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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION

Paper Code: UN426

Solutions for Class : 12 (PCM)

Mathematics

1. **(B)** Let S be the area and r be the radius of the plate. Then, $S = \pi r^2$

$$\Rightarrow \frac{dS}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

When r = 30 cms, we have:

$$\frac{dS}{dt} = \{(2\pi) \times 30 \times 0.025\} \text{ cm}^2/\text{sec}$$

$$\left[\therefore \frac{dr}{dt} = 0.025 \text{ cm/sec}\right]$$

$$= \frac{3\pi}{2} \text{ cm}^2/\text{sec}.$$

2. **(A)** Number of ways of forming Y such that Y has no element

= Number of ways of selecting 0 out of 5 elements of X $= {}^5C_0$;

Number of ways of forming Y such that Y has 1 element

= Number of ways of selecting 1 out of 5 elements of X $= {}^5C_1$;

Number of ways of forming Y such that Y has 2 element

= Number of ways of selecting 2 out of 5 elements of X $= {}^5C_2$;

Number of ways of forming Y such that Y has 3 element

= Number of ways of selecting 3 out of 5 elements of X $= {}^5C_3$;

Number of ways of forming Y such that Y has 4 element

= Number of ways of selecting 4 out of 5 elements of X $= {}^5C_4$;

Number of ways of forming Y such that Y has 5 element

= Number of ways of selecting 5 out of 5 elements of X $= {}^5C_5$;

∴ The total numbe rof ways of forming Y $= {}^5C_0 + {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 = 2^5$.

For each Y formed, there is only one way of forming Z i.e, Z is formed of all those elements of X not contained inY.

Hence, number of different ordered pairs (Y, Z) that can be dormed = 2^5.

3. **(A)** $A^2 - B^2 = (A-B)(A+B)$

$$\Rightarrow A^2 - B^2 = A(A+B) - B(A+B)$$

$$= A^2 + AB - BA - B^2$$

$$\Rightarrow AB - BA = O \Rightarrow AB = BA + O$$

$$\Rightarrow AB = BA \qquad [\square\ X + O = X]$$

4. **(B)** Let $\Delta = \begin{bmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{bmatrix} = xyz \begin{bmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{bmatrix}$

$\left[\begin{array}{c}\text{Taking } x, y, z \\ \text{common from} \\ R_1, R_2, R_3 \text{ resp.}\end{array}\right]$

$$= xyz \begin{bmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{bmatrix}$$

$\left[\begin{array}{c} R_2 \to R_2 - R_1, \\ R_3 \to R_3 - R_1 \end{array}\right]$

$$= xyz(y-x)(z-x) \begin{bmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+x \end{bmatrix}$$

$\left[\begin{array}{c}\text{Taking } (y-x) \\ (z-x)\text{common from} \\ R_2, \text{ and } R_3 \text{ respectively.}\end{array}\right]$

$= xyz(y-x)(z-x)(z-y)$

Now, $\Delta = 0 \quad xyz(y-x)(z-x)(z-y) = 0$

$\Delta \ xyz = 0 \quad [\square\ y - x \neq 0, z - x \neq 0, z - y \neq 0$ since $x \neq y \neq z]$

5. **(B)** Equation of the required parabola is of the form $y^2 = 4a(x - h)$. Differentiating we have

$$2y\frac{dy}{dx} = 4a \Rightarrow y\frac{dy}{dx} = 2a \Rightarrow \left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} = 0$$

The degree of this differential equation is 1 and the order is 2.

6. **(C)** Write $2ax + x^2 = (x + a)^2 - a^2$, and put $x + a = a\sec\theta$, so that $dx = a\sec\theta\tan\theta\, d\theta$

$$I = \int \frac{a\sec\theta\tan\theta}{a^3\tan^3\theta}\, d\theta$$

$$= \frac{1}{a^2}\int\frac{\cos\theta}{\sin^2\theta}\, d\theta = -\frac{1}{a^2\sin\theta} + C$$

$$= -\frac{1}{a^2}\frac{\sec\theta}{\tan\theta} + C = -\frac{1}{a^2}\frac{x+a}{\sqrt{2ax+x^2}} + C$$

7. **(B)** $-\pi/2 < \tan^{-1}x < \pi/2,\ -\pi/2 \le \sin^{-1}\frac{2x}{1+x^2} \le \pi/2$

$$\Rightarrow -3\pi/2 < 2\tan^{-1}x + \sin^{-1}\frac{2x}{1+x^2} < 3\pi/2$$

8. **(B)** $\dfrac{2(3\vec{a} - 2\vec{b}) + 3(2\vec{a} - 3\vec{b})}{2+3}$

$$= \frac{6\vec{a} - 4\vec{b} + 6\vec{a} - 9\vec{b}}{5}$$

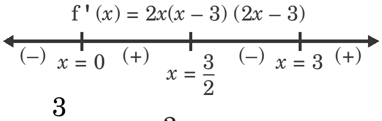
$$= \frac{12\vec{a}}{5} - \frac{13\vec{b}}{5}$$

9. **(B)** $f(x) = [x(x-3)]2$

$\Rightarrow f'(x) = 2x(x-3)(2x-3)$

Clearly, $f(x)$ increases

if $f'(x) > 0$



i.e., if $0 < x < \frac{3}{2}$ or $x > 3$

In particular, $f(x)$ increases for the values of x lying in the interval

$0 < x < \frac{3}{2}$.

10. **(B)** $Rf'(0) = \lim_{h\to0}\dfrac{f(0+h)-f(0)}{h}$

$$= \lim_{h\to0}\frac{\sqrt{1-\sqrt{1-h^2}}}{h}\times\frac{\sqrt{1+\sqrt{1-h^2}}}{\sqrt{1+\sqrt{1-h^2}}}$$

$$= \lim_{h\to0}\frac{h}{h\sqrt{1+\sqrt{1-h^2}}} = \lim_{h\to0}\frac{1}{\sqrt{1+\sqrt{1-h^2}}}$$

$$= \frac{1}{\sqrt{2}}.$$

$$Lf'(0) = \lim_{h\to0}\frac{f(0-h)-f(0)}{h}$$

$$= \lim_{h\to0}\frac{\sqrt{1-\sqrt{1-h^2}}}{-h}\times\frac{\sqrt{1+\sqrt{1-h^2}}}{\sqrt{1+\sqrt{1-h^2}}}$$

$$= \lim_{h\to0}\frac{h}{-h\sqrt{1+\sqrt{1-h^2}}} = \lim_{h\to0}\frac{-1}{\sqrt{1+\sqrt{1-h^2}}}$$

$$= \frac{-1}{\sqrt{2}}.$$

$\therefore Rf'(0) \ne Lf'(0)$.

So, $f(x)$ is not differentiable at $x = 0$.

But, $Rf'(0)$ and $Lf'(0)$ being finite, $f(x)$ is continuous at $x = 0$.

$\therefore$ The correct answer is (b).

11. **(A)** Using

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx,$$

we get

$$I = \int_0^{\pi/2}\frac{\sqrt{\operatorname{cosec} x}}{\sqrt{\sec x}+\sqrt{\operatorname{cosec} x}}dx \qquad (2)$$

Adding (1) and (2), we get

$$2I = \int_0^{\pi/2} dx = \frac{\pi}{2}$$

$$\Rightarrow I = \pi/4.$$

12. **(C)** f is one-one since for $x, y \in R$ such that $x \neq y$, we have:

$$f(x) = f(y) \Rightarrow \frac{x-m}{x-n} = \frac{y-m}{y-n}$$

$\Rightarrow xy - nx - my + mn = xy - mx - ny + mn$

$\Rightarrow (m - n)x = (m - n)y \Rightarrow x = y$

[∵ $m \neq n$ $m - n \neq 0$]

Now, $z = \dfrac{x-m}{x-n} zx - zn = x - m \Rightarrow (z-1)$

$x = zn - m$

$$\Rightarrow x = \frac{zn-m}{z-1}$$

Clearly, x is not defined when $z = 1$.

$1 \in R$ has no pre-image in R.

Thus, f is into,

Hence, f is one-one into.

13. **(B)** $\begin{bmatrix} a & 2 & 3 \\ b & 5 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} a+6-3 & 2a+8+3 \\ b+15+1 & 2b+20-1 \end{bmatrix}$$

$$= \begin{bmatrix} a+3 & 2a+11 \\ b+16 & 2b+19 \end{bmatrix}$$

$$\therefore \begin{bmatrix} a+3 & 2a+11 \\ b+16 & 2b+19 \end{bmatrix} = \begin{bmatrix} 4 & 13 \\ 12 & 11 \end{bmatrix}$$

Equating the corresponding elements of equal matrices, we get:

$a + 3 = 4, b + 16 = 12 \Rightarrow a = 1, b = -4$

$\therefore (a, b) = (1, -4)$

14. **(B)** For continuity at $x = 0$, we must have:

$$f(0) = \lim_{x \to 0} f(x) = \lim_{x \to 0} (x+1)^{\cot x}$$

Now, let $L = \lim_{x \to 0} (x+1)^{\cot x}$

Then, $\log L = \lim_{x \to 0} (\cot x) \log (x+1)$

$$= \lim_{x \to 0} \frac{\log(x+1)}{\tan x} \left[\frac{0}{1} \text{ form}\right]$$

$$= \lim_{x \to 0} \left[\frac{\left(\frac{1}{x+1}\right)}{\sec^2 x}\right] = \lim_{x \to 0} \frac{\cos^2 x}{(x+1)} = \frac{1^2}{(0+1)} = 1$$

$\therefore L = e^1 = e.$

Thus, $f(0) = e$.

Hence, we must define $f(x)$ as $f(0) = e$.

$\therefore$ The correct answer is (b).

15. **(D)** $s \propto \sqrt[3]{v}$

$\Rightarrow s = kv^{1/3}$ where k = constant

$\Rightarrow v = cs^3$ ...(i) where $c = \dfrac{1}{k^3}$ = constant

$$\Rightarrow \frac{ds}{dt} = cs^3$$

$$\Rightarrow \frac{d^2s}{dt^2} = 3cs^2 . \frac{ds}{dt}$$

$\Rightarrow a = 3cs^2.v$ [Using (i)]

$\Rightarrow a = 3c\,s^2.(cs^3)$

$\Rightarrow a = 3c^2\,s^5$

$\Rightarrow a \propto s^5$.

16. **(C)** Let $f(x) = \sin x + \cos x$.

Then, $f'(x) = \cos x - \sin x$

and $f''(x) = -\sin x - \cos x$.

Now, $f'(x) = 0 \Rightarrow \cos x - \sin x = 0 \Rightarrow \cos x = \sin x$

$$\Rightarrow x = \frac{\pi}{4} \text{ or } x = \frac{5\pi}{4}.$$

When $x = \dfrac{\pi}{4}$, we have : $f''(x) = f''\left(\dfrac{\pi}{4}\right) = \dfrac{-1}{\sqrt{2}} - \dfrac{1}{\sqrt{2}} = -\sqrt{2} < 0.$

When $x = \dfrac{5\pi}{4}$, we have : $f''(x) = f''\left(\dfrac{5\pi}{4}\right) = \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}$

$= \sqrt{2} > 0.$

Clearly, $x = \dfrac{\pi}{4}$ is the point of maxima.

$\therefore$ Maximum value of $f(x) = f\left(\dfrac{\pi}{4}\right) = \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}} = \sqrt{2}$

Another Method :

$f(x) = \sin x + \cos x = \sqrt{2}\left(\dfrac{1}{\sqrt{2}} \sin x + \dfrac{1}{\sqrt{2}} \cos x\right)$

$= \sqrt{2}\left(\sin x \cos\dfrac{\pi}{4} + \cos x \sin\dfrac{\pi}{4}\right) = \sqrt{2} \sin\left(x + \dfrac{\pi}{4}\right).$

Now, maximum value of the $\sin\left(x + \dfrac{\pi}{4}\right)$ is 1. [∵ $\sin\theta \leq 1 \ \forall\ \theta \in R$]

$\therefore$ Maximum value of $f(x) = \sqrt{2}.1 = \sqrt{2}$.

17. **(C)** $I = \int \frac{1-\sin x}{1-\sin^2 x}dx = \int(\sec^2 x - \sec x \tan x)dx$

$= \tan x - \sec x + C$

18. **(C)** We have :

If $a \in N$, then $a \not R\ a$ since a is not less than a.

$\therefore$ R is not relfexive.

If $a, b \in N$ then,

$a R b \Rightarrow a < b \not\Rightarrow b < a$ i.e. $b \not R\ a$.

$\therefore R$ is not symmetric.

If $a, b, c, \in N$ then,

$a R b, b R c \Rightarrow a < b$ and $b < c \Rightarrow a < c \Rightarrow a R c$.

$\therefore R$ is transitive.

19. **(B)** A is orthogonal $AA^T = 1$

$|AA^T| = |I| = 1$ [Taking determinant on both sides]

$|A|^2 = 1$ [□ $|AA|^T = |A|\ |A|^T = |A|^2$ since $|A|^T = |A|$]

$|A| = \pm 1$

20. **(B)** We know that if G is the centroid of the ΔABC, then $\overrightarrow{GA} + \overrightarrow{BG} + \overrightarrow{GC} = \overrightarrow{0}$

$\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} + 2\,\overrightarrow{BG} = 2\,\overrightarrow{GB}$

$\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} + (\overrightarrow{GB} + \overrightarrow{BG}) = 2\,\overrightarrow{BG}$

$\overrightarrow{GA} + \overrightarrow{BG} + \overrightarrow{GC} = 2\,\overrightarrow{BG}$

$[\because \overrightarrow{GB} + \overrightarrow{BG} = \overrightarrow{GG} = \overrightarrow{0}]$

21. **(B)** We have:

$f(x) = \frac{x}{x-1}$

$\Rightarrow (fof)(x) = f[f(x)]$

$= f\left(\frac{x}{x-1}\right) = \frac{\left(\frac{x}{x-1}\right)}{\left(\frac{x}{x-1}-1\right)} = \frac{\left(\frac{x}{x-1}\right)}{\frac{1}{x-1}} = x$

$\Rightarrow (fofof)(x) = f[(fof)(x)] = f(x) = \frac{x}{x-1}$

Proceeding in this manner we have:

$\underbrace{(f o f o \ldots . o f)}_{n \text{ times}}(x)$

$= \begin{cases} x, & \text{when } n \text{ in even} \\ \frac{x}{x-1}, & \text{when } n \text{ is odd} \end{cases}$

$\underbrace{(f o f o \ldots . o f)}_{19 \text{ times}}(x) = \frac{x}{x-1}.$

22. **(A)** We have: $(B^TAB) = B^TA^T(B^T)^T = B^TA^TB$ ..(i)

Clearly, $(B^TAB)^T = B^TAB$ iff $A^T = A$

i.e (B^TAB) is symmetric if and only if A is symmetric.

Also, $(B^TAB)^T \neq -(B^TAB)$ if B is skew symmetric. [Using B]

i.e. B^TAB cannot be skew-symmetric for every matrix A or if B is skew-symmetric.

23. **(B)** Differentiating we have

$\frac{dy}{dx} = c\,k\,x^{k-1} \Rightarrow c = \frac{1}{k}x^{1-k}\frac{dy}{dx}$

Putting this value in the given equation, we have

$y = \frac{1}{k}x^{1-k}\frac{dy}{dx}x^k = \frac{1}{k}x\frac{dy}{dx}.$

Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$, we get

$y = -\frac{1}{k}x\frac{dx}{dy} \Rightarrow k\,y\,dy + x\,dx$

$= 0 \Rightarrow k\,y^2 + x^2 = \text{const.}$

24. **(B)** Put $t = \frac{1}{2}(x - a + x - b) = x - \frac{1}{2}(a + b)$, so that

$(x - a)(b - x) = (t + c)(c - t) = c^2 - t^2$

where $c = \frac{1}{2}(b - a)$.

Thus,

$I = \int_{-c}^{c}\frac{dx}{\sqrt{(c^2 - t^2)}}$

$= 2\int_0^c \frac{dx}{\sqrt{(c^2-t^2)}} = 2\sin^{-1}\left(\frac{t}{c}\right)\Big]_0^c$

$= 2[\sin^{-1}(1) - 0] = \pi$

25. **(B)** Let $x_1 < x_2$

Then, $f(x_1) > f(x_2)$ [$\therefore$ f is decreasing]

$\Rightarrow g\{f(x_1)\} < g\{f(x_2)\}$ [$\therefore$ g is decreasing]

$\Rightarrow gof(x_1) < gof(x_2)$

$\therefore$ gof is increasing.

26. **(B)** $f(5) = f(5 + 0) = f(5).\ f(0) \Rightarrow f(0) = 1.$

$f'(5) = \lim_{h\to 0}\frac{f(5+h)-f(5)}{h}$

$= \lim_{h\to 0}\frac{f(5).f(h).-f(5)}{h}$

$= f(5).\lim_{h\to 0}\frac{f(h)-1}{h} = f(5).\lim_{h\to 0}\frac{f(h)-f(0)}{h}$

$= f(5) \times f'(0) = (2 \times 3) = 6$

$\therefore$ The correct answer is (B).

27. **(C)** $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$,

$\Rightarrow \cos^{-1} x + \cos^{-1} y = \pi - \cos^{-1} z$

$\Rightarrow \cos(\cos^{-1} x + \cos^{-1} y) = -z$

$\Rightarrow xy - \sqrt{1-x^2}\,\sqrt{1-y^2} = -z$

$\Rightarrow (xy+z)^2 = (1-x^2)(1-y^2)$

$\Rightarrow x^2 + y^2 + z^2 + 2xyz = 1$

28. **(C)** R is reflexive since $(x, x)\, R \;\forall\; x \in A$.

R is not symmetric since $(3, 6) \in R$ but $(6, 3) \notin R$.

Now, if $x, y, z \in A$, then $(x, y) \in R$, $(y, z) \in R \Rightarrow (x, z) \in R$.

$\therefore R$ is transitive.

29. **(D)** Let $\Delta = \begin{vmatrix} a & -b & -c \\ -a & b & -c \\ -a & -b & 0 \end{vmatrix} = abc \begin{vmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{vmatrix}$

$\begin{bmatrix} \text{Taking } a, b, c \\ \text{common from} \\ C_1, C_2, C_3 \text{ resp.} \end{bmatrix}$

$= abc \begin{vmatrix} 1 & 0 & 0 \\ -1 & 0 & -2 \\ -1 & -2 & 0 \end{vmatrix} = -4\,abc$

$[C_2 \to C_1 + C_2,\ C_3 \to C_1 + C_3]$

Now, $\Delta + \lambda\, abc = 0$

$\Rightarrow -4abc + \lambda\, abc = 0 \Rightarrow \lambda = 4.$

30. **(B)** Putting $p = \frac{dy}{dx}$, the given equation can be written as $y = px + 1/p$. Differentating w.r.t. x, we have

$p = \frac{dy}{dx} = p + x\frac{dp}{dx} - \frac{1}{p^2}\frac{dp}{dx}$

$\Rightarrow (x - (1/p^2))\frac{dp}{dx} = 0 \Rightarrow p^2 = \frac{1}{x}$ or $\frac{dp}{dx} = 0$

If $\frac{dp}{dx} = 0$ then p = constant = c putting this value in given equation, we get $y = cx + 1/c$ which represents a straight line. If $p^2 = 1/x$ then $y^2 = (px + 1/p)^2 = p^2x^2 + 1/p^2 + 2x = \frac{1}{x}x^2 + x + 2x = 4x$, which represents a parabola.

31. **Delete**

32. **(D)** $f(x) = x^3 - 3x^2 + 2x + 10$

$\Rightarrow f'(x) = 3x^2 - 6x + 2$

Now, $f'(x) = 0 \Rightarrow 3x^2 - 6x + 2 = 0$

$\Rightarrow x = \frac{6 \pm \sqrt{12}}{6} = \frac{3 \pm \sqrt{3}}{3}$

We neglect $x = \frac{3-\sqrt{3}}{3} \notin (1, 2)$

$\therefore x = \frac{3+\sqrt{3}}{3}$

33. **(C)** We have $\sqrt{1-\cos 2x} = \sqrt{2}\,|\sin x|$. Therefore, since $|\sin x|$ has the period π, we have

$\int_0^{100\pi} \sqrt{1-\cos 2x}\,dx$

$= \sqrt{2}$

$\int_0^{100\pi} |\sin x|\,dx$

$= 100\sqrt{2}\int_0^{\pi} \sin x\,dx = 200\sqrt{2}.$

$\therefore \sqrt{2}\int_0^{100\pi} \sqrt{1-\cos 2x}dx$

$= \sqrt{2}\left(200\sqrt{2}\right)$

$= 400$

34. **(D)** $A = \begin{bmatrix} i & 1-2i \\ -1-2i & 0 \end{bmatrix}$

We have : $\overline{A} = \begin{bmatrix} -i & 1+2i \\ -1+2i & 0 \end{bmatrix}$

$\therefore A^\theta = \left(\overline{A}\right)^T = \begin{bmatrix} -i & 1+2i \\ -1+2i & 0 \end{bmatrix}^T$

$= \begin{bmatrix} -i & -1+2i \\ 1+2i & 0 \end{bmatrix}$

Also, $-A = \begin{bmatrix} -i & -1+2i \\ 1+2i & 0 \end{bmatrix}$

Clearly, $A^\theta = -A$ and so A is a Skew-Hermitian Matrix.

35. **(B)** $f(x) = \sin^2 x$ and $g\{f(x)\} = |\sin x|$

$\Rightarrow g\{\sin^2 x\} = |\sin x| \Rightarrow g(x) = \sqrt{x}$

36. **(C)** Let $y = \frac{x^2}{2} - 2x + 5$

Then, rate of decrease of $y = \frac{dy}{dt} = (x-2)\frac{dx}{dt}$

Let $z = 2x$

Then, rate of decrease of z = $\frac{dz}{dt}$ = 2. $\frac{dx}{dt}$

Now, rate of decrease of y = 2 × (rate of decrease of z)

$\Rightarrow \frac{dy}{dt} = 2 \times \frac{dz}{dt}$

$\Rightarrow (x-2)\frac{dx}{dt} = 2.\left(2.\frac{dx}{dt}\right)$

$\Rightarrow x - 2 = 4 \Rightarrow x = 6$

37. **(A)** f (x) is an even function

$\Rightarrow f'(x)$ is odd

$\Rightarrow f'(-x) = -f'(x)$

$\Rightarrow f'(-0) = -f'(0) \Rightarrow 2f'(0) = 0 \Rightarrow f'(0) = 0.$

∴ The correct answer is (A).

38. **(C)** Let $\tan^{-1} 1/3 = \alpha$ and $\tan^{-1} 2\sqrt{2} = \beta$. Then $\tan\alpha = 1/3$ and $\tan\beta = 2\sqrt{2}$, so that $\sin(2\tan^{-1}(1/3)) + \cos(\tan^{-1} 2\sqrt{2}) = \sin 2\alpha + \cos\beta$

$= \frac{2\tan\alpha}{1+\tan^2\alpha} + \frac{1}{\sqrt{1+\tan^2\beta}}$

$[\because -\pi/2 < \beta < \pi/2]$

$= \frac{2(1/3)}{1+(1/9)} + \frac{1}{\sqrt{1+8}} = \frac{2}{3}.\frac{9}{10} + \frac{1}{3}$

$= \frac{3}{5} + \frac{1}{3} = \frac{14}{15}$

39. **(B)** Ellipses centred at origin are given by

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (i)

where a and b are unknown constants.

$\frac{2x}{a^2} + \frac{2y}{b^2}y_1 = 0 \Rightarrow \frac{x}{a^2} + \frac{y}{b^2}y_1 = 0.$ (ii)

Differentiating again, we have

$\frac{1}{a^2} + \frac{1}{b^2}\left(y_1^2 + yy_2\right) = 0$

Multiplying (iii) with x and then subtracting from (ii), we have

$\left(1/b^2\right)\left(yy_1 - xy_1^2 - xyy_2\right) = 0$

$\Rightarrow xyy_2 + xy_1^2 - yy_1 = 0$

40. **(D)** $f(x) = \frac{2x-1}{x+5}, x \neq -5 \Rightarrow y = \frac{2x-1}{x+5}$ where $y = f(x)$

$\Rightarrow xy + 5y + 2x - 1 \Rightarrow x(2-y) = (5y+1) \Rightarrow$

$x = \frac{5y+1}{2-y}, y \neq 2$

$\Rightarrow f^{-1}(y) = \frac{5y+1}{2-y}, y \neq 2$

$[y = f(x) \Rightarrow x = f{-}1(y)]$

$\Rightarrow f^{-1}(x) = \frac{5x+1}{2-x}, x \neq 2$

Physics

41. **(B)** Using Ampere's circuital law over a circular loop of any radius less than the radius of the pipe, we can see that net current inside the loop is zero. Hence, magnetic field at every point inside the loop will be zero.

42. **(C)** Electric field strength, $E = \frac{V}{d}$

$= \frac{10}{0.10 \sin 30^\circ} = \frac{10}{0.10 \times \frac{1}{2}}$ **=** 200 V/m

Perpendicular to line of force, i.e., at 120° with X-axis.

43. **(A)** Magnifying power of compound microscope $M = m_o \times m_e$

For objective, $\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$

$\frac{1}{1/4} = \frac{1}{v_o} - \frac{1}{-1/3.8}$

$4 = \frac{1}{v_o} + 3.8 \Rightarrow v_o = 5$ cm

$\therefore m_o = \frac{v_o}{u_o} = \frac{5}{1/3.8} = 19$

Hence, $m_e = \frac{m}{m_o} = \frac{95}{19} = 5$

44. **(C)** $l_1 : l_2 = 1 : 3$, $r_1 : r_2 = 3 : 1$,

Area $A = \pi r^2$

$A_1 : A_2 = r_1^2 : r_2^2 = 9 : 1$

They are made of the same material. So, the resistivity ρ is the same.

$\rho = \frac{RA}{l}, \frac{R_1 A_1}{l_1} = \frac{R_2 A_2}{l_2}$

$\frac{R_1}{R_2} = \frac{l_1}{l_2} \times \frac{A_2}{A_1} = \frac{1}{3} \times \frac{9}{1} = \frac{3}{1}$

$R_1 : R_2 = 3 : 1$

45. **(D)** Here, $\frac{dI}{dt} = 4\ A\ s^{-1}$;

$e = 20\ mV = 20 \times 10^{-3}\ V$

Now, $e = L\frac{dI}{dt}$ (in magnitude)

$L = \frac{e}{dI/dt} = \frac{20 \times 10^{-3}}{4} = 5 \times 10^{-3}$ H

46. **(A)** Velocity of electron in the nth orbit = $\frac{1}{137} \times \frac{c}{n}$.

Time taken to travel the first orbit = $\frac{\text{Distance}}{\text{Velocity}} = \frac{2\pi r}{V}$

$= \frac{2\pi r \times 137 \times n}{c}$

$= \frac{2 \times 3.14 \times 5.29 \times 10^{-11} \times 137 \times 1}{3 \times 10^{8}}$

$= 1.517 \times 10^{-16}$ s.

47. **(D)** There will be no force on electron due to magnetic field (because of parallel motion), but due to force applied by electric field, velocity of electron will decrease.

48. **(C)** Forces applied on the charge at the centre due to charges at A and C will cancel each other, so net force will be along diagonal BD.

49. **(A)** In a non-uniform magnetic field, the magnetic needle will experience both a force and a torque.

50. **(C)** Power P = 10 watt = 10 J/s.

Let n be the number of photons emitted per second

$P = nh\nu = \frac{nh\nu}{\lambda} = 10$

$n = \frac{10\,\lambda}{hc} = \frac{10 \times 6000 \times 10^{-10}}{6.62 \times 10^{-34} \times 3 \times 10^{8}}$

$= 3.02 \times 10^{10}$ per sec.

51. **(B)** Heat produced = $I^2 Rt$

or $mc\,\Delta\theta = I^2 Rt$

$\therefore$ Rise in temperature $\Delta\theta \propto I^2$

$\therefore \frac{\Delta\theta_1}{\Delta\theta_2} = \left(\frac{I_1}{I_2}\right)^2$

$\frac{3}{\Delta\theta_2} = \left(\frac{I}{2I}\right)^2$

$\therefore \Delta\theta_2 = 4 \times 3 = 12^\circ C$

52. **(D)** $\omega = \frac{\lambda D}{d}$

d is halved and D is doubled

$\therefore$ Fringe width ω will become four times.

53. **(B)** Initial amount of radioactive element

$= X + Y = 1 + 7 = 8$

Now, $N = N_0 \left(\frac{1}{2}\right)^n$

$\therefore\ 1 = 8\left(\frac{1}{2}\right)^n \Rightarrow \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^n$

or n = 3

Hence, required time period

$t = n\,T_{1/2} = 3 \times 2 = 6$ h

54. **(B)** Infrared spectrum lies between microwave and visible region.

55. **(C)** Here, $h = 6.6 \times 10^{-34}$ J s ;
$m = 9.1 \times 10^{-31}$ kg ;

$1\text{ eV} = 1.6 \times 10^{-19}$ J

$E = 400\text{ eV} = 400 \times 1.6 \times 10^{-19}$
$= 6.4 \times 10^{-17}$ J

If λ is de-Broglie wavelength of electron, then

$\lambda = \frac{h}{\sqrt{2\,m\,E}}$

$= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 6.4 \times 10^{-17}}}$

$= \frac{6.6 \times 10^{-34}}{10.8 \times 10^{-14}} = 0.61 \times 10^{-10}$ m

$= 0.61\ \text{Å}$

56. **(A)** Here, $R = 10\ \Omega$; $E_v = 220$ V ; f = 50 Hz ; $I_v = 2.0$ A ;

If Z is impedance of the CR-circuit, then

$Z = \frac{E_v}{I_v} = \frac{220}{2.0} = 110\ \Omega$

But $Z = \sqrt{R^2 + X_C^2}$

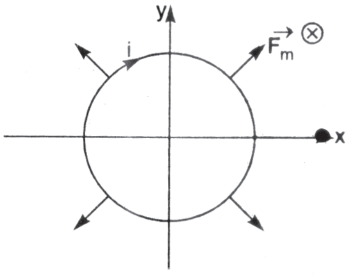
or $X_C = \sqrt{Z^2 - R^2} = \sqrt{110^2 - 10^2}$

$= \sqrt{120 \times 100} = 109.54\ \Omega$

57. **(A)** A junction diode conducts during alternate half cycles of a.c. input supply. During a half cycle of conduction, the capacitor will charge itself to peak value of the supply voltage. Therefore, voltage across the capacitor,

$V = E_0 = E_{r.m.s.} \times \sqrt{2} = 220 \times \sqrt{2}$
$= 311.1$ V

58. **(B)** Net force on a current carrying loop in uniform magnetic field is zero. Hence, the loop cannot translate. So, options (C) and (D) are wrong. From Fleming's left hand rule we can see that if magnetic field is perpendicular to paper inwards and current in the loop is clockwise (as shown) the magnetic force $\vec{F}_m$ on each element of the loop is radially outwards, or the loops will have a tendency to expand.



59. **(B)** Threshold wavelength

$= \lambda_0 = 6800\ \mathring{A} = 6800 \times 10^{-10}$ m.

Work function,

$$W = \frac{hc}{\lambda_0} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{6800 \times 10^{-10}}$$

$= 2.92 \times 10^{-19}$ J

$$= \frac{2.92 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.825 \text{ eV.}$$

60. **(C)** Here, mass of the copper penny, m = 3.11 g

Z = 29 ; A = 63.5 and N = 6.02×10^{23}

Number of atoms in the copper penny,

$$n = \frac{N}{A} \times m$$

$$= \frac{6.02 \times 10^{23} \times 3.11}{63.5} = 2.95 \times 10^{22}$$

Number of protons or electrons in the copper penny

$= n \times Z$

$= 2.95 \times 10^{22} \times 29$

Therefore, total positive or negative charge on the copper penny,

$q = 2.95 \times 10^{22} \times 29 \times 1.6 \times 10^{-19}$

$= 1.37 \times 10^{5}$ C

61. **(D)** Speed of X-rays does not depend on the applied voltage. It is same as speed of light i.e., 3×10^{8} m/s.

62. **(B)** Number of atoms in 1 g of U^{235}

$$= \frac{\text{Avogadro number}}{\text{Atomic weight}} = \frac{6.023 \times 10^{23}}{235}$$

Energy released per fission = 200 MeV

Therefore, energy released on fission of 1 g of U^{235}

$$= \frac{6.023 \times 10^{23} \times 200}{235} = 5.126 \times 10^{23} \text{ MeV}$$

$= 5.126 \times 10^{23} \times 1.6 \times 10^{-13}$ J

$= 8.2 \times 10^{10}$ W s ($\because$ 1 J = 1 W s)

$$= \frac{8.2 \times 10^{10}}{1000} = 8.2 \times 10^{7} \text{ kWs}$$

$$= \frac{8.2 \times 10^{7}}{3600} = 2.278 \times 10^{4} \text{ kWh}$$

63. **(C)** Here, $X_L = 160\ \Omega$; f= 50 Hz

Now, $X_L = 2\pi fL$ or $L = \frac{X_L}{2\pi f}$

$$= \frac{160}{2\pi \times 50} = 0.51 \text{ H}$$

64. **(D)** $R_0 = 10\ \Omega$, $R = 20\ \Omega$, $t = 273°$C

Temp. coefficient

$$= \alpha = \frac{R - R_0}{R_0 \times t} = \frac{20 - 10}{10 \times 273} = 0.00366/°C$$

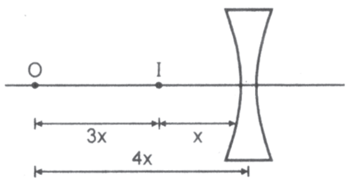
65. **(D)** Concave lens forms the virtual image of a real object. So let

$u = -4x$ and $v = -x$ then $3x = 10$ cm

or $x = \frac{10}{3}$ cm

$\therefore\ u = -\frac{40}{3}$ cm

and $v = -\frac{10}{3}$ cm



Substituting in $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

We get $\frac{1}{f} = \frac{-3}{10} + \frac{3}{40}$

or $f = \frac{-40}{9}$

or $f = -4.4$ cm

Chemistry

66. **(B)** Double salt of $Cu(CH_3COO)_2$ and copper arsenite $Cu_3(AsO_3)_2$ is called Paris green.

67. **(D)** Ferrocyanide ion is a complex ion. Hence, it is a complex salt.

68. **(D)** Aldehydes give silver mirror with ammoniacal $AgNO_3$ but ketones do not.

69. **(D)** $(NH_4)_2Cr_2O_7 \xrightarrow{heat} Cr_2O_3 + 4H_2O + N_2(g)$

70. **(A)** Amount of NaCl = $\frac{1.00\ g}{58.5\ g/mol}$

No. of unit cells in 1.0 g of NaCl

$$= \frac{6.02 \times 10^{23} mol^{-1}}{4} \times \frac{1.00}{58.5\ g\ mol^{-1}}$$

$= 2.57 \times 10^{21}$

71. **(C)** There are 4 possible isomeric amines of C_3H_9N. shown below.

$C_3H_7\ NH_2$,	$CH_3-\underset{NH_2}{\overset{H}{C}}-CH_3$,	$C_2H_5\ NH\ CH_3$	$(CH_3)_3N$
1-Propanamine and $(CH_3)_3$ N N, N-dimethyl methanamine.	2- propanamine	N-methyl ethane amine	N,N-dimethyl methanamine.

72. **(B)** Benzyl chloride and ethyl bromide both have labile halogen and hence are easily hydrolysed. Since a white ppt. soluble in NH_4OH is obtained, therefore, the compound X is benzyl chloride.

73. **(C)** Heterocyclic base at C_1' and carbohydrate unity at C_5'.

74. **(B)** $CH_3-CH_2-CH_2Br \xrightarrow[20^\circ C,\ -HBr]{NaOH\ (alc.)} CH_3-CH$ $= CH_2 \xrightarrow[20^\circ C]{HBr/acetic\ acid}$

$$CH_3-\underset{Br}{\underset{|}{CH}}-CH_3$$

75. **(B)** 3 d^5 configuration has five unpaired electrons. Hence highest magnetic moment is shown by the transition metal having this configuration.

76. **(C)** Their molar concentrations will be

Glucose = $\frac{10}{180} \times m = 0.05$ M,

Sucrose = $\frac{10}{342} \times M = 0.02$ M

NaCl = $\frac{10}{58.5} \times 2\ M = 0.34$ M

$CaCl_2 = \frac{10}{111} \times 3\ M = 0.27$ M

Thus, concentration of particles in NaCl is highest.

77. **(A)** Normality of oxalic acid solution

$= \frac{6.3\ g \times 4}{63\ g} = 0.4$

So, $0.4 \times 10\ mL = 0.1 \times V_{NaOH}$

So, $V_{NaOH} = \frac{0.4 \times 10\ mL}{0.1}$

= 40 mL

78. **(D)** For a first order reaction, Rate = k [Reactant]

So, $k = \frac{Rate}{[Reactant]}$

$= \frac{1.5 \times 10^{-2}\ mol\ L^{-1}\ min^{-1}}{0.5\ mol\ L^{-1}}$

$= 3 \times 10^{-2}\ min^{-1}$

and $t_{1/2} = \frac{0.693}{k} = \frac{0.693}{3 \times 10^{-2}\ min^{-1}}$

= 23.1 min

79. **(D)** Since C—I bond is the weakest and I^- is a better leaving group than Br^-, Cl^- and F^- ions, therefore, C_2H_5I is the most reactive alkyl halide.

80. **(A)** As in NaCl, Cl^- ions are arranged in ccp and Na^+ ions occupy octahedral voids and radius of octahedral void = 0.414 R, i.e.,

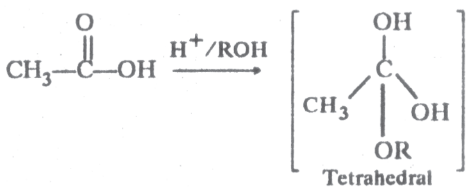
$r_{A+} = 0.414 \times r_{B-}$

or $r_{B-} = \frac{100}{0.414} = 241$ pm

81. **(B)** Ketones on reduction with Zn + HCl give the corresponding hydrocarbon.

82. **(A)** He—O_2 (80% : 20%) mixture is used by deep sea divers for artificial respiration. Because of low intermolecular forces in He, it is much less soluble in aqueous solutions (as compared to N_2) such as blood and does not cause "caisson sickness" or "bends" by bubbling out of blood when the worker moves from high pressure (while in deep sea) to atmospheric pressure.

83. **(A)** As the size of the hydrocarbon part (R) of the alcohol increases, the tetrahedral intermediate (I) formed during esterificaiton becomes more and more crowded and hence the rate of esterification decreases accordingly. Thus, the smallest alcohol, i.e., CH_3OH reacts at the fastest rate.



Tetrahedral intermediate (I)

84. **(C)** Lyophobic means solvent-hating. Metallic dipersions are lyophobic in nature.

85. **(C)** Wilkinson's catalyst is used for the hydrogenation of alkenes.

86. **(D)** Aromatic 1° amines give coupling reactions but aliphatic 1° amines do not.

87. **(A)** $Fe^{2+} + 6\,CN^- \rightarrow [Fe(CN)_6]^{4-}$.

88. **(C)** During acetylation, one H-atom (at. mass = 1 a.m.u.) of the NH_2 group is replaced by an acetyl group, i.e., 43 (mol. mass = 43 a.m.u.)

$$-NH_2 + CH_3COCl \rightarrow -NH-COCH_3 + HCl$$

In other words, acetylation of each NH_2 group increases the mass by 43—1 = 42 a.m.u. Now the mol. mass of the compound is 180 while that of the acetylated compound is 390, therefore, the number of NH_2 groups in the compound

$$= \frac{390-180}{42} = 5$$

89. **(C)** Halogens are coloured because their molecules absorb visible light causing the excitation of outer electrons to higher energy levels.

90. **(A)** $\Lambda^o_{(NaBr)} = \Lambda^o_{NaCl} + \Lambda^o_{KBr} - \Lambda^o_{KCl}$

$= (126 + 152 - 150)\ S\ cm^2\ mol^{-1}$

$= 128\ S\ cm^2\ mol^{-1}$

91. **(B)** 92. **(C)** 93. **(A)** 94. **(B)** 95. **(D)**

96. **(C)** 97. **(A)** 98. **(A)** 99. **(B)** 100. **(B)**