UNIFIED COUNCIL
An ISO 9001:2008 Certified Organisation
nstse
Test • Assess • Achieve

NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION

Paper Code: UN426 (UPDATED)

Solutions for Class : 11 (PCM)

Mathematics

1. **(C)** Since, P is one end of the focal chord of the parabola $y^2 = x$

Let coordinates of point P are $\left(\frac{1}{4}t^2, \frac{2}{4}t\right)$

$\therefore$ Coordinates of other vetex of focal chord i.e. Q is $\left(\frac{1}{4t^2}, \frac{2}{4t}\right)$ $\{\because t_1t_2 = -1\}$

Since, coordinates of P are (4, –2)

$\Rightarrow \frac{1}{4}t^2 = 4$ or $\frac{2}{4}t = -2$

$\Rightarrow \boxed{t = \pm 4}$ or $\boxed{t = -4}$

$\therefore \quad \boxed{t = -4}$

Hence, coordinates of Q are $\left(\frac{1}{64}, \frac{1}{8}\right)$

Now, equation of tangent at Q is

$yy_1 = \frac{1}{2}(x + x_1)$

$\Rightarrow \frac{1}{8}y = \frac{1}{2}\left(x + \frac{1}{64}\right)$

$\Rightarrow y = 4x + \frac{1}{16}$

$\therefore$ Slope to triangle at Q is 4.

2. **(A)** z =

$\frac{11-3i}{1+i} = \frac{11-3i}{1+i} \times \frac{1-i}{1-i} = \frac{14-14i}{2} = 7(1-i).$

Now, $z - i\alpha = 7 - 7i - i\alpha = 7 - (7+\alpha)i$

Clearly, $z - i\alpha$ is real if $7 + \alpha = 0$ i.e. if $\alpha = -7$

3. **(A)** Let the first term of the A.P. be a and its common difference be d.

Then,

$S_p = S_q \Rightarrow \frac{p}{2}\{2a + (p-1)d\}$

$= \frac{q}{2}\{2a + (q-1)d\}$

$\Rightarrow 2a(p - q) + \{(p^2 - q^2) - (p - q)\}d = 0$

$\Rightarrow 2a(1 - p - q)^d$

$\therefore S_{p+q} = \left(\frac{p+q}{2}\right)\{2a + (p+q-1)d\}$

$= \left(\frac{p+q}{2}\right)\{(1-p-q) + (p+q-1)d\} =$

$= \left(\frac{p+q}{2}\right).0 = 0$.

4. **(A)** When $n = 1$, we have : $3^{2^n} - 1 = 3^2 - 1 = 8$

Also, when $n = 1$, then :

$2^{n+2} = 2^3 = 8$; $3^{n+1} = 3^2 = 9$; $2^{n+3} = 2^4 = 16$; $5^n = 5^1 = 5$.

Clearly, $3^{2^n} - 1$ is divisible by 2^{n+2} only (amongst the given alternatives) for n = 1.

We shall show that $3^{2^n} - 1$ is divisible by 2^{n+2} for all integers $n \geq 1$.

Let $P(n)$: $3^{2^n} - 1$ is divisible by 2^{n+2}.

Then, P (1) is true (Proved above)

Let P (m) be true

Then, $3^{2^m} - 1$ is divisible by 2^{m+2}.

Let $3^{2^m} - 1 = k.2^{m+2} \Rightarrow 3^{2^m} = k.2^{m+2} + 1$...(i)

Now, $3^{2^m}-1=(3^{2^m})2-1 = (k.2^{m+2}+1)2-1$ [Using (i)]

$\{k^2(2^{m+2})^2+2k.2^{m+2}+1\}-1$

$2^{m+2}\{k\ 2.2^{m+2}+2k\}$, which is divisible by 2^{m+2}.

$P(m+1)$ is true for all integers $n \geq 1$.

Hence, $3^{2^m}-1$ is divisible by 2^{n+2} for all integers $n \geq 1$.

5. **(D)** The general term in the expansion of $(1+2\sqrt{x})^{40}$ is

$t_{r+1} = {}^{40}C_r(2\sqrt{x})^r = {}^{40}C_r 2^r . x^{r/2}$

Clearly, this term contains an integral power of x if $\frac{r}{2}$ is an integer and $0 \leq r \leq 40$ i.e. if $r = 0, 2, 4, 6, ..., 40$.

Sum of the coefficients of all the integral powers if x is

$S = {}^{40}C_0 + 2^2\,{}^{40}C_2 + 2^4\,{}^{40}C_4 + ... + 2^{40}\,{}^{40}C_{40}$ ...(i)

Now, by Binominal Expansion :

$= (1+2)^{40} = {}^{40}C_0 + {}^{40}C_1 .2 + {}^{40}C_2 .2^2 + {}^{40}C_3 .2^3 + ... + {}^{40}C_{40} .2^{40}$ ...(ii)

and $(1-2)^{40} = {}^{40}C_0 - {}^{40}C_1 .2 + {}^{40}C_2 .2^2 - {}^{40}C_3 .2^3 + ... + {}^{40}C_{40} .2^{40}$ ...(iii)

Adding (ii) and (iii), we get:

$(1+2)^{40} + (1-2)^{40} = 2\{{}^{40}C_0 + 2^2.{}^{40}C_2 + 2^4.{}^{40}C_4 + ... + {}^{40}C_{40} .2^{40}\}$

$\Rightarrow {}^{40}C_0 + 2^2.{}^{40}C_2 + 2^4.{}^{40}C_4 + ... + {}^{40}C_{40} . 2^{40}$

$= \frac{1}{2}(3^{40}+1)$ ....(iv)

From (i) and (iv) we get: $S = \frac{1}{2}(3^{40}+1)$.

6. **(C)** a number formed from the digit 4, 5, 6, 7, 8, 9 will be divisible by 5 if the digit at the units place is 5. Now, we have to fill six places with the given six digits (without repetition) such that the sixth place always has the digit 5 i.e. there is only 1 way of filling 6th place.

Clearly, the number of ways filling the remaining five places with the remaining five digits = 5! = 120.

Total number of ways of filling all the six places = 1×120 = 120.

Hence, there are 120 numbers formed from the given digits and divisible by 5.

7. **(D)** **Case 1.** When $x - 3 \geq 0$ i.e. when $x \geq 3$.

In this case: $|x-3| = (x-3)$.

$\therefore 2 \leq |x-3| < 4 \Rightarrow 2 \leq x-3 < 4$

$\Rightarrow 2+3 \leq x < 4+3$

[$\because a > b \Rightarrow a+m > b+m \ \forall \ m \in N$]

$\Rightarrow 5 \leq x < 7$

Thus, in this case, we have: $x \in [5, 7)$

Case 2. when $x - 3 < 0$ i.e. when $x < 3$.

In this case: $|x-3| = -(x-3)$.

$\therefore 2 \leq |x-3| < 4 \Rightarrow 2 \leq -(x-3) < 4$

$\Rightarrow 2 \leq -x+3 < 4$

$\Rightarrow 2-3 \leq -x < 4-3$

[$\because a > b \Rightarrow a-m > b-m \ \forall \ m \in N$]

$\Rightarrow -1 \leq -x < 1$

$\Rightarrow -1 < x \leq 1$

[Multiplying throughout by -1]

[Note: $a > b \Rightarrow am < bm \forall m < 0$]

Thus, in this case, we have : $x \in (-1, 1]$

Hence, from both the cases, we get:

$x \in (-1, 1] \cup [5, 7)$.

8. **(C)** **Given limit**

$$\lim_{x \to 0} 6\left(\frac{e^{2\sqrt{x}}-1}{2\sqrt{x}}\right).\left(\frac{\tan 3\sqrt{x}}{3\sqrt{x}}\right).\left(\frac{x}{\sin x}\right)$$

$= (6 \times 1 \times 1 \times 1) = 6$

Hence, the correct answer is (a).

9. **(C)** The given equation can be written as

$\Rightarrow$ (1/2) (sin 8x + sin 2x) = (1/2) (sin 8x + sin 4x)

$\Rightarrow$ sin 2x - sin 4x = 0 $\Rightarrow$ - 2 sin x cos 3x = 0

$\Rightarrow$ sin x = 0 or cos 3x = 0. That is, x = nπ (n$\in$ **I**), or 3x = k π + π/2 (k $\in$ **I**). Therefore, since x $\in$ [0, π], the given equation is satisfied if x = 0, π, π/6, π/2 or 5π/6.

10. **(B)**

11. **(B)** A = $\{x : x$ is a multiple of 4$\}$ and

B = $\{x : x$ is a multiple of 6$\}$

$\Rightarrow$ A $\cap$ B = $\{x : x$ is a multiple of both 4 and 6$\}$

= $\{x : x$ is a multiple of the L.C. M. of 4 and 6$\}$

= $\{x : x$ is a multiple of 12$\}$

Thus, A $\cap$ B consists of all multiples of 12.

12. **(C)** The given equation reduces to

$$\frac{(x-1)^2}{9}-\frac{y^2}{3}=1$$

$\therefore$ $3 = 9\,(e^2 - 1)$

$\Rightarrow$ $e^2 = \dfrac{4}{3}$

$\therefore$ $\boxed{e=\dfrac{2}{\sqrt{3}}}$

13. **(C)** We have:

$f(1) = 1$, $f(2) = 2$, $f(3) = 3$, $f(4) = 4$, $f(5) = 6$, $f(6) = 7$, ...

$\therefore$ Each natural number except 5 is the image of one of the natural numbers.

So, the image of the function F is N – {5}.

14. **(A)** a, b, c are in A.P. $\Rightarrow$ $2b = a + c$

$\Rightarrow$ $a - 2b + c = 0$

On comparing equation (i) with the given equation of the system of lines i.e, $ax + by + c = 0$, we conclude that these lines always pass through the point (x, y) = (1, –2).

15. **(C)** The general term in the expansion of

$$\left(\frac{2x^2}{3}+\frac{3}{2x^2}\right)^{10}$$

$$t_{r+1} = {}^{10}C_r\left(\frac{2x^2}{3}\right)^{10-r}\left(\frac{3}{2x^2}\right)^{r}$$

$$= {}^{10}C_r\,(2)^{10-2r}\,(3)^{2r-10}\,(x)^{20-4r} \qquad \text{....(i)}$$

Now, the index is 10, which is even, so that the middle term of the expansion is

$\left(\dfrac{10}{2}+1\right)$th i.e., the 6th term.

Putting $r = 5$ in (i), we get:

$$t_6 = {}^{10}C_5\,(2)^0\,(3)^0\,(x)^0\;\frac{10\times9\times8\times7\times6}{5\times4\times3\times2\times1} = 252$$

$\therefore$ The middle term in the given expansion is 252.

16. **Delete**

17. **(A)** $n(A\cup B) = n(A) + n(B) - n(A\cap B)$

$= 12 + 9 - 4 = 17.$

$\therefore$ $n[(A\cup B)^C] = n(U) - n(A\cup B)$

$= 20 - 17 = 3.$

18. **(C)** $\displaystyle\sum_{n=1}^{13}(i^n+i^{n+1}) = \sum_{n=1}^{13}i^n+\sum_{n=1}^{13}i^{n+1}$

$= (i^1 + i^2 + i^3 + i^4 + i^5 + i^6 + i^7 + i^8 + i^9 + i^{10} + i^{11} + i^{12} + i^{13})$

$+ (i^2 + i^3 + i^4 + i^5 + i^6 + i^7 + i^8 + i^9 + i^{10} + i^{11} + i^{12} + i^{13} + i^{14})$

$= \{(i^1 + i^2 + i^3 + i^4) + (i^5 + i^6 + i^7 + i^8) +$

$(i^9 + i^{10} + i^{11} + i^{12}) + i^{13}\}$

$= \{i^2 + i^3 + i^4 + (i^5 + i^6 + i^7 + i^8) + (^9 + i^{10} + i^{11} + i^{12}) + i^{13} + i^{14}\}$

$= \{(i - 1 - i + 1) + (i - 1 - i + 1) + (i - 1 - i + 1) + i\}$

$+ \{-1 - i + 1 + (i - 1 - i + 1) + (i - 1 - i + 1) +$

$\left[\because i^1 = i^5 = i^9 = i^{13} = \ldots i;\ i^2 = i^6 = i^{10} = i^{14} = \ldots = -1;\ i3 = i7 = i11 = \ldots = -i;\ i4 = i8 = i12 = \ldots = 1\right]$

$= \{0 + 0 + 0 + i\} + \{-1 - i + 1 + 0 + 0 + i - 1\} = i - 1.$

19. **(A)** Let d be the common difference to the A.P.

Then, $a_7 = 15$ $\Rightarrow$ $a_2 = 15 - 5d$ and $a_{12} = 15 + 5d$

$$\left[\begin{array}{l}\text{Note}: a_2 = a_1 + d \text{ and } a_7 = a_1 + 6d\\ \quad \Rightarrow a_7 - a_2 = 5d \Rightarrow a_2 = a_7 - 5d = 15 - 5d\\ \quad \text{Similarly, } a_{12} = a_7 + 5d = 15 + 5d\end{array}\right]$$

$\therefore$ $a_2a_7a_{12} = (15 - 5d)(15)(15 + 5d)$ $= 15\,(225 - 25d^2)$

$= (15)(25)(9 - d^2)$

Clearly $a_2a_7a_{12}$ is greatest when $(9 - d^2)$ is greatest i.e, when $d^2 = 0$ i.e. when $d = 0$.

20. **(B)** $a^x + a^y = a^{x+y}$

$\Rightarrow a^{-y} + a^{-x} = 1$

$\Rightarrow -\,a^{-y}\log a.\dfrac{dy}{dx} - a^{-x}\log a = 0$

$\Rightarrow \dfrac{1}{a^y}.\dfrac{dy}{dx}+\dfrac{1}{a^x} = 0$

$\Rightarrow \dfrac{dy}{dx} = -\dfrac{a^y}{a^x} = -\,a^{(y-x)}$

$\Rightarrow \left.\dfrac{dy}{dx}\right]_{(1,1)} = -\,a^0 = -\,1.$

$\therefore$ The correct answer is (B).

21. **Delete**

22. **(B)** Let the required G.P. be $1, r, r^2, r^3, \ldots \infty$

Then, $1 = (r + r^2 + r^3 + \ldots \infty) = \dfrac{r}{1-r}$

$\left[\begin{array}{l}\text{Note}: |r| < 1 \text{ Since it is an} \\ \text{inf inite G.P. with a finite} \\ \text{sum of its terms}\end{array}\right]$

$\Rightarrow 1 - r = r \Rightarrow 2r = 1 \Rightarrow r = \dfrac{1}{2}$.

Fourth term of the G.P. $= r_4 = r^3$

$= \left(\dfrac{1}{2}\right)^3 = \dfrac{1}{8}$.

23. **(B)** $x^2 + 6x - 27 > 0 \Rightarrow (x + 9)(x - 3) > 0 \Rightarrow x < -9$ or $x > 3$

Also, $-x^2 + 3x + 4 > 0$

$\Rightarrow -x^2 - 3x - 4 < 0$

$[\because a > b \Rightarrow am < bm \ \forall \ m < 0]$

$\Rightarrow (x - 4)(x + 1) < 0 \Rightarrow 1 < x < 4$

$\therefore x^2 + 6x - 27 > 0$ and $-x^2 + 3x + 4 > 0$

$\Rightarrow (x < -9$ or $x > 3)$ and $(1 < x < 4)$

$\Rightarrow x > 3$ and $x < 4 \Rightarrow x \in (3, 4)$.

24. **(A)** $\dfrac{SP}{PM} = e = \dfrac{1}{2}$

$\Rightarrow \quad 4SP^2 = PM^2$

$\Rightarrow \quad PM = \dfrac{x - y + 3}{\sqrt{2}}$

$\Rightarrow \quad 4\{(x+1)^2 + (y-1)^2\} = \left(\dfrac{x-y+3}{\sqrt{2}}\right)^2$

$\Rightarrow 8(x^2 + y^2 + 2x - 2y + 2) = x^2 + y^2 + 9 - 2xy + 6x - 6y$

$\therefore \quad \boxed{7(x^2 + y^2) + 2xy + 10x\ 10y + 7 = 0}$

25. **(A)** From $\sin x + \sin^2 x = 1$, we get $\sin x = \cos^2 x$. Now the given expression is equal to

$\cos^6 x(\cos^6 x + 3\cos^4 x + 3\cos^2 x + 1) - 1$

$= \cos^6 x(\cos^2 x + 1)^3 - 1$

$= \sin^3 x(\sin x + 1)^3 - 1$

$= (\sin^2 x + \sin x)^3 - 1 = 1 - 1 = 0$

26. **(A)** Clearly, n(A) = 8 i.e, the number of elements in the given set is 8.

$x^2 = 16 \Rightarrow x = -4$ or 4.

And, $2x = 6 \Rightarrow x = 3$

Clearly, there is no real number x that satisfies both the conditions $x^2 = 16$ and $2x = 6$

$\therefore A = \phi$

27. **(B)** $y = \sin^{-1}(\tanh ax)$

$= \dfrac{1}{\sqrt{1 - \tanh^2 ax}}.(a\,\text{sech}^2\,ax)$

= a sech ax.

$\therefore$ The correct answer is (B).

28. **Delete**

29. **(A)** Let A, B, C be the angles of ΔABC and a, b, c be their corresponding sides.

Then, by sine Formula :

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = k$ (say) ...... (i)

Now, $a + b + c = 6\left(\dfrac{\sin A + \sin B + \sin C}{3}\right)$

$\Rightarrow a + b + c = 2\left(\dfrac{a}{k} + \dfrac{b}{k} + \dfrac{c}{k}\right) \Rightarrow k = 2$

[Using (1)]

Putting $k = 2$ in(i), we get:

$\dfrac{a}{\text{Sin } A} = 2 \Rightarrow \text{Sin } A = \dfrac{a}{2} = \dfrac{1}{2} = \sin 30^{\circ}$

$\Rightarrow \angle A = 30^{\circ}$ $[a = 1]$

30. **(C)** $d = \pm 2$ and $S_n = 6n - n^2$

31. **(B)** $\dfrac{(2n)!}{n!} = \dfrac{1.2.3.....(2n-1)(2n)}{n!}$

$= \dfrac{\{1.3.5.....(2n-1)\}\{2.4.6...(2n)\}}{n!}$

$= \dfrac{\{1.3.5.....(2n-1)\}.2^n\{1.2.3...n\}}{n!}$

$= \dfrac{\{1.3.5.....(2n-1)\}.2^n.n!}{n!}$

$= \{1.3.5....(2n-1)\}.2^n$.

32. **(A)** The general term in the expansion of $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$

$$t_{r+1} = {}^{10}C_r\,(-1)^r\left(\frac{x}{2}\right)^{10-r}\left(\frac{3}{x^2}\right)^r$$

$$= {}^{10}C_r\,(-1)^r\,(2)^{r-10}\,(3)^r\,x^{10-3r} \quad ....(i)$$

This term contains x^4 if $10 - 3r = 4$ i.e. if $r = 2$.

∴ The term containing x^4 is

$t_3 = {}^{10}C_2\,(-1)^2\,2^{(-8)}\,3^2\,x^4$

[Putting $r = 2$ in (i)]

$$\Rightarrow t_3 = \frac{10\times 9}{2}\times 1\times\frac{9}{256}\times x^4 = \frac{405}{25}x^4$$

Thus, the coefficient of x^4 in the expansion of $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$ is $\frac{405}{256}$.

33. **(C)** 5 boys can be arranged in a row in = 5! ways.

Now, since no two girls can sit next to each other, they have to be arranged in any of the 5 places out of the 6 places which are formed in the gaps between 5 boys and on the extremes.

Number of ways of arranging 5 girls in 6 places = ${}^6P_5$ = 6!

∴ Total number of all possible arrangements = 5! 6!

34. **(B)** The n(s) = 200, n(p) = 120, n(m) = 140

$n(p'\cap m') = 40$

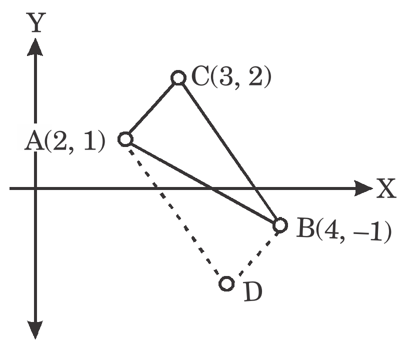
Now, $p'\cap m' = (p\cup m)'$

∴ $n(p\cup m) = 160$

$\Rightarrow n(p\cap m) = 120 + 140 - 160$

$= 100$

35. **(A)**



ABCD is a parallellogram and diagonal AC divides into two equal triangles. Area of ABCD

= 2(–1 –2)–1(4–3) + 1(8 + 3)

= –6 – 1 + 11 = 4

36. **(C)** k = $\pi/18$ sin $5\pi/18$ sin 7 $\pi/18$ = sin 10° sin 50° sin 70°

$$= \frac{1}{2}\,[\cos 40° - \cos 60°]\,\sin 70°$$

$$= \frac{1}{2}\cos 40°\sin 70° - \frac{1}{4}\sin 70°$$

$$= \frac{1}{4}\,[\sin 110° + \sin 30°] - \frac{1}{4}\sin 70°$$

$$= \frac{1}{4}\sin(180° - 70°) + \frac{1}{4}\times\frac{1}{2} - \frac{1}{4}\sin 70° = \frac{1}{8}$$

37. **(C)**

38. **(B)** **Give limit**

$$= \lim_{x\to a}\frac{\left(\sqrt{a+2x}-\sqrt{3x}\right)\left(\sqrt{3a+x}+2\sqrt{x}\right)}{\left(\sqrt{a+2x}+\sqrt{3x}\right)\left(\sqrt{3a+x}-2\sqrt{x}\right)}$$

$$\frac{\left(\sqrt{a+2x}+\sqrt{3x}\right)}{\left(\sqrt{3a+x}+2\sqrt{x}\right)}$$

$$= \lim_{x\to a}\frac{(a-x)\left(\sqrt{3a+x}+2\sqrt{x}\right)}{\left(\sqrt{a+2x}+\sqrt{3x}\right)3(a-x)}$$

$$= \lim_{x\to a}\frac{\left(\sqrt{3a+x}+2\sqrt{x}\right)}{3\left(\sqrt{a+2x}+\sqrt{3x}\right)} = \frac{1}{3}\times\frac{4\sqrt{a}}{2\sqrt{3a}} = \frac{2}{3\sqrt{3}}.$$

Hence, the correct answer is (b).

Use of Expansions:

Method: Expand each of the expandable functions in the given limit. Simplify the rational expression by cancelling the common factors, if any. Now, put x = a.

39. **(D)** Let r be the common ratio of the G.P.

x, $2x + 2$, $3x + 3$, ...

$$\text{Then, } r = \frac{3x+3}{2x+2} = \frac{3(x+1)}{2(x+1)} = \frac{3}{2}$$

$$\text{Now, } 2x + 2 = (x)\left(\frac{3}{2}\right) \Rightarrow \left(\frac{x}{2}\right) - 2$$

$\Rightarrow x = -4$

∴ The fourth term of the G.P. is

$$t_4 = (x)\,(r)^3 = (-4)\left(\frac{3}{2}\right)^3 = \frac{-27}{2} = -13.5$$

40. **(C)** The given expression is equal to

$$\frac{\sin^2 A}{\cos A(\sin A-\cos A)}+\frac{\cos^2 A}{\sin A(\cos A-\sin A)}$$

$$-\sec A \operatorname{cosec} A$$

$$=\frac{\sin^3 A-\cos^3 A}{\sin A\cos A(\sin A-\cos A)}$$

$$-\sec A \operatorname{cosec} A$$

$$=\frac{\sin^2 A+\cos^2 A+\sin A\cos A}{\sin A\cos A}$$

$$-\sec A \operatorname{cosec} A$$

= sec A cosec A + 1 – sec A cosec A = 1.

Physics

41. **(B)** As each car travels one circle in the same time; and $\omega = \frac{2\pi}{T}$, therefore, their angular velocities must be same, i.e., $\omega_1 : \omega_2 = 1 : 1$.

42. **(B)** Maximum stress = Young's modulus × maximum strain

$= 2\times 10^{11}\times 10^{-3} = 2\times 10^{8}\ Nm^{-2}$

∴ Maximum force (F) = maximum stress × area

$= 2\times 10^{8}\times 3\times 10^{-6} = 600$ N

Maximum mass $= \frac{F}{g} = \frac{600}{10} = 60$ kg

43. **(A)** Here mass of the α-particle may be written as 4.

Hence, applying the conservation of momentum we find :

(A – 4) V – 4 v = 0

Hence, V = 4 v/(A – 4)

44. **(A)** (i) $\sqrt{\text{Energy / mass}} = \sqrt{ML^2T^{-2} / M}$

$= [L^1 T^{-1}]$

(ii) $\sqrt{\text{Pressure / density}}$

$= \sqrt{ML^{-1} T^{-2} / ML^{-3}}$

$= [L^1 T^{-1}]$

(iii) $\sqrt{\text{Force / linear density}}$

$= \sqrt{ML T^{-2} / ML^{-1}}$

$= [L^1 T^{-1}]$

45. **(C)** $\frac{G\times 100}{x^2} = \frac{G\times 10{,}000}{(1-x)^2}$

This gives $\frac{10}{x} = \frac{100}{1-x}$

That is $x = \frac{1}{11}$ m.

46. **(B)** Radius of semi-circular path = r = l

As P.E = K.E.

∴ mgr $= \frac{1}{2}$ m v^2

$r = \frac{v^2}{2g} = \frac{7\times 7}{2\times 9.8} = 2.5$ m

47. **(A)** A vector has both magnitude and direction whereas a scalar has only magnitude but no direction.

48. **(C)** Lateral pressure exerted by water on the face of the door in contact with water is

$P_w = h\rho_w g = 4\times 1000\times 10$

$= 4\times 10^4\ Nm^{-2}$

Lateral pressure exerted by acid on the face of the door in contact with acid is (□ $\rho_a = 1.5\times 1000 = 1500\ kg\ m^{-3}$)

$P_a = h\rho_w g = 4\times 1500\times 10 = 6\times 10^4$ N

∴ Net pressure on the door

$= P_a - P_w = 2\times 10^4$ N.

Now, the face area of the door = 20 cm^2

$= 20\times 10^{-4}\ m^2$.

∴ Force on the door = pressure × area

$= 2\times 10^4\times 20\times 10^{-4} = 40$ N

Hence, a force of 40 N must be applied on the door in order to counterbalance the force due to the two liquids.

49. **(C)** In this case net pulling force

$= m_A g \sin 60^\circ + m_B g \sin 60^\circ - m_C g \sin 30^\circ$

$= (1)(10)\frac{\sqrt{3}}{2} + (3)(10)\left(\frac{\sqrt{3}}{2}\right) - (2)(10)\left(\frac{1}{2}\right)$

= 24.64 N

Total mass being pulled

= 1 + 3 + 2 = 6 kg

∴ Acceleration of the system a $= \frac{24.64}{6}$

$= 4.1\ m\ s^{-2}$

50. **(D)** The bursting of tyre is very fast. So the gas fails to gain or lose heat. Hence, the process is an adiabatic process.

51. **(B)** Here only the potential gradient has non zero dimensions. Others are dimensionless.

52. **(C)** Gravitational field is a gravitational force experienced by unit mass and depends on the value of g which decreases with height and depth.

53. **(B)** Here, $v = 20$ m/s, $r = 10$ m, $a_t = 30$ m/s^2, $a = ?$

$$a_r = \frac{v^2}{r} = \frac{20 \times 20}{10} = 40 \text{ m/s}^2$$

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{40^2 + 30^2} = 50 \text{ m/s}^2$$

54. **(B)** $x = (20.15 \pm 0.05)$ g; $\Delta x = \pm 0.05$ g

$y = (20.17 \pm 0.02)$ g; $\Delta y = \pm 0.02$ g

Difference = $z = y - x$

$= (20.17 - 20.15) = 0.02$ g

$\Delta z = \pm(\Delta y + \Delta x) = \pm(0.02 + 0.05)$

$= \pm 0.07$ g

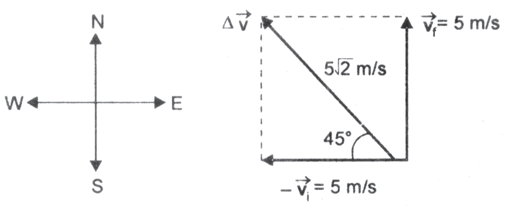
$\therefore z = (0.02 \pm 0.07)$ g

55. **(C)** Both the spring balances will show the actual weight, 4 kg each.

56. **(A)** $p = mv$, $v = \frac{p}{m} = \frac{500}{5} = 100$ m s^{-1}

K.E. $= \frac{1}{2} mv^2 = \frac{1}{2} \times 5 \times 100^2 =$ 2.5×10^4 J

57. **(C)** $\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$



$\Delta\vec{v} = 5\sqrt{2}$ m/s in north-west direction.

$\vec{a}_{av} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}$ m/s^2 (in north-west direction)

58. **(B)** When a body is orbiting around a heavier body in elliptical orbit, its angular momentum is conserved.

59. **(B)** As $\eta = 1 - \frac{T_2}{T_1} = \frac{W}{Q_1}$

$\therefore 1 - \frac{300}{600} = \frac{800}{Q_1}$

$Q_1 = 1600$ J

60. **(C)** True value = Measured value ± Error.

61. **(C)** Weight of sphere = weight of mercury displaced + weight of oil displaced

or $V \rho g = \frac{V}{2} \times 13.6 \times g + \frac{V}{2} \times 0.8 \times g$

or $\rho = \frac{13.6 + 0.8}{2} = 7.2$ g cm^{-3}

62. **(B)** $F = 500$ N,

Total mass = m = 10 + 20 = 30 kg;

$a = \frac{F}{m} = \frac{500}{30} = 16.66$ m s^{-1}

(i) When the pull is applied on 20 kg mass, T_1 is the tension in the string. Thus, 10 kg is acted upon by T_1 and its acceleration is 16.66 m s^{-1}.

$\therefore T_1 = 10a = 10 \times 16.66 = 166.6$ N

(ii) When the pull is applied on 10 kg mass, $T_2 = 20\,a = 20 \times 16.66 = 333.2$ N. Thus, tension depends on the end where pull is applied.

63. **(B)** Three vectors of unequal magnitude, which can be represented by the three sides of a triangle taken in order, produce zero resultant.

64. **(A)** The effective weight inside a satellite is given by :

$W' = mg' = m(g - a)$ ———(1)

Because the frame of reference attached to the satellite is an accelerated frame whose acceleration towards centre of earth is provided by gravitation pull of earth over the satellite.

Hence, $a_c = \frac{v^2}{r}$ $G\frac{Me}{r^2} = g$

(from, $F_c = F_g \Rightarrow \frac{mv^2}{r} = G\frac{M_e m}{r^2}$)

Therefore, eqn. (1) becomes $W' = 0$

65. **(A)** I (ring) = MR^2, I (sphere) = $\frac{2}{5} MR^2$,

I (disc) = $MR^2/2$.

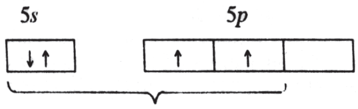
Chemistry

66. **(A)** Due to high ionization enthalpy and weak metallic bonding, mercury is a liquid at room temperature.

67. **(D)** CCl_4 under the name pyrene is incombustible and is used as a fire extinguisher.

68. **(A)** Calorific value is the heat of combustion per gram C.V. for H_2 = –285/2, for CO = –284/28, for CH_4 = –890/16.

69. **(D)** In $SnCl_2$, Sn is sp^2 hybridized. As such it has two bond pairs and one lone pair of electrons.



sp^2–hybridization

70. **(B)** The reaction is

$$4\,NH_3\,(g) + 5O_2\,(g) \rightarrow 4\,NO\,(g) + 6\,H_2O\,(g)$$

$4\,NH_3$	$5O_2$	$4\,NO$
4 × 17 g	5 × 32 g	4 × (14 + 16) g
68 g	160 g	120 g
10.0 g	20.0 g	?

Based on of the given data, it shows that in the given reaction, oxygen is the limiting reagent.
Therefore,
Maximum mass of nitric oxide formed

$$= \frac{120\,g \times 20\,g}{160\,g} = 15\,g$$

71. **(D)** X and Y are different atoms if they have different number of electrons, protons and /or neutrons. If X and Y have the same total number of nucleons but yet are different, they must have a different number of protons and neutrons, and hence, cannot belong to the same element.

72. **(D)** To convert covalent compounds into ionic compounds (NaCN, Na_2S, NaX)

73. **(C)** Using the relationship,

$$p = \rho \frac{RT}{M}$$

For the given data, if M is the molar mass of the gaseous oxide, we can write

$$2\text{ bar} = \rho.\frac{RT}{M}$$

and for nitrogen,

$$5\text{ bar} = \rho.\frac{RT}{28\,g/mol}$$

From these equations, one can write

$$\frac{5\,\text{bar}}{2\,\text{bar}} = \frac{M}{28\,g/mol}$$

This gives,

$$M = \frac{5\,\text{bar}}{2\,\text{bar}} \times 28\,g\,mol^{-1} = 70\,g\,mol^{-1}$$

74. **(C)** No. of atoms in different flasks depends upon atomicity of gases present in flasks. The ratio is 2 : 1 : 2 : 3.

75. **(B)** Water molecule can form two hydrogen bonds through its two hydrogen atoms and another two with two lone pairs of electrons on the O atom.

76. **(B)** Li is least reactive due to high ionization enthalpy.

77. **(D)** According to the reaction stoichiometry,

$$I_2\,(g) \rightleftharpoons 2I\,(g)$$

Then $P_{I(g)} = P_{total} \times$ Fraction of I atoms

$= 10^5\,Pa \times 0.3$

and $P_{I_2(g)} = P_{total} \times$ Fraction of I_2 molecules

$= 10^5\,Pa \times 0.7$

$$K_p = \frac{(P_{I(g)})^2}{P_{I_2(g)}} = \frac{(0.3\times10^5\,Pa)^2}{0.7\times10^5\,Pa}$$

$= 1.28 \times 10^4$ Pa

78. **(A)** Alkenes do not contain a divalent functional group on the either side of which the alkyl chain can differ and hence alkenes do not show metamerism.

79. **(B)** Equilibrium shifts backward.

80. **(B)** When cement is mixed with water, it absorbs water to form a gelatinous mass which sets to a hard mass. This is called setting of cement. The setting of cement involves a series of hydration and hydrolysis reactions leading to the formation of colloidal gels. These gels soon begin to harden due to the formation of interlocking crystals of hydrated silicated gels. The process of hydration and hydrolysis are exothermic. Water is sprinkled over it to keep it cool also.

81. **(D)** The strength of a sample of hydrogen peroxide solution is expressed in terms of volume of oxygen at STP that one volume of hydrogen peroxide gives on heating. For example, '20 volumes of H_2O_2' means 1 litre of this solution liberates 20 L of O_2 at STP. Hydrogen peroxide acts both as an oxidising and a reducing agent. As an oxidising agent, H_2O_2 is converted to H_2O and as reducing agent, it is converted to O_2.

82. **(A)** Average atomic mass $= \dfrac{19\times10 + 81\times11}{100}$

$= 10.81$

83. **(C)** Given : Average velocity = 400 m/s

We know

$$u_{rms} = \sqrt{\frac{3\,RT}{M}} \quad \text{and} \quad u_{av} = \sqrt{\frac{8\,RT}{\pi M}}$$

So $$\frac{u_{rms}}{u_{av}} = \sqrt{\frac{3\,RT/M}{8\,RT/\pi M}} = \sqrt{\frac{3\,\pi}{8}}$$

$$= \sqrt{1.178} = 1.085$$

Then

u_{rms} = $1.085 \times u_{av}$ = 1.085×400 m/s

= 434.2 m/s

84. **(A)** Displacement of σ - electrons.

85. **(D)** Protons and neutrons have relatively the same mass, while an electron is much less lighter. Neutrons are neutral, while protons and electrons are charged particles.

86. **(C)** $BaO_2 + H_2SO_4 \longrightarrow BaSO_4 + H_2O_2$

In this reaction, none of the elements undergoes a change in oxidation number or valency.

87. **(A)**

```
            H   H
            |   |
C2H6 = H – C – C – H
            |   |
            H   H
```

$C_2H_4 \rightarrow H_2C = CH_2$

$C_2H_2 \rightarrow HC \equiv CH$

88. **(D)** Thallium metal marks the paper like lead metal.

89. **(C)** The reaction for the oxidation of one mole of benzene is

$$C_6H_6(l) + \frac{15}{2}O_2(g) \longrightarrow 6CO_2(g) + 3H_2O(l)$$

$\Delta H = -781.0$ k cal

Δn for the reaction = $6 - \frac{15}{2} = -1.5$

Therefore, $\Delta H = \Delta E + \Delta n\,RT$

$\Delta E = \Delta H - \Delta n\,RT$

= –781000 cal –(–1.5 mole)×

(2 cal K^{-1} mol^{-1})× 298 K

= –780106 cal

= –780.1 k cal

90. **(B)** trans-2-Butene has zero dipole moment.

91. **(C)** 92. **(C)**

93. **(B)** 94. **(C)**

95. **(D)** 96. **(C)**

97. **(A)** 98. **(A)**

99. **Del** 100. **(A)**