrt{x^2-1}}dx$

$$= \int\left[\frac{1}{\sqrt{x^2-1}} - \frac{1}{x\sqrt{x^2-1}}\right]dx$$

$$= \mathbf{Cosh^{-1}}\ \boldsymbol{x} - \mathbf{Sec^{-1}}\ \boldsymbol{x} + \mathbf{c}$$

4. **(A)** $\log k = \log \underset{x \to 0}{\text{Lt}} f(x) = \log \underset{x \to 0}{\text{Lt}} (\cos x)^{1/x}$

$$= \underset{x \to 0}{\text{Lt}} \frac{\log \cos x}{x} = \underset{x \to 0}{\text{Lt}} \{-\tan x\} = 0$$

$$\Rightarrow \mathbf{k = 1}$$

5. **(B)** $AB = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 5 & 9 & 13 \\ -1 & 2 & 4 \\ -2 & 2 & 4 \end{bmatrix}$$

6. **(B)** Given curve is $(y-2)^2 = x - 1$

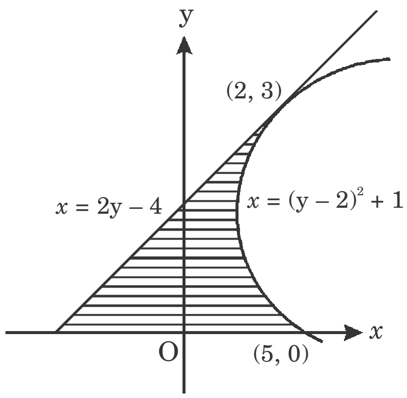
$$\Rightarrow \frac{d}{dx}\{(y-2)^2\} = \frac{d}{dx}\{x-1\}$$

$$\Rightarrow 2(y-2)\frac{dy}{dx} = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2(y-2)} \Rightarrow \left(\frac{dy}{dx}\right)_{(2,3)} = \frac{1}{2}$$

The equation of the tangent to the parabola at (2, 3) is

$$y - 3 = \frac{1}{2}(x-2) \Rightarrow x - 2y + 4 = 0$$



The area of the bounded region

$$= \int_0^3 [(y-2)^2 + 1 - (2y-4)]dy$$

$$= \int_0^3 (y^2 - 6y + 9)\,dy$$

$$= \int_0^3 (y-3)^2\,dy = \left[\frac{(y-3)^3}{3}\right]_0^3 = \mathbf{9}$$

7. **(C)** Equation of the tangent at $(1, \sqrt{3})$ is

$x + \sqrt{3}\,y - 4 = 0 \rightarrow (1)$

Equation of the normal at $(1, \sqrt{3})$ is

$\sqrt{3}\,x - y = 0 \rightarrow (2)$,

Equation of the x - axis is $y = 0 \rightarrow (3)$

Point of intersection of (1) and (2) is $(1, \sqrt{3})$

Point of intersection of (1) and (3) is (4, 0)

Point of intersection of (2) and (3) is (0, 0).

Area of the triangle

$$= \frac{1}{2}\left|0 - 4\sqrt{3}\right| = \mathbf{2\sqrt{3}\ sq.units}.$$

8. **(B)** $\frac{xy}{zr}\cdot\frac{yz}{xr} = \frac{y^2}{r^2} = \frac{y^2}{x^2+y^2+r^2} < 1$

$$\text{G.E.} = \text{Tan}^{-1}\left(\frac{\frac{xy}{zr}+\frac{yz}{xr}}{1-\frac{xy}{zr}\cdot\frac{yz}{xr}}\right) + \text{Tan}^{-1}\left(\frac{xz}{yr}\right)$$

$$= \text{Tan}^{-1}\left[\frac{\frac{y}{r}\left(\frac{x^2+z^2}{xz}\right)}{\frac{(r^2-y^2)}{r^2}}\right] + \text{Tan}^{-1}\left(\frac{xz}{yr}\right)$$

$$= \text{Tan}^{-1}\left[\frac{yr.\left(\frac{x^2+z^2}{xz}\right)}{x^2+z^2}\right] + \text{Tan}^{-1}\left(\frac{xz}{yr}\right)$$

$$= \text{Tan}^{-1}\left(\frac{yr}{xz}\right) + \cot^{-1}\left(\frac{yr}{xz}\right) = \frac{\pi}{\mathbf{2}}$$

9. **(C)** $S = \{(x, y) : y = x + 1, 0 < x < 2\} \Rightarrow$ S is not symmetric

$T = \{(x, y) : x - y \text{ is an integer}\} \Rightarrow$ clearly T is an equivalence

10. **(D)** Given $x = 2r$.

Surface of the cube $A = 6x^2 \Rightarrow \frac{dA}{dt} = 12x\frac{dx}{dt}$.

Surface of the sphere S

$= 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r\frac{dr}{dt}$.

Ratio $= 12x\frac{dx}{dt} : 8\pi r\frac{dr}{dt} = 12\,(2r) : 8\pi r = \mathbf{3 : \pi}$

11. **(A)** $k(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$

$$= \begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$$

$$= \begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix}$$

$= [(1-\alpha)(\alpha-\beta)(\beta-1)]^2$

$= (1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$.

$\therefore$ **k = 1**.

12. **(D)** Let $I = \int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}\,dx$

$$= \int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\cot x}+\sqrt{\tan x}}\,dx$$

$$\Rightarrow 2I = \int_0^{\pi/2} dx = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{\mathbf{4}}$$

13. **(B)** $\frac{k}{2} = f(0) = \lim_{x \to 0} \frac{\sin 3x}{x} = 3 \Rightarrow \mathbf{k = 6}$

14. **(B)** A satisfies $x^2 + 4x - p = 0$

$\Rightarrow A^2 + 4A - pI = 0$

$$\Rightarrow pI = A^2 + 4A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}^2 + 4\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix} + \begin{bmatrix} -32 & -12 \\ 8 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} 42 & 0 \\ 0 & 42 \end{bmatrix} = 42I$$

$\Rightarrow$ p = **42**

15. **(A)** Put $x = \tan\theta$. Then $dx = \sec^2\theta \, d\theta$;

$x = 0, \infty \Rightarrow \theta = 0, \pi/2$

$$\int_0^{\infty} \frac{\log(1+x^2)}{1+x^2} dx$$

$$= \int_0^{\pi/2} \frac{\log(1+\tan^2\theta)}{1+\tan^2\theta} \sec^2\theta \, d\theta$$

$$= \int_0^{\pi/2} \log(\sec^2\theta) d\theta$$

$$= 2\int_0^{\pi/2} \log(\sec\theta) d\theta$$

$$= -2\int_0^{\pi/2} \log(\cos\theta) d\theta$$

$$= -2\left[-\frac{\pi}{2}\log 2\right]$$

$= \boldsymbol{\pi \log 2}$

16. **(D)** The given curve is

$y = 2x - x^2$ or $y = -(x^2 - 2x + 1) + 1$

or $y - 1 = -(x - 1)^2$, which is a downward parabola. It meets the x-axis where

$y = 0 \Rightarrow 2x - x^2 = 0 \Rightarrow x = 0, 2$

$\therefore$ Required area $= \int_0^2 |y| dx = \int_0^2 (2x - x^2) dx$

$$= \left[2\left(\frac{x^2}{2}\right) - \frac{x^3}{3}\right]_0^2$$

$$= \left(2^2 - \frac{2^3}{3}\right) = 4 - \frac{8}{3} = \mathbf{\frac{4}{3}}$$

17. **(B)** $h(x) = f[g(x)] = e^{\sin^{-1}x}$

$$\Rightarrow h'(x) = e^{\sin^{-1}x} \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{h'(x)}{h(x)} = \mathbf{\frac{1}{\sqrt{1-x^2}}}$$

18. **(D)** Since $x - x + \sqrt{2} = \sqrt{2}$ which is an irrational number so x R x for all $x \in$ R. Hence R is reflexive.

R is not symmetric as $(\sqrt{2}, 1) \in$ R but $(1, \sqrt{2}) \notin$ R.

Again R is not transitive since $(\sqrt{2}, 1) \in$ R and $(1, 2\sqrt{2}) \in$ R but $(\sqrt{2}, 2\sqrt{2}) \notin$ R.

19. **(D)** $y = a^2 \sec^2\theta + b^2 \operatorname{cosec}^2\theta \Rightarrow \frac{dy}{d\theta} = 2a^2 \sec\theta$.

$\sec\theta \tan\theta - 2b^2 \operatorname{cosec}\theta . \operatorname{cosec}\theta \cot\theta$

$= 2a^2 \sec^2\theta \tan\theta - 2b^2 \operatorname{cosec}^2\theta \cot\theta$

$\frac{dy}{d\theta} = 0 \Rightarrow 2a^2 \sec^2\theta \tan\theta - 2b^2 \operatorname{cosec}^2\theta \cot\theta = 0 \Rightarrow a^2 \sec^2\theta \tan\theta = b^2 \operatorname{cosec}^2\theta \cot\theta$

$\Rightarrow \tan^4\theta = b^2/a^2 \Rightarrow \tan^2\theta = b/a$

Minimum value of $y = a^2(1 + \tan^2\theta) + b^2(1 + \cot^2\theta) = a^2(1 + b/a) + b^2(1 + a/b)$

$= a^2 + ab + b^2 + ab$

$= \mathbf{(a + b)^2}$

20. **(D)** $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is singular $\Rightarrow$ Det = 0

$\Rightarrow \cos^2\theta - \sin^2\theta = 0$

$\Rightarrow \cos 2\theta = 0 \Rightarrow 2\theta = (2n + 1)\frac{\pi}{2}, n \in z$

$\Rightarrow \boldsymbol{\theta = (2n + 1)\frac{\pi}{4}}, n \in z$

21. **(B)** $f(x) = x^2 - 4x + 5 = (x - 2)^2 + 1$

$x \in (2, \infty) \Rightarrow x \geq 2 \Rightarrow x - 2 \geq 0$

$\Rightarrow (x - 2)^2 + 1 \geq 1 \Rightarrow$ B = **[1, ∞)**.

22. **(D)** m[–3, 4] = n[4 – 3] = [10 – n]

$\Rightarrow$ –3m + 4n = 10

4m – 3n = –11

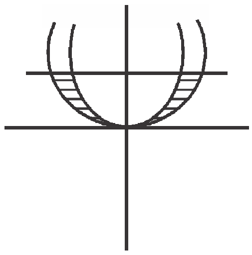
$\Rightarrow$ m = 2, n = 1

$\therefore$ 3m + 7n = –6 + 7 = **1**

23. **(A)** Required area = $\int_0^2 (x_1 - x_2)\,dy$

$= 2\int_0^2 (3\sqrt{y} - \frac{1}{2}\sqrt{y})\,dy$

$= 2\int_0^2 \frac{5}{2}\sqrt{y}\,dy$



$= 5\left[\frac{y^{3/2}}{3/2}\right]_0^2 = \mathbf{\frac{20\sqrt{2}}{3}}$

24. **(C)**

25. **(B)** $x = \frac{1-\sqrt{y}}{1+\sqrt{y}} \Rightarrow \frac{1+x}{1-x} = \frac{1+\sqrt{y}+1-\sqrt{y}}{1+\sqrt{y}-1+\sqrt{y}} = \frac{1}{\sqrt{y}}$

$\Rightarrow y = \left[\frac{1-x}{1+x}\right]^2$

$\Rightarrow \frac{dy}{dx} = 2\left[\frac{1-x}{1+x}\right] \times \frac{(1+x)(-1)-(1-x)1}{(1+x)^2}$

$= \frac{2(1-x)}{(1+x)3}(-1-x-1+x)$

$= \mathbf{\frac{4(x-1)}{(1+x)^3}}$

26. **(B)** $\cos^{-1}\frac{3}{5} = A, \sin^{-1}\frac{4}{5} = B$

$\Rightarrow \cos A = \frac{3}{5}, \sin B = \frac{4}{5}$

$\Rightarrow \sin A = \frac{4}{5}, \cos B = \frac{3}{5}$

$\cos^{-1}\frac{3}{5} - \sin^{-1}\frac{4}{5} = \cos^{-1}x$

$\Rightarrow A - B = \cos^{-1}x$

$\therefore x = \cos(A - B) = \cos A \cos B + \sin A \sin B$

$= \frac{3}{5}\times\frac{3}{5} + \frac{4}{5}\times\frac{4}{5} = \frac{9}{25} + \frac{16}{25} = \frac{25}{25} = \mathbf{1}$

27. **(A)** $|a - a| = 0 < 1.$

$\therefore$ a R a $\forall$ a $\in$ R. $\therefore$ R is reflexive

Again a R b $\Rightarrow |a - b| \le 1 \Rightarrow |b - a| \le 1$

$\Rightarrow$ b R a.

$\therefore$ R is symmetric

Again 1R $\frac{1}{2}$ and $\frac{1}{2}$ R1 but $\frac{1}{2} \ne 1$.

$\therefore$ R is not anti-symmetric

Further, 1R2 and 2R3 but 1 R 3

$[\because |1 - 3| = 2 > 1]$.

$\therefore$ R is not transitive

28. **(D)** $y^2 = 4x + 4$, $y^2 = 36(9 - x) \Rightarrow 4x + 4 = 36(9 - x) \Rightarrow 4x = 320 \Rightarrow x = 8; y = 6$

$y^2 = 4x + 4 \Rightarrow 2y\frac{dy}{dx} = 4$

$\Rightarrow m_1 = \frac{2}{y} = \frac{2}{6} = \frac{1}{3}$

$y^2 = 36(9 - x) \Rightarrow 2y\frac{dy}{dx} = -36$

$\Rightarrow m_2 = \frac{18}{y} = -\frac{18}{y} = -3$

$m_1 m_2 = \frac{1}{3}\times(-3) = -1$

$\Rightarrow$ Required angle = **90°**

29. **(D)** $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$

$(C_1 + C_2 + C_3)$

$\Rightarrow \begin{vmatrix} a+b+c-x & c & b \\ a+b+c-x & b-x & a \\ a+b+c-x & a & c-x \end{vmatrix} = 0$

$\Rightarrow (a + b + c - x)\begin{vmatrix} 1 & c & b \\ 1 & b-x & a \\ 1 & a & c-x \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 1 & c & b \\ 1 & b-x & a \\ 1 & a & c-x \end{vmatrix} = 0$ or $a + b + c - x = 0$

$\Rightarrow x$ = a + b + c or (bc – bx – cx + x^2 – a^2) – c(c – x – a) + b(a – b + x) = 0

$\Rightarrow x = 0$ or $x^2 + bc - bx - cx - a^2 - c^2 + cx + ac + ab - b^2 + bx = 0$

$\Rightarrow x = 0$ or $x^2 = a^2 + b^2 + c^2 - ab - bc - ca =$

$a^2 + b^2 + c^2 - \frac{1}{2}(a + b + c)^2 + \frac{1}{2}(a^2 + b^2 + c^2)$

$= \frac{3}{2}(a^2 + b^2 + c^2)$

$\Rightarrow x = 0, \pm\sqrt{\mathbf{\frac{3}{2}(a^2 + b^2 + c^2)}}$

30. **(D)** If f(x) = x^3, then f : R $\to$ R is a function.

Let $x_1, x_2 \in$ R and $f(x_1) = f(x_2)$.

Now $f(x_1) = f(x_2) \Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2$.

$\therefore$ f : R $\to$ R is one one.

Let y $\in$ R. Then $x = \sqrt[3]{y} \in$ R.

Now $f(x) = x^3 = (\sqrt[3]{y})^3 = y \Rightarrow$ f : R $\to$ R is onto.

$\therefore$ f : R $\to$ R is **one one onto**.

31. **(D)** $(a + b + c)^2 = 0$

$\Rightarrow a^2 + b^2 + c^2 + 2a.b + 2b.c + 2c.a = 0$

$\Rightarrow |a|^2 + |b|^2 + |c|^2 + 2(a.b + b.c + c.a) = 0$

$\Rightarrow 1 + 1 + 2(a.b + b.c + c.a) = 0$

$\Rightarrow a.b + b.c + c.a = \mathbf{\frac{-3}{2}}$

32. **(A)** $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$A = \begin{bmatrix} a^2 + b^2 & 2ab \\ 2ab & a^2 + b^2 \end{bmatrix}$

$= \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$ (Given)

$\Rightarrow$ **$\alpha = a^2 + b^2$, $\beta = 2ab$**

33. **(B)** $I = \int\left[\sum_{r=0}^{\infty} \frac{x^r 2^r}{r!}\right]dx$

$= \int e^{2x}\,dx = \frac{1}{2}e^{2x} + c$

34. **(B)** The equation of the family of parabolas with vertex at (0, –1) and having axis along the y-axis is

$x^2 = 4a\,(y + 1) \Rightarrow 2x = 4a\frac{dy}{dx} \Rightarrow 4a = \frac{2x}{y'}$

$\therefore x^2 = 4a\,(y + 1) \Rightarrow x^2 = \frac{2x}{y'}(y + 1)$

$\Rightarrow xy' = 2(y + 1)$

$\Rightarrow xy' - 2y - 2 = 0$

$\therefore$ The required differential equation is

$xy' - 2y - 2 = 0$.

35. **(B)** For f to be continuous at $x = \frac{\pi}{4}$,

we must have $f\left(\frac{\pi}{4}\right) = \underset{x \to \pi/4}{\text{Lt}} f(x)$

$= \underset{x \to x/4}{\text{Lt}} [\tan(\pi/4 - x)/\cot 2x]$

$= \underset{x \to \pi/4}{\text{Lt}} \left[\frac{1 - \tan x}{1 + \tan x} \cdot \frac{2\tan x}{1 - \tan^2 x}\right]$

$= \underset{x \to \pi/4}{\text{Lt}} \left[\frac{2\tan x}{(1 + \tan x)^2}\right] = \mathbf{\frac{1}{2}}$

36. **(B)** $\text{Tan}^{-1}(1/7) = \alpha$, $\text{Tan}^{-1}(1/3) = \beta$

$\Rightarrow \tan\alpha = 1/7, \tan\beta = 1/3$

$\cos 2\alpha = \frac{1 - \tan^2\alpha}{1 + \tan^2\alpha} = \frac{1 - 1/49}{1 + 1/49} = \frac{48}{50} = \frac{24}{25}$

$\tan 2\beta = \frac{2\tan\beta}{1 - \tan^2\beta} = \frac{2(1/3)}{1 - (1/9)} = \frac{2/3}{8/9} = \frac{3}{4}$

$\sin 4\beta = \frac{2\tan 2\beta}{1 + \tan^2 2\beta} = \frac{2(3/4)}{1 + (3/4)^2} = \frac{24}{25}$

$\therefore \cos 2\alpha =$ **$\sin 4\beta$**

37. **(A)** $\frac{dy}{dx} = \frac{(x+5)1 - (x+9)1}{(x+5)^2} = \frac{-4}{(x+5)^2}$

Equation of tangent at $P(x_1, y_1)$ is $y - y_1$

$= \frac{-4}{(x_1 + 5)^2}(x - x_1)$

It passes through the origin

$\Rightarrow y_1 = \frac{-4}{(x_1 + 5)^2}$

P lies on the curve

$\Rightarrow y_1 = \frac{x_1+9}{x_1+5} \Rightarrow \frac{-4}{(x_1+5)^2}\frac{x_1+9}{x_1+5}$

$\Rightarrow (x_1+9)(x_1+5)+4x_1 = 0$

$\Rightarrow x_1^2 + 18x_1 + 45 = 0$

$\Rightarrow x_1 = -3 \text{ or } -15 \Rightarrow y_1 = 3 \text{ or } 3/5$

Equation of the tangent at (–3, 3) is y – 3 = –(x + 3) $\Rightarrow$ **x + y = 0**

38. **(A)** $\int\left(\frac{2-\sin 2x}{1-\cos 2x}\right)e^x\,dx$

$= \int\left(\frac{2-2\sin x\cos x}{2\sin^2 x}\right)e^x\,dx$

$= \int(\text{cosec}^2 x - \cot x)e^x\,dx$

$= \mathbf{-e^x \cot x + c}$.

39. **(A)** $f(-x) = \frac{\cos(-x)\sin(-x)}{\tan(-x)+\cot(-x)} = \frac{\cos x \sin x}{\tan x + \cot x}$

$= f(x) \Rightarrow f(x)$ is even.

40. **(C)** If a = xi + yj + 2k then a × i + 2a – 5j = 0

$\Rightarrow$ –yk + 2j + 2xi + 2yj + 2zk – 5j = 0

$\Rightarrow$ 2xi + (z + 2y – 5)j + (–y + 2z)k = 0

$\Rightarrow$ 2x = 0, 2y + z – 5 = 0, –y + 2z = 0

$\Rightarrow$ x = 0, y = 2z, 2y + z – 5 = 0

$\Rightarrow$ x = 0, y = 2, z = 1

$\therefore$ **a = 2j + k**

Physics

41. **(B)** Magnetic moment = M = 0.9 Am^2

Volume of the magnet = Area of cross-section × length,

$V = 10^{-4} \times 0.05 = 5\times10^{-6}\ m^2$

Magnetisation = $I = \frac{M}{V}$

$= \frac{0.9}{5\times10^6}$

$= 1.8\times10^5$ A/m

42. **(C)** $q_1 = 1.6\times10^{-19}$ C

$q_2 = 1.6\times10^{-19}$ C (The magnitudes are equal)

$r = 5.3\times10^{-11}$ m

$\frac{1}{4\pi\varepsilon_0} = 9\times10^9\ Nm^2\ C^{-2}$

Force of attraction between the electron and proton

$F = \frac{1}{4\pi\varepsilon_0}\times\frac{q_1\times q_2}{r^2}$

$= \frac{9\times10^9\times(1.6\times10^{-19})^2}{(5.3\times10^{-11})^2}$

$= 8.202\times10^{-8}$ N.

43. **(C)** Here, $\omega = 0.031$; $\mu_r = 1.645$;

$\mu_b = 1.665$

It μ is refractive index of glass for yellow ray, then

$\omega = \frac{\mu_b - \mu_r}{\mu - 1}$

or $\mu = 1 + \frac{\mu_b - \mu_r}{\omega}$

$= 1 + \frac{1.665 - 1.645}{0.031}$

= 1 + 0.645 = 1.645

44. **(D)** Internal resistance of a cell depends on the given factors.

45. **(A)** The resistance of the coil is

$R = \frac{6\ V}{12\ A} = 0.5\,\Omega$.

In the AC circuit, the current is in phase with the emf. This means that the net reactance of the circuit is zero. The impedance is equal to the resistance, i.e., $Z = 0.5\,\Omega$.

The rms current = $\frac{\text{rms voltage}}{Z}$

$= \frac{6\ V}{0.5\Omega} = 12$ A.

46. **(A)** Since the nucleus A_3 is ${}_{69}A_3^{172}$ and β-particle is an electron (${}_{-1}e^0$) and α-particle is ${}_2He^4$ (nucleus of helium), the given decay series may be represented as

$${}_{72}A^{180} \xrightarrow{\beta} {}_{73}A_1^{180} \xrightarrow{\alpha} {}_{71}A_2^{176} \xrightarrow{\alpha} {}_{69}A_3^{172}$$

i.e. the mass number and atomic number of the radioactive nucleus A are 180 and 72 respectively.

47. **(A)** From Maxwell's electromagnetic theory, the electromagnetic wave propagation contains electric and magnetic fields vibrating perpendicularly to each other. Hence, changing of electric field gives rise to magnetic field.

48. **(C)** Common potential,

$$V = \frac{\text{Total charge}}{\text{Total capacitance}}$$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$$= \frac{0 + CV_0}{KC + C}$$

$$= \frac{CV_0}{C(1+K)}$$

$$V = \frac{V_0}{1+K}$$

$$\therefore\ K + 1 = \frac{V_0}{V}$$

$$\Rightarrow\ K = \frac{V_0}{V} - 1$$

$$= \frac{V_0 - V}{V}$$

49. **(C)** Magnetic field at a distance r = $B = \frac{\mu_0 I}{2\pi r}$

Let the resultant magnetic field be zero at a distance r from the wire.

$$r = \frac{\mu_0 I}{2\pi B},\ I = 10\ \text{A},\ B = 5\times 10^{-4}\ \text{T},$$

$$r = \frac{4\pi\times 10^{-7}\times 10}{2\pi\times 5\times 10^{-4}} = 4\times 10^{-3}\ \text{m}$$

50. **(A)** The energy of each photon is

$$E = \frac{hc}{\lambda}$$

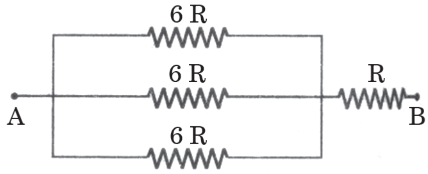
$$= \frac{(6.63\times 10^{-34}\ \text{J-s})\times(3\times 10^{8}\ \text{m/s})}{(632.8\times 10^{-9}\text{m})}$$

$$= 3.14\times 10^{-19}\ \text{J.}$$

The energy of the laser emitted per second is 5×10^{-3} J. Thus the number of photons emitted per second

$$= \frac{5\times 10^{-3}\,\text{J}}{3.14\times 10^{-19}\,\text{J}} = 1.6\times 10^{16}.$$

51. **(A)** The given network of resistors is equivalent to the arrangement shown in the figure.



Let R' be the equivalent resistance of the parallel combination of the three resistors, each of resistance 6 R. Then

$$\frac{1}{R'} = \frac{1}{6R} + \frac{1}{6R} + \frac{1}{6R} = \frac{3}{6R} \text{ or } R' = 2\,R$$

Therefore, the equivalent resistance of the network between the points A and B,

$$R_{equi} = 2\,R + R = 3\,R$$

52. **(C)** $Z_F = \frac{a^2}{\lambda} = \frac{(4\times 10^{-3})^2}{400\times 10^{-9}} = 40$ m

53. **(C)** If N be the number of atoms of a radioactive substance left at some instant of time, then

$$N = N_0\left(\frac{1}{2}\right)^n$$

where N_0 is the initial number of atoms and n is number of half lives.

$$\therefore\ n = \frac{t}{T/2} = \frac{30}{10} = 3$$

$$\therefore\ \frac{N}{N_0} = \left(\frac{1}{2}\right)^3$$

$$= \left(\frac{1}{2}\right)^3 = \frac{1}{8} = 0.125$$

54. **(B)** X-rays do not carry any charge.

X-rays are not deflected by electric and magnetic fields because they do not carry any charge. X-rays are electromagnetic waves, they show all properties of light rays hence, they undergo reflection, refraction, interference, diffraction and polarisation.

55. **(C)** The linear momentum of the photon

$$= \frac{h}{\lambda} = \frac{6.63\times 10^{-34}\ \text{J-s}}{122\times 10^{-9}\,\text{m}}$$

$$= 5.43\times 10^{-27}\ \text{kg-m/s.}$$

As the photon is absorbed and the atom stops, the total final momentum is zero. From conservation of linear momentum, the initial momentum must be zero. The atom should move opposite to the direction of motion of the photon and they should have the same magnitudes of linear momentum. Thus,

$(1.67\times 10^{-27}\ \text{kg})v = 5.43\times 10^{-27}\ \text{kg m/s}$

or, $v = \dfrac{5.43\times 10^{-27}}{1.67\times 10^{-27}}$ m/s = 3.25 m/s.

56. **(D)** Induced emf can be produced by the given methods.

57. **(D)** The given truth table can have the output as $Y = A.B$

Therefore, the given truth table is of AND gate.

58. **(D)** We know, $S = \dfrac{I_g G}{I - I_g}$

or $\dfrac{I_g}{I} = \dfrac{S}{G+S}$

Here, $G = 50\,\Omega$ and $S = 5\,\Omega$

$\therefore \dfrac{I_g}{I} = \dfrac{5}{50+5} = \dfrac{1}{11}$

59. **(C)** $\lambda = \dfrac{h}{mv} = \dfrac{6.62\times 10^{-27}}{0.5\times 400}$

(Take $h = 6.62\times 10^{-27}$ erg s)

$= 3.31\times 10^{-29}$ cm

60. **(D)** Initial charges on capacitor

$q_1 = C_1V_1, \quad q_2 = C_2V_2$

$= 2\times 30 \quad = 3\times 20$

$= 60$ pC $\quad = 60$ pC

When switches S_1 and S_3 are closed then capacitors get connected in series, initially there are same charge on capacitors, so there will be no redistribution, hence potential difference will remain as such, i.e., $V_1 = 30$ V, $V_2 = 20$ V.

61. **(A)** Ozone layer blocks the radiations of wavelength less than 3×10^{-7} m on reaching the earth i.e., between 200 – 280 nm.

62. **(B)** Possible nuclear fusion reaction is :

$${}_6C^{12} + {}_1H^1 \rightarrow {}_7N^{13} + 2\text{MeV}$$

63. **(B)** When the flux is reversed, the change in flux $d\phi$ is $\phi - (-\phi) = 2\phi$.

The average e.m.ϕ. induced

$= E = -\dfrac{Nd\phi}{dt} = \dfrac{-50\times 2\times 0.3}{0.03} = 1000$ V.

64. **(A)** Here, e.m.f. of each battery, E = 5 V and internal resistance of each battery, $r = 0.2\ \Omega$

Let I be the current in the circuit.

Then,

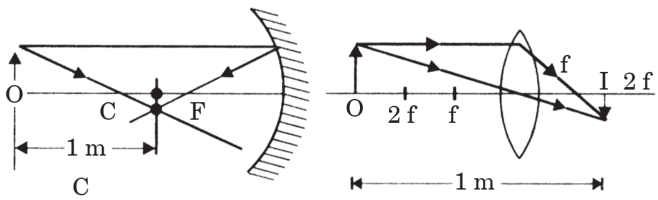
$$I = \frac{\text{total e.m.f. of all the eight batteries}}{\text{total internal resistance of all batteries}}$$

$$= \frac{8\times 5}{8\times 0.2} = 25\ \text{A}$$

Now, the reading of the voltmeter,

$V = E - Ir = 5 - 25\times 0.2 = 5 - 5 = 0$

65. **(C)** Image can be formed on the screen if it is real. Real image of reduced size can be formed by a concave mirror or a convex lens as shown in figure.



A diminished real image is formed by a convex lens when the object is placed beyond 2f and the image of such object is formed beyond 2f on the other side.

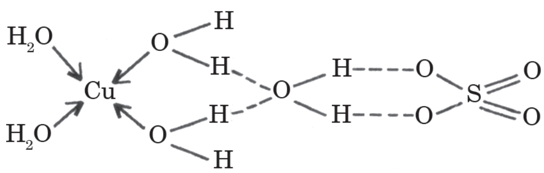
Thus, $d > (2f + 2f)$

or $4f < 0.1$ m

or $f < 0.25$ m

Chemistry

66. **(B)** In solid $CuSO_4.5H_2O$, copper is coordinated to four water molecules as $[Cu(H_2O)_4]SO_4.H_2O$ and fifth water molecule is held between SO_4^{2-} and two of the coordinated water molecules through hydrogen bonding as shown below



67. **(C)** Fe(II) contains 4 unpaired electrons. So, $\mu = \sqrt{4(4+2)}$ BM = 4.9 BM

Fe(III) contains 5 unpaired electrons

$\mu = \sqrt{5(5+2)}$ BM = 5.9 BM

68. **(B)** Acetaldehyde and acetone react differently with ammonia. Whereas acetaldehyde gives acetaldehydeammonia, acetone gives diacetoneamine.

69. **(C)** Oxygen has a higher b.p. (90 K) than nitrogen (77 K) and therefore, can be separated from air by fractional distillation (Claude's process).

70. **(C)** There are eight corners and six faces in a cube. A corner atom is shared by eight cubes, and the face-centered atom by two cubes. Thus,

Effective number of X atoms in a cube
$= 1/8 \times 8 = 1$

and Effective number of Y atoms in a cube
$= 1/2 \times 6 = 3$

Therefore, formula of the compound is XY_3.

71. **(C)** Acetonitrile has sp^2–sp σ bond, sp–sp σ bond and two π bonds due to the overlap of unhybridised p-orbitals on carbon and nitrogen.

72. **(B)** Putting a substituent at position 3 will make the molecule chiral

```
1      2     3*    4     5     6     7
CH3—CH2—CH—CH2—CH2—CH2—CH3
              |
              X
```

73. **(C)** Lactose $\xrightarrow{H_2O}$ glucose + galactose

74. **(C)** $-NO_2$ and –COOH group being powerful electron-withdrawing groups, reduce the electron-density in the benzene ring and hence do not favour F.C. alkylation while CH_2CH_3 and OH groups being electron-donating increase the electron-density in the benzene ring and thus favour F.C. alkylation.

75. **(B)** The outer electronic configuration of these ions are given below:

Ti	$3d^2 4s^2$	Ti^{2+}	$3d^2$
V	$3d^3 4s^2$	V^{3+}	$3d^2$
Cr	$3d^5 4s^1$	Cr^{4+}	$3d^2$
Mn	$3d^5 4s^2$	Mn^{5+}	$3d^2$

76. **(B)** $\Delta T = K_b \times \text{molality} = \dfrac{K_b \times 0.6 \times 1000}{M \times 100}$

$$\therefore M = \frac{5.2 \times 0.6 \times 1000}{0.52 \times 100} = 60$$

77. **(B)** Due to larger size of P atom it has d-orbitals and thus can show + 5 oxidation state which is more stable as compared to N.

78. **(D)** $r = k[N_2O_5]$

2.40×10^{-5} mol l^{-1} sec^{-1}
$= 3.0 \times 10^{-5}$ sec^{-1} $[N_2O_5]$

$$[N_2O_5] = \frac{2.40 \times 10^{-5} \text{ mol } l^{-1}}{3.0 \times 10^{-5}} = 0.8 \text{ mol } l^{-1}$$

79. **(D)** RX + KOH $\rightarrow$ ROH + KX is an example of nucleophilic substitution reaction.

80. **(A)** From the valence numbers, it is clear that three Fe^{2+} are equivalent to two Fe^{3+}. So, three Fe^{2+} ions would be replaced by two Fe^{3+} ions. In other words, two Fe^{3+} ions replace three Fe^{2+} ions, then there is a net loss of 1 iron ion. From the given formula,

Total loss of iron from one molecule of FeO
$= (1 – 0.93) = 0.07$

Therefore, Total Fe^{3+} ions present in one molecule of FeO $= 2 \times 0.07 = 0.14$

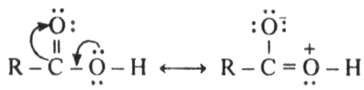
Total number of Fe^{2+} and Fe^{3+} ions in one molecule of FeO = 0.93

So, out of 0.93 atoms of iron, only 0.14 atoms are present as Fe^{3+}. Therefore, Percentage of iron as Fe^{3+} =

$$\frac{0.14}{0.93} \times 100 = 15\%$$

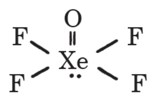
i.e., 15% of the total iron in $Fe_{0.93}O$ is present as Fe^{3+}.

81. **(B)** The carboxilate iron shows resonance as shown below.



Hence, carboxylic acids undergo ionisation in water.

82. **(B)** The structure of $XeOF_4$ is



83. **(D)** $C_4H_{10}O \equiv CH_3CH_2CH_2OCH_3$

84. **(C)** Amount of CH_3COOH adsorbed on 1 g of charcoal

$$= (0.5 – 0.49 \text{ mol } L^{-1}) \times \frac{100 \text{ mL}}{1000 \text{ mL/L}}$$

$= 1 \times 10^{-3}$ mol

So, No. of CH_3COOH molecules in 1×10^{-3} mol of acetic acid
$= 6.02 \times 10^{23}$ $mol^{-1} \times 10^{-3}$ mol

$= 6.02 \times 10^{20}$

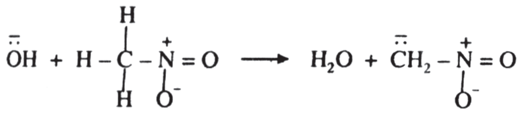
Area occupied by one CH_3COOH molecule

$= \frac{3.01 \times 10^2\,m^2}{6.02 \times 10^{20}} = 5 \times 10^{-19}\ m^2$

Thus the surface area of the charcoal covered by each CH_3COOH molecule is $5 \times 10^{-19}\ m^2$.

85. **(A)** Chlorophylls are green pigments which contain magnesium.

86. **(B)** Nitro alkanes having α-hydrogen have weakly acidic character due to high electron withdrawing effect (–I effect), caused by $-NO_2$ group. Hence undergo nucleophilic substitution reaction in presence of NaOH.



87. **(B)** As ΔG^o is more negative for the reaction (ii), therefore, C(gr) will reduce ZnO to zinc metal.

88. **(D)** $CH_3CH_2COOH + NaHCO_3 \rightarrow CH_3CH_2COONa + H_2O + CO_2(g)$

89. **(B)** $Na_2SO_3 + S \xrightarrow{\text{alkaline solution}} Na_2S_2O_3$

90. **(A)** Since (iii) has the largest oxidation potential and (ii) has the largest reduction potential. The half-cells, (iii) anode and (ii) cathode will give largest potential i.e.,
$-(-1.25) + 1.25 = 2.50\ V$