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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION

Paper Code: **UN436 (UPDATED)**

Solutions for Class : 11-PCM

MATHEMATICS

1. (B) OA = OA, OA = OB
 \Rightarrow OB = OA OA = OB, OB = OC
 \Rightarrow OA = OC
 Hence R, S, T i.e. an equivalence relation.

2. (A) Since, $5x = 3x + 2x$
 $\Rightarrow \tan 5x = \tan (3x + 2x)$
 $\Rightarrow \tan 5x = \frac{\tan 3x + \tan 2x}{1 - \tan 3x \tan 2x}$
 $\Rightarrow \tan 5x - \tan 5x \tan 3x \tan 2x$
 $= \tan 3x + \tan 2x$
 $\Rightarrow \tan 5x \tan 3x \tan 2x = \tan 5x$
 $- \tan 3x - \tan 2x.$

3. (A) $\sin \alpha + \cos \alpha = m \Rightarrow 1 + \sin 2 \alpha = m^2$
 $\Rightarrow \sin 2 \alpha = m^2 - 1$
 Now, $\sin^6 \alpha + \cos^6 \alpha$
 $= (\sin^2 \alpha + \cos^2 \alpha)^3 - 3 \sin^2 \alpha \cos^2 \alpha (\sin^2 \alpha + \cos^2 \alpha)$
 $= 1 - 3 \frac{(-1 + m^2)^2}{4} = \frac{4 - 3(m^2 - 1)^2}{4}$

4. (B) Let P (n):
 $\frac{1}{1} + \frac{1}{1+2} + \frac{1}{1+2+3} + \dots (n+1) \text{ terms}$
 $t_n = \frac{1}{1+2+3+\dots+n} = \frac{2}{n(n+1)}$
 $\Rightarrow t_n = 2 \left[\frac{1}{n} - \frac{1}{n+1} \right]$
 $\Rightarrow P(n): \left\{ \left(1 - \frac{x}{2} \right) + \left(\frac{x}{2} - \frac{x}{3} \right) + \dots \right\}$

$$\dots + \left(\frac{x}{n} - \frac{1}{n+1} \right) \}$$

$$\therefore P(n) = 2 \left(1 - \frac{1}{n+1} \right) = \frac{2n}{n+1}$$

5. (A) $|\sqrt{2i} - \sqrt{-2i}| = |\sqrt{2i} - i\sqrt{2i}| = |\sqrt{2i}(1-i)|$
 $= \sqrt{2} |i| |1-i| = \sqrt{2} \times 1 \sqrt{1^2 + (-1)^2} = \sqrt{2} \times \sqrt{2} = 2$

6. (B) Given inequations are
 $2x - 1 \leq 3, 3x + 1 \geq -5$
 $\Rightarrow 2x \leq 4, 3x \geq -6$
 $\Rightarrow x \leq 2, x \geq -2$
 $\Rightarrow -2 \leq x \leq 2$

7. (C) We have,
 $\frac{\left(\sin \frac{\pi}{8} + i \cos \frac{\pi}{8} \right)^8}{\left(\sin \frac{\pi}{8} - i \cos \frac{\pi}{8} \right)^8}$
 $\frac{i^8 \left(\cos \frac{\pi}{8} - i \sin \frac{\pi}{8} \right)^8}{(-i)^8 \left(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8} \right)^8}$
 $\frac{\cos \pi - i \sin \pi}{\cos \pi + i \sin \pi} = 1$

8. (C) The required number
 $= \text{coeff. of } x^{2m} \text{ in } (x^0 + x^1 + \dots + x^m)^4$
 $= \text{coeff. of } x^{2m} \text{ in } \left(\frac{1-x^{m+1}}{1-x} \right)^4$

$$\begin{aligned}
&= \text{coeff. of } x^{2m} \text{ in } (1-x^{m+1})^4 (1-x)^{-4} \\
&= \text{coeff. of } x^{2m} \text{ in } (1-4x^{m+1}+6x^{2m+2}+\dots) \\
&\left(1+4x+\dots+\frac{(r+1)(r+2)(r+3)}{3!}x^r+\dots\right) \\
&= \frac{(2m+1)(2m+2)(2m+3)}{6} - 4m \frac{(m+1)(m+2)}{6} \\
&= \frac{(m+1)(2m^2+4m+3)}{3}
\end{aligned}$$

9. (B) $(1+x-2x^2)^6 = 1+a_1x+a_2x^2+\dots+a_{12}x^{12}$
substituting $x=1$ and $x=-1$, we get

$$\begin{aligned}
1+a_1+a_2+\dots+a_{12} &= 0 \\
1-a_1+a_2-\dots+a_{12} &= 2^6 \\
\hline
2+2a_2+\dots+2a_{12} &= 2^6 \\
2[1+a_2+a_4+\dots+a_{12}] &= 2^6 \\
\text{or } 1+a_2+a_4+\dots+a_{12} &= 2^5 \text{ or } (32)
\end{aligned}$$

$$a_2+a_4+\dots+a_{12} = 32-1 = 31$$

10. (D)
$$\begin{aligned}
&\frac{1}{\sqrt{4x+1}} \left\{ \left(\frac{1+\sqrt{4x+1}}{2} \right)^7 - \left(\frac{1-\sqrt{4x+1}}{2} \right)^7 \right\} \\
&= \frac{1}{2^7 \sqrt{4x+1}} \{ 2^7 C_1 \sqrt{4x+1} + 7 C_3 (\sqrt{4x+1})^3 \\
&\quad + 7 C_5 (\sqrt{4x+1})^5 + 7 C_7 (\sqrt{4x+1})^7 \} \\
&= \frac{1}{2^6} \{ 7 C_1 + 7 C_3 (4x+1) + 7 C_5 (4x+1)^2 + 7 C_7 (4x+1)^3 \}
\end{aligned}$$

Clearly, it is a polynomial of degree 3.

11. (B) If the common difference of the A.P. be D , then $b = a + D$, $c = a + 2D$, $d = a + 3D$ and $e = a + 4D$.
 $\therefore a - 4b + 6c - 4d + e = a - 4(a + D) + 6(a + 2D) - 4(a + 3D) + a + 4D = 0$.
12. (C) The line joining the points $(0,0)$ and $(1,1)$ is perpendicular to the line $x + y = 2$
13. (C) Slope of the two lines are $-\frac{1}{k-1}$ and $-\frac{2}{k^2}$
For the lines to be at right angles $\left(-\frac{1}{k-1}\right) \times \left(-\frac{2}{k^2}\right) = -1$
i.e., if $2 = -k^2(k-1)$, i.e., if

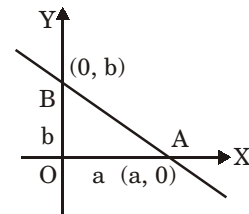
$$k^3 - k^2 - 2 = 0$$

$$\text{i.e., if } k^3 + k^2 - 2k^2 - 2 = 0,$$

$$\text{i.e., if } k^2(k+1) - 2(k^2-1) = 0$$

$$\text{i.e., if } (k+1)(k^2-2k+2) = 0,$$

$$\text{i.e., if } k = -1.$$



Clearly, the triangle in reference is a right triangle and circumcentre of a right triangle is at the mid point of the hypotenuse i.e. at the mid point of the join of $(a, 0)$ and $(0, b)$.

14. (D) Slope of the lines are respectively $-\frac{a}{b}$ and $\frac{a+b}{a-b}$. Hence, the acute angle θ between them is given by $\tan \theta =$

$$\left| \frac{\frac{a+b}{a-b} + \frac{a}{b}}{1 + \left(\frac{a+b}{a-b}\right)\left(-\frac{a}{b}\right)} \right| = \left| \frac{b(a+b+a(a-b))}{b(a-b)-a(a+b)} \right|$$

$$\left| \frac{b^2+a^2}{-b^2-a^2} \right| = \frac{a^2+b^2}{b^2+a^2} = 1$$

$$\Rightarrow \theta = 45^\circ$$

15. (A) $\|PF_1| - |PF_2|\|$ is a fixed constant = length of transverse axis = $2a$.
16. (D) We have,
$$\begin{aligned}
E &= \frac{31!}{2^{31}(32!)} = \frac{1}{2^{31}(32)} = \frac{1}{2^{36}} \\
&= 2^{-36} = (2^3)^{-12} = 8^{-12}
\end{aligned}$$
Thus, $x = -12$
17. (A) If the given points A, B, C and D are taken in order and $|AC|$ and $|BD|$ bisect each other then A B C D is a parallelogram.
Note that $|AB| \neq |BC|$ and $|AC| \neq |BD|$.

18. (A) $y^2 = 12x$
 $4a = 12 \Rightarrow a = 3$
Let $P \equiv (3t_1^2, 6t_1)$ and $Q = (3t_2^2, 6t_2)$
Given, $\frac{6t_1}{6t_2} = \frac{1}{2} \Rightarrow t_2 = 2t_1$
Let $R(\alpha, \beta)$ be the point of intersection of the normals to parabola at P and Q, then
 $\alpha = 2a + a(t_1^2 + t_2^2 + t_1t_2) = 6 + 21t_1^2 \dots(1)$
and $\beta = -at_1t_2(t_1 + t_2) = -18t_1^3 \dots(2)$

From (1), $t_1^6 = \left(\frac{\alpha-6}{21}\right)^3$ and from (2),

$$t_1^6 = \frac{\beta^2}{324}$$

$$\therefore \beta^2 = \frac{324(\alpha-6)^3}{(21)^3} \Rightarrow 343\beta^2 = 12(\alpha-6)^3$$

Hence locus of R is: $343y^2 = 12(x-6)^3$

19. (C) Let the line makes an angle θ with the z-axis.

We have,

$$\cos^2 \frac{\pi}{4} + \cos^2 \frac{\pi}{4} + \cos^2 \theta = 1$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} + \cos^2 \theta = 1$$

$$\Rightarrow \cos \theta = 0$$

$$\Rightarrow \theta = \frac{\pi}{2}$$

20. (B) Given, latus rectum = $\frac{1}{2}$.minor axis

$$\Rightarrow \frac{2b^2}{a} = \frac{1}{2} \cdot 2b$$

$$\Rightarrow a = 2b$$

$$\therefore e^2 = \frac{a^2 - b^2}{a^2} = \frac{4b^2 - b^2}{4b^2} = \frac{3b^2}{4b^2} = \frac{3}{4}$$

$$\Rightarrow e = \frac{\sqrt{3}}{2}$$

21. (A) $6x + 1 > 7 - 4x$

$$\Rightarrow x > \frac{3}{5}$$

$$\therefore \frac{3}{5} < x \leq 2$$

22. (A) Let us take

$$\text{A.P.: } 2, 9.5, 17, 24.5, 32$$

$$\text{G.P.: } 2, 4, 8, 16, 32$$

$$A_s = 2 + 9.5 + 17 + 24.5 + 32 = 85$$

$$G_s = 2 + 4 + 8 + 16 + 32 = 62$$

$$\therefore A_s > G_s$$

23. (A) If $P(x_1, y_1)$ is the middle point of the chord, then the equation of the chord is,

$$3xx_1 - 2yy_1 + 2(x + x_1) - 3(y + y_1) = 3x_1^2 - 2y_1^2 + 4x_1 - 6y_1$$

If this chord is parallel to $y = 2x$, then

$$\frac{3x_1 + 2}{2} = \frac{2y_1 + 3}{1} \Rightarrow 3x_1 + 2 = 4y_1 + 6$$

$$\Rightarrow 3x_1 - 4y_1 = 4$$

\therefore The equation to the locus of P is $3x - 4y = 4$

24. (A) $\lim_{x \rightarrow \frac{\pi}{4}} \frac{2\sqrt{2} - (\cos x + \sin x)^3}{1 - \sin 2x} \quad \left(\frac{0}{0} \text{ form}\right)$

Using L' Hospital's rule,

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{-3(\cos x + \sin x)^2(-\sin x + \cos x)}{-2 \cos 2x}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{-3(\cos x + \sin x)(\cos^2 x - \sin^2 x)}{-2 \cos 2x}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{-3(\cos x + \sin x) \cos 2x}{-2 \cos 2x}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{3(\cos x + \sin x)}{2} = \frac{3}{2} \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) = \frac{3}{\sqrt{2}}$$

25. (C) Since, $i^{n+2} \cos[45 + 90(n+2)]^\circ$

$$= -i^n [-\cos(45 + 90n)^\circ]$$

$$= i^n [\cos(45 + 90n)^\circ]$$

Every other term has the same value. The

first is $\frac{\sqrt{2}}{2}$, and these are 2 terms with this value ($n = 0, 2, 4, \dots, 40$).

The second term is $i \cdot \cos 135^\circ = -\frac{i\sqrt{2}}{2}$ and there are 20 terms with this value ($n = 1, 3, \dots, 39$)

Thus the sum is $\frac{i\sqrt{2}}{2} (21 - 20i)$

26. (C) Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \text{ As it passes through } (4, -1)$$

$$\frac{16}{a^2} + \frac{1}{b^2} = 1 \text{ (or) } a^2 + 16b^2 = a^2b^2$$

since $x + 4y - 10 = 0$ touches the ellipse

$$\left(\frac{10}{4}\right)^2 = a^2\left(\frac{1}{4}\right)^2 + b^2 \Rightarrow a^2 + 16b^2 = 100$$

$a^2b^2 = 100$ and the required equation can

$$\text{be } \frac{x^2}{20} + \frac{y^2}{5} = 1$$

27. (D) $x^2 - (1 - 2i)x = \left(\frac{1}{2} + i\right)$
 $\Rightarrow 2x^2 + (-2 + 4i)x - 1 - 2i = 0$
 $x = \frac{-(-2 + 4i) \pm \sqrt{(-2 + 4i)^2 - 4(2)(-1 - 2i)}}{4}$
 $= \frac{2 - 4i \pm \sqrt{-4}}{4} = \frac{2 - 4i \pm 2i}{4}$
 $\Rightarrow x = \frac{1 - i}{2}$ or $\frac{1 - 3i}{2}$
28. (B) By substitution,
 $7\cos x + 3\sin x = 7\sin x + 3\cos x$
 $\Rightarrow \sin x = \cos x$
 $\Rightarrow x = \frac{\pi}{4}$
 $\therefore y = 7 \cos \frac{\pi}{4} + 3 \sin \frac{\pi}{4} = 7 \times \frac{1}{\sqrt{2}} + 3 \times \frac{1}{\sqrt{2}}$
 $= \frac{10}{\sqrt{2}} = 5\sqrt{2}$
29. (D) $\frac{12 - 4}{6 - 1} = \frac{10 - 12}{c - 6}$
 $\Rightarrow 8c - 48 = -10$
 $\Rightarrow c = 4.75$
30. (A) With $z = x + iy$,
we have $\frac{1}{1 - z} = \frac{1 - x + iy}{2 - 2x} = \frac{1}{2} + i \frac{y}{2 - 2x}$
31. (D) For $n = 1$, $P(1)$ is $2^1 < 1$, which is false
Also $P(2) = 2^2 < 1 \times 2$ is false
 $P(3) = 2^3 < 1 \times 2 \times 3$ is false
 $P(4) = 2^4 < 1 \times 2 \times 3 \times 4$ is true.
32. (C) For $f: A \rightarrow B$ to be surjective,
we have $n(A) \geq n(B)$.
33. (D) $(1, 1) \Rightarrow 2(1) - 3(1) > -5$
 $-1 > -5$ (true)
 $= (1, 1)$ satisfies $2x - 3y > -5$
 $(-1, 1) \Rightarrow 2(-1) - 3(1) > -5$
 $-5 > -5$ (false)
 $(-1, 1)$ does not satisfy $2x - 3y > -5$

$$(1, -1) \Rightarrow 2(1) - 3(-1) > -5$$

$$2 + 3 > -5$$

$$5 > -5 \text{ (true)}$$

$$(1, -1) \text{ satisfies } 2x - 3y > -5$$

$$(-1, -1) \Rightarrow 2(-1) - 3(-1) > -5 \Rightarrow$$

$$1 > -5 \text{ (true)}$$

$$(2, -1) \Rightarrow 2(2) - 3(-1) > -5$$

$$7 > -5 \text{ (true)}$$

$$\therefore (2, -1) \text{ satisfies } 2x - 3y > -5$$

$$(-2, -1) \Rightarrow 2(-2) - 3(-1) > -5$$

$$-4 + 3 > -5$$

$$-1 > -5 \text{ (true)}$$

$$(-2, -1) \text{ satisfies } 2x - 3y > -5$$

\therefore '5' points satisfies the given condition.

34. (D) $x^2y^2 = a^2(\sec \theta + \tan \theta)^4 \cdot b^2(\sec \theta - \tan \theta)^4$
 $= a^2b^2(\sec^2 \theta - \tan^2 \theta)^4$
 $= a^2b^2$

35. (B) If 'z' is represented by $A(x, y)$, 'iz' is represented by $B(-y, x)$, 'z + iz' is represented by $C(x - y, x + y)$, then

$$\text{Area of } \Delta ABC = \frac{1}{2} \left| \begin{array}{ccc} x & -y & x-y \\ y & x & x+y \\ y & x & y \end{array} \right|$$

$$= \frac{1}{2}(x^2 + y^2)$$

$$= \frac{1}{2}|z|^2$$

36. (A) Given $\tan(3x - 2x) = 1$

$$\Rightarrow \tan x = 1$$

$$\Rightarrow x = n\pi + \frac{\pi}{4}, n \in \mathbb{Z}$$

If $x = n\pi + \frac{\pi}{4}$, then $\tan 2x$ is not defined.

$$\therefore \text{There is no } x \text{ such that } \frac{\tan 3x - \tan 2x}{1 + \tan 3x \tan 2x} = 1.$$

37. (A) Let $f(x) = lx^2 + mx + n$

$$\text{Now } f(1) = f(-1) \Rightarrow l + m + n = l - m + n$$

$$\Rightarrow m = 0, n = -1$$

$$\therefore f(x) = l(x^2 - 1) \text{ and } f'(x) = 2lx$$

a, b, c are in A.P. $\Rightarrow 2la, 2lb, 2lc$ are in A.P.

$$\Rightarrow f'(a), f'(b), f'(c) \text{ are in A.P.}$$

38. (B)
$$\frac{dy}{dx} = x^{\sin x} \left\{ \frac{1}{x} \sin x + \log x \cdot \cos x \right\}$$

$$+ (\sin x)^x \left\{ \frac{\cos x}{\sin x} \cdot x + \log(\sin x) \right\}$$

$$= x^{\sin x} \left(\frac{\sin x}{x} + \log x \cdot \cos x \right) + (\sin x)^x [x \cdot \cot x + \log(\sin x)]$$

39. (C) Given ${}^4C_2 \cdot \alpha^2 = {}^6C_3 (-\alpha)^3 \Rightarrow 6\alpha^2 = -20\alpha^3$

$$\Rightarrow \alpha = \frac{-3}{10}$$

40. (D) The relation R in option (A) contains (1, -1) and (1, 1). So it is not a function.
 The relation R in option (B) contains (1, 1) and (1, 3). So it is not a function.
 Again the relation R in option (C) contains (1, 1) and (1, 2). So it is not a function.

PHYSICS

41. (C) A metal ball and a rubber ball of the same mass are dropped from the same height. After hitting the floor, the rubber ball rises higher than the metal ball. There is a greater change in momentum in case of the rubber ball.

42. (B) During uniform rotatory motion along a circular path magnitude of acceleration is constant.

43. (B) $y = 12x - \frac{3}{4}x^2$

Therefore, $\frac{dy}{dt} = 12 \frac{dx}{dt} - \frac{3}{2}x \frac{dx}{dt}$

At $x = 0$, we find $\frac{dy}{dt} = 12 \frac{dx}{dt}$. If θ be the angle of projection, then $(dy/dx)/(dx/dt) = 12 = \tan \theta$. (slope)

Also if u = initial velocity, then $u \cos \theta = 3$. Therefore, $(\tan \theta)(u \cos \theta) = 36$, or $u \sin \theta = 36$.

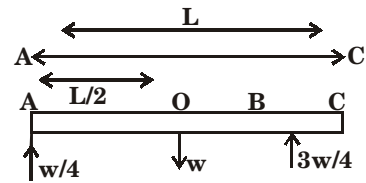
And range $R = \frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin \theta \cos \theta}{g}$

$$= \frac{2(u \sin \theta)(u \cos \theta)}{10} = \frac{2 \times 36 \times 3}{10} = 21.6 \text{ m}$$

44. (C) $v_x = 2ct$ and $v_y = 2bt$. Speed after one second will be

$$\left[(2b)^2 + (2c)^2 \right]^{1/2} = 2(b^2 + c^2)^{1/2}$$

45. (A)



Here AC is the bar. Sunil is holding the bar at A and the Anil is holding it at B. The load on Sunil is $w/4$ where W is weight of the bar. Then load on M_2 is $3W/4$. The weight is acting at point 'O'. Taking moments about A, we find :

$$W \times \frac{L}{2} = \frac{3w}{4} \times AB$$

Hence, $AB = \frac{2L}{3}$

46. (B) $v_{es} = \sqrt{2} v_0$. Kinetic energy in orbit is $\frac{Mv_0^2}{2} = E$

Kinetic energy to escape is $Mv_{es}^2 / 2$

$$= \frac{M(\sqrt{2}v_0)^2}{2} = 2E$$

47. (B) The inclination of the string is $\tan^{-1}(a/g)$ opposite to the direction of motion.
 48. (A) Viscous force is temperature dependent and velocity dependent.
 49. (B) If the force of cohesion is greater than adhesion, then the liquid will not wet the solid.
 50. (B) Moment of inertia of a thin circular disc $MR^2 / 4$. So, the moment of inertia is minimum through the centre parallel to the surface.

51. (C) Volume of raft $V = \frac{M}{\rho} = \left(\frac{30}{750} \right) \text{m}^3$
 $= 0.04 \text{ m}^3$

Maximum water the raft can displace $= V \times 1000 \text{ kg} = 0.04 \times 1000 = 40 \text{ kg}$
 So, we can place $40 - 30 = 10 \text{ kg}$ mass on the raft.

52. (C) Maximum possible strain $= \frac{0.4}{100}$

$$\therefore A = \frac{F}{Y \times \text{strain}} = \frac{2 \times 10^4 \times 100}{7 \times 10^9 \times 0.4}$$

$$= 7.15 \times 10^{-4} \text{ m}^2$$

$$\approx 7.1 \times 10^{-4} \text{ m}^2$$

53. (A) For a liquid - solid interface, if the angle of contact is acute, then

- (i) the liquid will wet the solid.
 (ii) the liquid will rise in the capillary tube made of such a solid and

(iii) Meniscus of the liquid will be concave.

54. (B) Velocity at the top is \sqrt{gr} and that at the bottom is $\sqrt{5} gr$.
Difference in kinetic energy

$$\begin{aligned} &= \frac{1}{2} M (5gr - gr) \\ &= 2 M gr \\ &= 2 \times 1 \times 10 \times 1 \\ &= 20 \text{ J} \end{aligned}$$

55. (D) The centre of mass of system of particles depends upon the :
(i) masses of the particles
(ii) position of the particles
(iii) relative separation between the particles.

56. (A) $V_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

Given V_{rms} of argon at $T_1 = V_{\text{rms}}$ of He at T_2

$$\sqrt{\frac{3RT_1}{M_a}} = \sqrt{\frac{3RT_2}{M_h}}; T_2 = -20 + 273 = 253 \text{ K}$$

Squaring, $\frac{3RT_1}{M_a} = \frac{3RT_2}{M_h}; T_1 = \frac{M_a}{M_h} \times T_2$
 $= \frac{39.9 \times 253}{4} = 2523.7 \text{ K}$

57. (C) $\eta = \frac{P_0}{P_1} \Rightarrow P_1 = \frac{P_0}{\eta}$

(Power) $P_1 = \frac{100}{60} \times \frac{mgh}{t} = \frac{5}{3} \times \frac{100 \times 10 \times 10}{5}$
 $= 3.3 \times 10^3 \text{ W} = 3.3 \text{ kW}$

58. (B) Acceleration due to gravity, $g =$ electric intensity = rate of change of potential

$$= \frac{10}{100} = \frac{1}{10} \text{ m s}^{-2}$$

The work done in moving a body of 5 kg upwards through 40 m will be = mgh
 $= 5 \times (1 / 10) \times 40 = 20 \text{ J}$

59. (A) $K = \frac{r_1^2 + r_2^2 + \dots}{n}$, radius of gyration depends on the distribution of mass about the axis of rotation and it is independent of the mass of the body.

60. (A) Here $m = 8000 \text{ kg}$, $u = 800 \text{ m s}^{-1}$

$$g = 10 \text{ m s}^{-2}, \frac{dm}{dt} = ?$$

To overcome the weight of the rocket,

$$F = mg = u \frac{dm}{dt}$$

$$8000 \times 10 = 800 \frac{dm}{dt}$$

$$\frac{dm}{dt} = \frac{8000 \times 10}{800} = 100 \text{ kg s}^{-1}$$

61. (B) Mean diameter =

$$\frac{0.39 + 0.38 + 0.39 + 0.41 + 0.38 + 0.37 + 0.40 + 0.39}{8}$$

$$\bar{d} = 0.38875 \text{ mm}$$

= 0.39 mm (rounded off to two significant figures)

Absolute error in the first reading = $0.39 - 0.39 = 0.00 \text{ mm}$

Similarly finding the absolute error in the other seven readings and taking the mean;

$$\text{Mean absolute error} = \overline{\Delta d} =$$

$$\frac{0.00 + 0.01 + 0.00 + 0.02 + 0.01 + 0.02 + 0.01 + 0.00}{8} = 0.00875 = 0.01 \text{ mm}$$

$$\text{Relative error} = \frac{\overline{\Delta d}}{d} = \frac{0.01}{0.39} = 0.0256$$

62. (B) A raw egg behaves like a spherical shell and a half boiled egg behaves like a solid sphere.

$$\therefore \frac{I_r}{I_s} = \frac{2/3 MR^2}{2/5 MR^2} = \frac{5}{3} > 1$$

63. (D) $\frac{4S}{r_1} - \frac{4S}{r_2} = \frac{4S}{r}$

$$\text{or } \frac{1}{r} = \frac{1}{r_1} - \frac{1}{r_2} = \frac{1}{4} - \frac{1}{5} = \frac{1}{20} \text{ or } r = 20 \text{ cm}$$

64. (B) $C_m = \frac{3}{2} R$, $C_{di} = \frac{5}{2} R$.

If change in temperature is ΔT , then

$$1 \times \frac{3}{2} R \Delta T + 1 \times \frac{5}{2} R \Delta T = 2 \times C_v \times \Delta T$$

This gives $C_v = 2 R$

65. (B) Here $dx_1 = dx_2$, $A_1 = A_2$, $\frac{K_1}{K_2} = \frac{2}{3}$
- Let θ be the temp. of the junction.
- As $\frac{dQ_1}{dt} = \frac{dQ_2}{dt}$
- $\therefore K_1 A_1 \frac{dT_1}{dx_1} = K_2 A_2 \frac{dT_2}{dx_2}$
- $K_1 = (100 - \theta) = K_2 (\theta - 0)$
- or $\frac{K_1}{K_2} = \frac{\theta}{100 - \theta} = \frac{2}{3}$
- $3\theta = 200 - 2\theta$; $5\theta = 200$; $\theta = 40^\circ\text{C}$

CHEMISTRY

66. (B) HCl, a strong acid, decreases the sulphide ion concentration by common ion effect. Secondly, dil. HCl is used to keep the sulphide ion concentration at a minimum level. Thus, products of their respective sulphides precipitate out.
67. (D) Wave mechanical model of the atom depends on:
- (i) Heisenberg's uncertainty principle
- (ii) De Broglie concept of dual nature of electron.
- (iii) Schrodinger wave equation.
68. (D) $M + X O \rightarrow MO + X + Q$
- $$\begin{array}{r} M + \frac{1}{2} O_2 \rightarrow MO + 351.4 \text{ kJ} \\ XO \rightarrow \frac{1}{2} O_2 - 90.8 \text{ kJ} \\ \hline M + XO \rightarrow MO + X \\ \Delta H = +260.61 \text{ kJ} \end{array}$$
69. (D) Both alkenes and alkynes decolourise alkaline $KMnO_4$ but alkanes do not react with ammoniacal cuprous chloride.
70. (D) For an ideal gas, $PV = \frac{1}{3} mNu^2$

$$\text{Average Kinetic Energy } (E_k) = \frac{1}{2} mNu^2$$

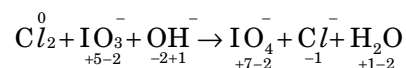
$$\text{Thus, } PV = \frac{2}{3} N \left(\frac{1}{2} m u^2 \right) = \frac{2}{3} N E_k$$

For 1 mol of a gas, $V = V_m$ and $N = N_A$

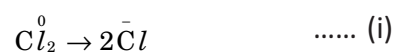
$$\text{Hence } PV_m = \frac{2}{3} N_A E_k$$

71. (D) Solubility of carbonates and bicarbonates of alkali metals increases as one goes down the group. In the Solvay process sodium bicarbonate precipitate out. $KHCO_3$ is relatively more soluble than $NaHCO_3$ but is slightly soluble in water.

72. (B) Writing oxidation numbers of all atoms,

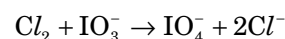


Oxidation numbers of Cl and I have changed.

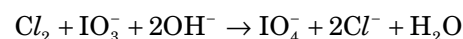


Decrease in oxidation no. of Cl = 2 units per Cl_2 molecule.

Increase oxidation number of I = 2 units per IO_3^- molecule



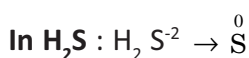
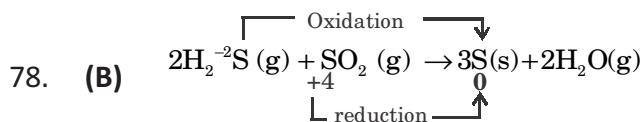
To balance oxygen $2OH^-$ ions be added on L.H.S. and one H_2O molecule on R.H.S. Hence, the balanced equation is:



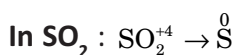
73. (D) Pauli's principle cannot conclude about the number of unpaired electrons in a subshell.
74. (C) In option (A) No violation of any rule
In option (B) Violation of Aufbau principle
In option (C) Violation of both Aufbau principle & Hund's rule.
In option (D) Violation of Aufbau principle.
75. (D) When cement is mixed with water, it absorbs water to form a gelatinous mass sets to a hard mass. This is called setting of cement. The setting of cement involves a series of hydration and hydrolysis reactions leading to the formation of colloidal gels. These gels soon begin to harden due to the formation of interlocking crystals of hydrated silicated gels. The process of hydration and hydrolysis are exothermic. Water is sprinkled over it to keep it cool also.

76. (C) Half filled or completely filled orbitals are found to be more stable. Therefore, the ionisation enthalpy is higher when an electron is to be removed from a fully filled or half filled orbitals.

77. (C) $P_1 V_1 = P_2 V_2 = P_3 V_3 = P_4 V_4$
 $P_2 = 125 \text{ Torr} ; P_3 = 200 \text{ Torr} ;$
 $V_2 = 64 \text{ ml} ; V_3 = ?$
 $P_2 V_2 = P_3 V_3$
 or $V_3 = \frac{P_2 V_2}{P_3} = \frac{125 \times 64}{200} = 40 \text{ ml}$



The oxidation number of S (in H_2S) is -2 and it changes to 0 in the reaction. Thus, H_2S gets oxidised to S.



The oxidation number of S (in SO_2) is $+4$ and it changes to 0 in the reaction. Thus, SO_2 gets reduced to S.

79. (B) BeH_2 cannot be prepared by direct action of H_2 on Be. BeH_2 is prepared by the action of LiAlH_4 on BeCl_2
 $2 \text{BeCl}_2 + \text{LiAlH}_4 \rightarrow 2\text{BeH}_2 + \text{LiCl} + \text{AlCl}_3$

80. (B) $780 \text{ mm of Hg} = \frac{780}{760} \text{ atm}$

$w = 22 \text{ g} ; M = 44 \text{ g mol}^{-1}$

$T = 27^\circ\text{C} = 27 + 273 = 300 \text{ K}$

Volume occupied

$$= \frac{w}{M} \cdot \frac{RT}{P} = \frac{22}{44} \times \frac{0.0821 \times 300 \times 760}{780}$$

$$= 12 \text{ l.}$$

81. (C) Mass of the organic compound taken
 $= 0.244 \text{ g}$

Mass of CO_2 formed $= 0.616 \text{ g}$

Mass of H_2O formed $= 0.108 \text{ g}$

(i) Mass of C in the CO_2 formed

$$= \frac{0.616 \times 12}{44} \text{ g}$$

Percentage of C in the compound

$$= \frac{0.616 \times 12}{44} \times \frac{100}{0.244} = 68.85$$

(ii) Mass of H in the H_2O formed

$$= \frac{0.108 \times 2}{18} \text{ g}$$

Percentage of H in the compound

$$= \frac{0.108 \times 2 \times 100}{18 \times 0.244} = 4.92$$

(iii) Percentage of O in the compound
 $= 100 - (68.85 + 4.92) = 26.23$

The percentage composition of the compound is, C = 68.85, H = 4.92 and O = 26.23.

82. (B) Wavelength, $\lambda = 580 \text{ nm} = 580 \times 10^{-9} \text{ m}$

Velocity of light, $c = 3 \times 10^8 \text{ m s}^{-1}$

Then,

$$\text{Frequency, } \nu = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m s}^{-1}}{580 \times 10^{-9} \text{ m}}$$

$$= 5.17 \times 10^{14} \text{ s}^{-1}$$

$$= 5.17 \times 10^{14} \text{ Hz}$$

$$\text{Wavenumber, } \bar{\nu} = \frac{1}{\lambda} = \frac{1}{580 \times 10^{-9} \text{ m}}$$

$$= 1.72 \times 10^6 \text{ m}^{-1}$$

83. (D) Addition of helium gas to an equilibrium mixture at constant volume does not disturb the chemical equilibrium. As such there is no effect on the relative amount of SO_3 , O_2 and SO_2 gases respectively.

84. (A) Zeolite used for softening of hard water is $\text{Na}_2\text{O} \cdot \text{Al}_2\text{O}_3 \cdot 2\text{SiO}_2 \cdot x\text{H}_2\text{O}$, which is hydrated sodium aluminium silicate.

85. (A) Mass of the substance taken $= 0.316 \text{ g}$

Mass of BaSO_4 formed $= 0.466 \text{ g}$

From stoichiometry, $\text{BaSO}_4 = \text{S}$

$$\frac{233}{32}$$

(molecular mass of BaSO_4

$$= 137 + 32 + 64 = 233)$$

Then, mass of S in 0.466 g of BaSO_4

$$= \frac{0.466 \times 32}{233} \text{ g}$$

Percentage of S in the compound

$$= \frac{0.466 \times 32}{233} \times \frac{100}{0.316} = 20.25 \%$$

86. (D) All the alkali metals and their salts impart colour to bunsen flame. The colours imparted by different alkali metals are as follows.

Element	Li	Na	K	Rb	Cs
Colour	Crimson red	Golden yellow	Pale violet	Red violet	Bluish violet

When heat energy is supplied to alkali metal atom or ion in salt, the electronic excitation occurs in which electron jumps to higher energy level. When this excited electron de-excites to ground state, the energy is emitted in the form of electromagnetic radiation which lies in visible region thereby imparting colour to the flame. The colour of flame depends upon the wavelength of radiation emitted e.g., yellow D-line of Na-spectra arises from $3s^1 \rightarrow 3p^1$ transition.

87. (A) $C:H = \frac{12 \times 100}{13 \times 12} = \frac{1 \times 100}{13 \times 1} = 1:1$

\therefore E.F. = CH

Since, P decolourises Br_2-H_2O , but Q does not, therefore, P = C_2H_2 (acetylene) and Q = C_6H_6 (benzene).

90. (C) The sum of mass % is 99.8. Hence, there is no oxygen in the given compound.

Element	Mass %	Atomic mass	Atomic ratio	Simplest ratio	Simplest whole number ratio
C	64.4	12	$64.4 / 12 = 5.37$	$5.37 / 0.53 = 10.1$	10
H	5.5	1	$5.5 / 1 = 5.5$	$5.5 / 0.53 = 10.4$	10
Fe	29.9	56	$29.9 / 56 = 0.53$	$0.53 / 0.53 = 1$	1

Thus, the empirical formula of the compound is $C_{10}H_{10}Fe$.

GENERAL AWARENESS

91. (B) 92. (C) 93. (A) 94. (A) 95. (A) 96. (A) 97. (B) 98. (B) 99. (C)
100. (B)

=====*The End*=====

88. (D) (a) It is exact neutralisation. Hence, pH = 7.

(b) After neutralisation, $\frac{M}{10}$ HCl left = 10 ml.

Total volume = 100 ml

Dilution = 10 times.

$\therefore [H^+] = 10^{-2}$

or pH = 2

(c) After neutralisation, $\frac{M}{10}$ NaOH left = 80 ml.

Total volume = 100 ml. pH > 7.

(d) After neutralisation, $\frac{M}{5}$ HCl left = 50 ml.

Total volume = 100 ml

Dilution = 2 times

$\therefore [H^+] = \frac{1}{10} = 10^{-1} M$ or pH = 1

89. (B) Due to the poor shielding (screening) effect of d-electrons in case of Ga, the valence electrons are attracted more strongly and hence, the size is not increased.