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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION

Paper Code: UN436 (UPDATED)

Solutions for Class : 12-PCM

MATHEMATICS

1. **(A)** We have, $\tan^{-1}(x-1)+\tan^{-1}(x+1) = \tan^{-1}3x - \tan^{-1}x$

$$\Rightarrow \tan^{-1}\frac{x-1+x+1}{1-(x-1)(x+1)} = \tan^{-1}\frac{3x-x}{1+(3x)(x)}$$

$$\Rightarrow \frac{2x}{2-x^2} = \frac{2x}{1+3x^2} \Rightarrow x+3x^3 = 2x-x^3$$

$$\Rightarrow 4x^3 - x = 0$$

$$\Rightarrow x = 0 \text{ or } \pm\frac{1}{2}$$

□ $1 < x < \sqrt{2}$, there is no solution.

2. **(C)** $A^2 = \begin{bmatrix}1 & 0\\ \frac{1}{2} & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{1}{2} & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\ 2\times\frac{1}{2} & 1\end{bmatrix}$

$$= A^2.A = \begin{bmatrix}1 & 0\\ 2\times\frac{1}{2} & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{1}{2} & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\ 3\times\frac{1}{2} & 1\end{bmatrix}$$

Continuing in this way, we get

$$A^{100} = \begin{bmatrix}1 & 0\\ 100\times\frac{1}{2} & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\ 50 & 1\end{bmatrix}$$

3. **(C)** $|1-x^4|$ is an even function.

So, $\int_{-2}^{2}|1-x^4|.dx = 2\int_{0}^{2}|1-x^4|.dx$

$$= 2\int_0^1 (1-x^4).dx + 2\int_1^2 (x^4-1)dx$$

$$= 2\left(x-\frac{x^5}{5}\right)\Big]_0^1 + 2\left(\frac{x^5}{5}-x\right)\Big]_1^2$$

$$= 2\left(1-\frac{1}{5}\right)+2\left(\frac{32}{5}-2-\frac{1}{5}+1\right) = 12$$

4. **(B)** According to the given condition,

$$y.\frac{dy}{dx} = a \Rightarrow \int y.dy = \int a.dx$$

$$\Rightarrow \frac{y^2}{2} = ax + b$$

$$\Rightarrow y^2 = 2ax + 2b$$

5. **(C)** Let $\tan^{-1}\frac{1}{3} = \alpha$ and $\tan^{-1}2\sqrt{2} = \beta$.

Then $\tan\alpha = \frac{1}{3}$ and $\tan\beta = 2\sqrt{2}$, so that

$$\sin(2\tan^{-1}\frac{1}{3}) + \cos(\tan^{-1}2\sqrt{2})$$

$$= \sin 2\alpha + \cos\beta$$

$$= \frac{2\tan\alpha}{1+\tan^2\alpha} + \frac{1}{\sqrt{1+\tan^2\beta}}$$

$$= \frac{2\left(\frac{1}{3}\right)}{1+\frac{1}{9}} + \frac{1}{\sqrt{1+8}} = \frac{3}{5}+\frac{1}{3} = \frac{14}{15}$$

6. **(B)** Given A' = A and B' = B

We have, (AB – BA)' = (AB)' – (BA)'

= B'A' – A'B'

= BA – AB

= – (AB – BA)

Hence, AB – BA is skew symmetric.

7. **(A)** $\int_{\sqrt{2}}^{x}\frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{2}$

$$\Rightarrow \left[\sec^{-1}t\right]_{\sqrt{2}}^{x} = \frac{\pi}{2}$$

$\Rightarrow \sec^{-1}x - \frac{\pi}{4} = \frac{\pi}{2}$

$\Rightarrow \sec^{-1}x = \frac{3\pi}{4} \Rightarrow x = \sec\frac{3\pi}{4} = -\sqrt{2}$

8. **(A)** $(\vec{a}+\vec{b})\times(\vec{a}\times\vec{b}) = [(\vec{a}+\vec{b}).\vec{b}]\vec{a} - [(\vec{a}+\vec{b}).\vec{a}]\vec{b}$

$= (\vec{a}.\vec{b})\vec{a} + (\vec{b}.\vec{b})\vec{a} - (\vec{a}.\vec{a})\vec{b} - (\vec{b}.\vec{a})\vec{b}$

$= (\vec{a}\vec{b})(\vec{a}-\vec{b}) + (\vec{a}-\vec{b})$

$= [(\vec{a}.\vec{b})+1](\vec{a}-\vec{b})$

$\therefore (\vec{a}+\vec{b})\times(\vec{a}\times\vec{b})$ is parallel to $\vec{a}-\vec{b}$.

9. **(C)** ∵ Given equation contains only one parameter, its order is 1.

$y^2 = 2c(x+\sqrt{c}) \Rightarrow 2y.y_1 = 2c \Rightarrow c = yy_1$

$\therefore$ The given equation is $y^2 = 2yy_1(x+\sqrt{yy_1})$

$\Rightarrow y - 2xy_1 = 2y_1\sqrt{yy_1}$

$\Rightarrow (y-2xy_1)^2 = 4y.y_1^3$

$\therefore$ Degree = 3

10. **(B)** A(adj A) = |A|I $\Rightarrow$ |A| = 4

|adj A| = $(|A|)^2 = (4)^2 = 16$

11. **(C)** $3\tan^{-1}\left(\frac{1}{2+\sqrt{3}}\right) = \tan^{-1}\frac{1}{x} + \tan^{-1}\frac{1}{3}$

Put $\frac{1}{2+\sqrt{3}} = t$

LHS $= \tan^{-1}\left(\frac{3t-t^3}{1-3t^2}\right) = \tan^{-1}1 = \frac{\pi}{4}$

RHS $= \tan^{-1}\frac{1}{x} + \tan^{-1}\frac{1}{3} = \tan^{-1}\left(\frac{3+x}{3x-1}\right)$

$\therefore \frac{\pi}{4} = \tan^{-1}\left(\frac{3+x}{3x-1}\right) \Rightarrow 1 = \frac{3+x}{3x-1} \Rightarrow x=2$

12 **(C)** $dx + dy = (x+y)(dx-dy)$

$\Rightarrow \frac{dx+dy}{x+y} = dx - dy$

$\Rightarrow \log(x+y) = x - y + c$

13. **(C)** (cx i – 6 j + 3k.xi + 2j + 2cx k) is obtuse

$\Rightarrow (cxi - 6j + 3k).(xi + 2j + 2cxk) < 0$

$\Rightarrow cx^2 - 12 + 6cx < 0$

Since this is ture for all real x, it follows that c < 0 and the roots of $cx^2 + 6cx - 12 = 0$ are imaginary.

$\therefore c < 0$ and $36c^2 + 48c < 0 \Rightarrow c < 0, 3c + 4 > 0$

$\Rightarrow c < 0, c > -4/3 \Rightarrow -4/3 < c < 0$

14. **(A)** Augmented matrix is

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -1 & 3 \\ 3 & 2 & k & 4 \end{bmatrix} \begin{matrix} R_2 - 2R_1 \\ R_3 - 3R_1 \\ : \end{matrix} \left|\begin{matrix} 1 & 1 & 1 & 2 \\ 0 & -1 & -3 & -1 \\ 0 & -1 & k-1 & -2 \end{matrix}\right|$$

$$\begin{matrix} R_3 - R_2 \\ : \end{matrix} \left|\begin{matrix} 1 & 1 & 1 & 2 \\ 0 & -1 & -3 & -1 \\ 0 & 0 & k & -1 \end{matrix}\right|$$

Given system of equationshas a unique solution $\Rightarrow k \neq 0$.

15. **(A)** $\frac{dy}{dx} - \frac{2xy}{1+x^2} = 0$

$\Rightarrow \frac{1}{y}dy - \frac{2x}{1+x^2}dx = 0$

$\Rightarrow \log y - \log(1+x^2)$

$= \log(1+x^2) = \log A$

$\Rightarrow y = A(1+x^2)$

16. **(C)** $(\bar{i}+\bar{j}+\bar{k}).(5\bar{i}+2\bar{j}+2\bar{k})$

$= 5 + 2 + 2 = 9;$

$(\bar{i}+\bar{j}+\bar{k}).\left(\frac{5}{2}\bar{i}+\bar{j}+\bar{k}\right) = \frac{5}{2}+1+1 = \frac{9}{2}$

$(\bar{i}+\bar{j}+\bar{k}).\left(\frac{5}{2}\bar{i}+\bar{j}+\bar{k}\right) = \frac{5}{3}+\frac{2}{3}+\frac{2}{3} = \frac{9}{3} = 3,$

$(\bar{i}+\bar{j}+\bar{k}).\left(\bar{i}+\frac{2}{3}\bar{j}+\frac{2}{5}\bar{k}\right) = 1+\frac{2}{5}+\frac{2}{5} = \frac{9}{5}$

$\therefore \bar{b} = \frac{5}{3}\bar{i}+\frac{2}{3}\bar{j}+\frac{2}{3}\bar{k}$

17. **(B)** $\frac{dy}{dx} = \frac{\frac{d}{d\theta}(\sec^n\theta - \cos^n\theta)}{\frac{d}{d\theta}(\sec\theta - \cos\theta)}$

$= \frac{n\sec^{n-1}\theta\sec\theta\tan\theta - n\cos^{n-1}\theta(-\sin\theta)}{\sec\theta\tan\theta + \sin\theta}$

$= \frac{n\tan\theta(\sec^n\theta + \cos^n\theta)}{\tan\theta(\sec\theta + \cos\theta)}$

$\left(\frac{dy}{dx}\right)^2 = \frac{n^2(\sec^n\theta + \cos^n\theta)^2}{(\sec\theta+\cos\theta)^2}$

$= \frac{n^2\left[(\sec^n\theta - \cos^n\theta)^2 + 4\sec^n\theta\cos^n\theta\right]}{(\sec\theta-\cos\theta)^2 + 4\sec\theta\cos\theta}$

$\frac{n^2[y^2+4]}{x^2+4}$

18. **(A)** Let

$y = 27^{\cos 2x} 81^{\sin 2x} = 3^{3\cos 2x}.3^{4\sin 2x} = 3^{3\cos 2x + 4\sin 2x}$

$\therefore$ y will be minimum when 3 cos 2x + 4sin 2x is minimum.

Let z = 3 cos 2x + 4 sin 2x.

Put 3 = r cos α ; 4 = r sin 2x

$\therefore$ $r^2 = 3^2 + 4^2 = 25$. $\therefore$ r = 5

$$\tan\alpha = \frac{4}{3} \Rightarrow \alpha = \tan^{-1}\frac{4}{3}$$

$$\therefore z = r\sin(2x+\alpha)$$

$$= 5\sin\left(2x + \tan^{-1}\frac{4}{3}\right) \Rightarrow -5 \le z \le 5$$

$\therefore$ minimum z = –5 $\Rightarrow$ minimum y = 3^{-5}

$$= \frac{1}{243}$$

19. **(B)** Put x = tan θ, we have

$$\frac{\sqrt{1+x^2}-1}{x} = \frac{\sec\theta - 1}{\tan\theta} = \frac{1-\cos\theta}{\sin\theta} = \tan\frac{\theta}{2}$$

$$\therefore\ y = \frac{\theta}{2} = \frac{1}{2}\tan^{-1}(x)$$

$$\Rightarrow y'(x) = \frac{1}{2(1+x^2)} \Rightarrow y'(1) = \frac{1}{2(1+1^2)} = \frac{1}{4}$$

20. **(D)** Let $\bar{a} + 2\bar{b} = x\bar{c}$ and $\bar{b} + 3\bar{c} = y\bar{a}$

Then $\bar{a} + 2\bar{b} + 6\bar{c}$ = (x + 6) $\bar{c}$ and also

$\bar{a} + 2\bar{b} + 6\bar{c}$ = (1 + 2y) $\bar{a}$

So (x + 6) $\bar{c}$ = (1 + 2y) $\bar{a}$

Since $\bar{a}$ and $\bar{c}$ are non-zero and non-collinear, we have x + 6 = 0 and 1 + 2y = 0,

i.e. x = – 6 and y = $-\frac{1}{2}$

In either case, we have $\bar{a} + 2\bar{b} + 6\bar{c} = \bar{0}$

21. **(C)** Putting sin x = t in the given integral, we get

$$\int \frac{1-t^2+(1-t^2)^2}{t^2+t^4}\,dt$$

$$= \int \frac{(1-t^2)+(2-t^2)^2}{t^2+t^4}\,dt$$

$$= \int\left(1 + \frac{2}{t^2} - \frac{6}{1+t^2}\right)dt$$

$$= t - \frac{2}{t} - 6\tan^{-1}(t) + C$$

= sin x – 2 (sin x)$^{-1}$ – 6 $\tan^{-1}$ (sin x) + C(C)

Domain of $\sin^{-1} x$ is [– 1, 1] and domain of $\sec^{-1}x$ is (– ∞ , – 1] ∪ [1, ∞], therefore, the domain of f (x) is [– 1, 1] ∩ {(– ∞ , – 1] ∪ [1, ∞)} = {– 1, 1}.

22. **(B)** $\sin^2\left(\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right) = \sin^2\theta$,

where $\theta = \tan^{-1}\sqrt{\frac{1+x}{1-x}}$

$$\Rightarrow \tan^2\theta = \frac{1+x}{1-x}$$

$$= \tan^2\theta\cos^2\theta = \frac{\tan^2\theta}{\sec^2\theta},$$

$$= \frac{\tan^2\theta}{1+\tan^2\theta},$$

$$= \frac{\frac{1+x}{1-x}}{1+\frac{1+x}{1-x}} = \frac{1+x}{1-\cancel{x}+1+\cancel{x}} = \frac{1+x}{2}$$

23. **(C)** ABC = (AB) C

AB is of order 1 × 3 ($\therefore$ A is of order 1×3 B is of order 3×3)

$\therefore$ (AB) C is of order 1 × 1.

24. **(B)** If, |A| ≠ 0 So, A^{-1} exists

$\therefore$ AB = AC

$$\Rightarrow A^{-1}(AB) = A^{-1}(AC)$$

$$\Rightarrow (A^{-1}A)B = (A^{-1}A)C$$

$\therefore$ B = C.

25. **(B)** The required determinant is obtained by the successive operations

$$C_1 \to 2C_1 \text{ and } C_1 \to C_1 + 3C_2 + 4C_3$$

$\therefore$ The value of the determinant is multiplied by 2 (since of the first operation),

second operation does not affect the value of the determinant.

26. **(A)** Expanding with R1, we obtain the value of the given determinant as

1 ($\cos^2 x - \sin^2 x$) + 0 + 0 = cos 2x.

27. **(D)** $\left.\begin{array}{l}R_2 \to R_3 - R_1\\ R_3 \to R_3 - R_1\end{array}\right\} \Rightarrow \begin{vmatrix} x & -6 & -1 \\ 2-x & -3x+6 & x-2 \\ -3-x & 2x+6 & x+3 \end{vmatrix} = 0$

$$\Rightarrow (x-2)(x+3)\begin{vmatrix} x & -6 & -1 \\ -1 & -3 & 1 \\ -1 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ $(x-2)(x+3) \Rightarrow (x-2)(x+3)$

$$\begin{vmatrix} x-1 & -6 & -1 \\ 0 & -3 & 1 \\ 0 & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (x-2)(x+3)(x-3)(-1)^{1+1}\begin{vmatrix} -3 & 1 \\ 2 & 1 \end{vmatrix} = 0$$

[Expand C_1]

$\Rightarrow (x-2)(x+3)(x-1)(-5) = 0$

$\Rightarrow x = 2, -3, 1$

28. **(A)** Since, f (x) is continuous at x = 0

$\therefore \lim\limits_{x \to 0} f(x) = f(0) = k$

$\Rightarrow \lim\limits_{x \to 0} (\cos x)^{1/x} = k$

$\Rightarrow \lim\limits_{x \to 0} \frac{1}{x} \log \cos x = \log k$

$\Rightarrow \lim\limits_{x \to 0} \dfrac{\frac{-\sin x}{\cos x}}{1} = \log k$

$\Rightarrow \log k = 0$

$\therefore k = e^0 = 1$

29. **(B)** $f(x) = \log_a x$ (a > 0, a ≠ 1)is strict monotomically increasing if a > 1 and strict monotonically decreasing if 0 < a < a < 1. Note that $f'(x) = \dfrac{1}{x \log a}$ < 0 for 0 < a < 1 and x > 0.

30. **(A)** Let $f(x) = x^2 + \dfrac{250}{x}$

$f'(x) = 2x - \dfrac{250}{x^2}$ and $f''(x) = 2 + \dfrac{500}{x^3}$

Now $f'(x) = 0 \Rightarrow \dfrac{2x - 250}{x^2} = 0$

$\Rightarrow 2x^3 = 250 \Rightarrow x^3 = 125 \Rightarrow x = 5$

$f''(5) = 2 + \dfrac{500}{5^3} = 2 + 4 = 6 > 0$

$\therefore f$ has a local minimum at $x = 5$ and local minimum value of $x = 5$ is

$f(5) = 5^2 + \dfrac{250}{5} = 25 + 50 = 75$

Since f is continuous in $(0, \infty)$ and has only one extremum at $x = 5$, therefore, $f(5)$ is absolutely minimum.

31. **(D)** Given $f(x) = x$, $D_f = R$

$\Rightarrow f'(x) = 1 > 0$, for all $x \in R$.

$\therefore$ $f(x)$ is strictly increasing on R. So, $f(x)$ has neither a maximum nor a minimum value.

32. **(B)** Let $I = \displaystyle\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx$

then $I = \displaystyle\int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{a-(a-x)}} dx$

Adding (1) and (2), we get

$21 = \displaystyle\int_0^a 1\,dx = [x]_0^a$

$\Rightarrow 2I = a - 0 \Rightarrow I = \dfrac{a}{2}$.

33. **(A)** Let $I = \int x \sin x \sec^3 x dx$

$I = \int x(\tan x \sec^2 x)dx$

$\Rightarrow I = x\dfrac{\tan^2 x}{2} - \int \dfrac{\tan^2 x}{2} dx$

$\Rightarrow I = x\dfrac{\tan^2 x}{2} - \int (\sec^2 x - 1) dx$

$\Rightarrow I = x\dfrac{\tan^2 x}{2} - \dfrac{1}{2}(\tan x - x) + c$

$\Rightarrow I = \dfrac{x}{2}(\sec^2 x - 1) - \dfrac{1}{2}\tan x + \dfrac{x}{2} + c$

$\therefore I = \dfrac{1}{2}(x\sec^2 x - \tan x) + c$

34. **(A)** Required area = $\displaystyle\int_{-\pi/2}^{\pi/2} \cos x\, dx$

Note that cos x is non – negative in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

35. **(D)** Required area =

$$\int_{-1}^{1} |x^3|\, dx = \int_{-1}^{0} |x^3|\, dx + \int_{0}^{1} |x^3|\, dx$$

$$= \int_{-1}^{0} (-x^3)\, dx + \int_{0}^{1} x^3\, dx$$

$$= -\frac{x^4}{4}\Big]_{-1}^{0} + \frac{x^4}{4}\Big]_{0}^{1} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

(∵ $x^3 \le 0$ for $x \le 0$ and $x^3 \ge 0$ for $x \ge 0$)

36. **(D)** $y = (x + c)\, e^{-x}$ ------- (1)

$\Rightarrow ye^{x} - x = c$ ------ (2)

Differentiating (1), we have

$$\frac{dy}{dx} = e^{-x} - (x + c)(e^{-x})$$

$$\frac{dy}{dx} = e^{-x} - ye^{x}e^{-x} \text{......[using (2)]}$$

$$\frac{dy}{dx} = e^{-x} - y$$

$$\therefore \frac{dy}{dx} + y = e^{-x}$$

37. **(B)** Given, $\frac{d}{dx}\left(\frac{dy}{dx}\right) = e^{-2x}$

Integrating, $\frac{d}{dx} = \frac{e^{-2x}}{-2} + c$

Again integrating, we get

$$y = \frac{e^{-2x}}{(-2)(-2)} + cx + d$$

38. **(D)** As $\vec{v} = 2\hat{i} + 3\hat{j} - 6\hat{k}$, therefore,

$$|\vec{v}| = \sqrt{2^2 + 3^2 + (-6)^2} = \sqrt{4 + 9 + 36} = 7$$

$$\therefore \hat{v} = \frac{1}{|\vec{v}|}\vec{v} = \frac{1}{7}(2\hat{i} + 3\hat{j} - 6\hat{k}) = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}$$

∴ d.c. of $\vec{v}$ are $< \frac{2}{7}, \frac{3}{7}, \frac{-6}{7} >$.

39. **(A)** $|\vec{a} \cdot \vec{b}| = ||\vec{a}||\vec{b}|| \cos\theta| = |\vec{a}||\vec{b}||\cos\theta| \le |\vec{a}||\vec{b}|$

(since $|\cos\theta| \le 1|$)

This is called Cauchy Schwarz's inequality.

40. **(C)** $(\bar{i} + \bar{j} + \bar{k}).(5\bar{i} + 2\bar{j} + 2\bar{k})$

$= 5 + 2 + 2 = 9;$

$$(\bar{i} + \bar{j} + \bar{k}).\left(\frac{5}{2}\bar{i} + \bar{j} + \bar{k}\right) = \frac{5}{2} + 1 + 1 = \frac{9}{2}$$

$$(\bar{i} + \bar{j} + \bar{k}).\left(\frac{5}{2}\bar{i} + \bar{j} + \bar{k}\right) = \frac{5}{3} + \frac{2}{3} + \frac{2}{3} = \frac{9}{3} = 3,$$

$$(\bar{i} + \bar{j} + \bar{k}).\left(\bar{i} + \frac{2}{3}\bar{j} + \frac{2}{5}\bar{k}\right) = 1 + \frac{2}{5} + \frac{2}{5} = \frac{9}{5}$$

$$\therefore \bar{b} = \frac{5}{3}\bar{i} + \frac{2}{3}\bar{j} + \frac{2}{3}\bar{k}$$

PHYSICS

41. **(C)** If the plane of the coil makes an angle with the magnetic field, the magnitude of the torque exerted on it is given by τ = nIAB sinθ. For the given value of θ, when A is doubled τ value doubles.

42. **(A)** Let half life of sample be $t_{1/2}$. If $\frac{7}{8}$ of the radioactive atoms had decayed.

Atoms remaining = $1 - \frac{7}{8} = \frac{1}{8}$

∴ For $\frac{1}{8}^{th}$ $\left(\text{i.e.,} \frac{1}{2^3}\right)$ the no. of half lives undergone are 3.

∴ $3\, T_{1/2}$ = 60 days

$T_{1/2}$ = 20 days

43. **(C)** The dispersion of light in a medium imply:

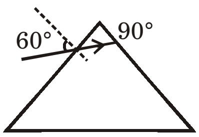
(i) Lights of different wavelengths travel with different speeds in the medium.

(ii) Lights of all frequencies does not travel with the same speed in the medium.

(iii) The refractive index of the medium is different for different wavelengths of light.

44. **(A)** The phase angle between voltage V and current I is $\pi/2$. Therefore, power factor cos = cos $(\pi/2)$ = 0. Hence the power consumed is zero.

45. **(C)**



Given, A = 30°

i_1 = 60° & r_2 = 0

Since A = $r_1 + r_2$

A = r_1 = 30°

$$\mu = \frac{\text{Sin}\, i_1}{\text{Sin}\, r_1} = \frac{\text{Sin}\,60°}{\text{Sin}\,30°} = \frac{(\sqrt{3}/2)}{(1/2)} = \sqrt{3}$$

46. **(C)** If the junction is forward biased in a p – n junction diffusion current is greater than the drift current in magnitude**.**

47. **(A)** The following statements reason out that photo electric effect supports quantum nature of light,

(i) Electric charge of the photo electrons is quantized.

(ii) The maximum KE of photo electrons depends only on the frequency of light and not on intensity.

(iii) There is a minimum frequency.

48. **(B)** Here, A = 5º; $\delta = 3.2°$

Now, for a prism of small angle,

$$\delta = A(\mu - 1)$$

or $$\mu = 1 + \frac{\delta}{A} = 1 + \frac{3.2°}{5°} = 1.64$$

49. **(D)** d = 0.20 m, $\theta = 45°$

$$F = B_H \tan\theta, \quad \frac{\mu_0}{4\pi} \times \frac{2M}{d^3} = B_H \tan\theta$$

$$M = \frac{4\pi}{\mu_0} \times \frac{d^3}{2} B_H \tan\theta$$

$$= 10^7 \times \frac{(0.20)^3}{2} \times 0.4 \times 10^{-4} \times \tan 45°$$

$$= 1.6 \text{ A m}^{-2}$$

50. **(A)** Let R be the radius of curvature of the refracting surface. When object lies in the rarer medium,

$$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

Here, $\mu_1 = 1$; $\mu_2 = 1.5$; u = – 10 cm;

v = – 40 cm

$$\therefore \quad -\frac{1}{-10} + \frac{1.5}{-40} = \frac{1.5 - 1}{R}$$

or $$\frac{0.5}{R} = \frac{1}{10} - \frac{1.5}{40}$$

R = +8 cm (convex)

51. **(D)** Here, d = 0.12 mm = 0.012 cm; D = 1 m = 100 cm;

λ = 6000 Å = 6000 × 10^{-8} cm

For (n + 1)th dark fringe, $y_n' = (2n+1)\frac{D\lambda}{2d}$

For, third dark fringe, n = 2

$$\therefore y_2^1 = (2 \times 2 + 1) \times \frac{100}{2 \times 0.012} \times 6000 \times 10^{-8}$$

$$= 1.25 \text{ cm}$$

52. **(B)** Let L_1 and L_2 be the self-inductances of two inductors and M, the mutual inductance between them. Further, let e_1 and e_2 be the induced e.m.f.s produced in two inductors and e, the total e.m.f. produced in two inductors and e, the total e.m.f. produced. Then,

$e = e_1 + e_2$ ...... (i)

If $\frac{dI}{dt}$ is the rate of change of current through the two inductors in series, then

$$e_1 = -L_1 \frac{dI}{dt} - M\frac{dI}{dt} \quad \text{and}$$

$$e_2 = -L_2 \frac{dI}{dt} - M\frac{dI}{dt}$$

If L is the self-inductance of the two inductors in series, then

$$e = -L\frac{dI}{dt}$$

Therefore, the equation (i) becomes

$$-L\frac{dI}{dt} = \left(-L_1\frac{dI}{dt} - M\frac{dI}{dt}\right) + \left(-L_2\frac{dI}{dt} - M\frac{dI}{dt}\right)$$

$$L = L_1 + L_2 + 2M$$

53. **(A)** Z = $\sqrt{R^2 + (X_L - X_C)^2}$

On added a capacitor in series ($X_L - X_C$) decreases and the value of 'Z' decreases.

54. **(D)** Phase modulation is required for digital communication using a fibre optic link set up.

55. **(D)** In series (minimum value of capacitance) :

$$C = \frac{C_1C_2C_3}{C_1C_2 + C_2C_3 + C_3C_1}$$

$$= \frac{{}^{3}\cancel{9} \times \cancel{9} \times \cancel{9}}{\cancel{3} \times \cancel{81}_{9}} = 3\,\mu F$$

In parallel (maximum value of capacitance)

$C = C_1 + C_2 + C_3$

$= 9 + 9 + 9 = 27\ \mu F$

56. **(C)** Here, $\frac{dI}{dt} = 6 A\,s^{-1}$; e = 18 mV = 18×10^{-3} V

$$e = \frac{\angle dI}{dt}$$

$$L = \frac{e}{(dI/dt)} = \frac{18 \times 10^{-3}}{6} = 3 \times 10^{-3}\,H = 3\,mH$$

57. **(A)** Radio waves are electromagnetic waves whose wavelength ranges from 10^{-3} m to 10^{5} m and has the longest wavelength compared to X - rays, ultraviolet rays and visible light.

58. **(B)** Given : q = 1 C, $E_0 = 8.854 \times 10^{-12} C^2N^{-1}m^{-2}$

From Gauss' theorem, electric flux

$$\phi = \frac{q}{E_o}$$

Electric lines of force originating from a charge of 1 C, $\phi = \frac{1}{8.854 \times 10^{-12}}$ = 1.129×10^{11}.

59. **(C)** If e is the magnitude of charge on an electron or a proton, then

$$F = 9 \times 10^9\ \frac{e^2}{r^2} = 9 \times 10^9 \times \frac{(1.6 \times 10^{-19})^2}{(0.53 \times 10^{-10})^2}$$

$= 8.2 \times 10^{-8}$N

60. **(A)** A thermistor is a heat sensitive resistor, whose value varies appreciably with temperature more so than in standard resistors. Therefore, its temperature coefficient of resistivity should be high. Thermistors are usually made up of metal oxides.

61. **(B)** Binding energy = $\Delta m \times 931.5$ MeV

$\Delta m = 3 \times 4.0026032 - 12$ a.m.u.

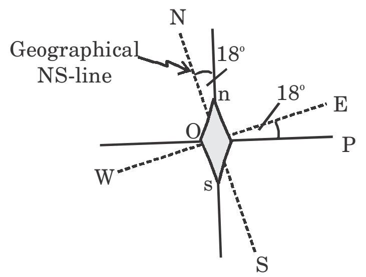
= 0.007806 a.m.u.

As 1 a.m.u. = 931.5 MeV

= 0.007806×931.5 MeV

= 7.27 MeV

62. **(A)** The compass needle *n s* points along the magnetic N S-line and the ship is sailing along OP i.e., towards east (as indicated by compass needle). As the declination of that place is 18° east of north, the geographical N-S line is shown by a dotted N-S line. Thus, the true direction of ship is 18° south of east.



63. **(A)** de-Broglie wavelength,

$$\lambda = \frac{h}{m\upsilon} = \frac{6.62 \times 10^{-34}}{3 \times 2} = 1.1 \times 10^{-34}\ m$$

64. **(B)** As the image formed is erect and hence virtual, the magnification produced by the lens is positive i.e. m = + 4.

Also, f = + 20 cm

Now, $m = \frac{f}{u+f}$

$\therefore\ 4 = \frac{20}{u+20}$ or u + 20 = 5

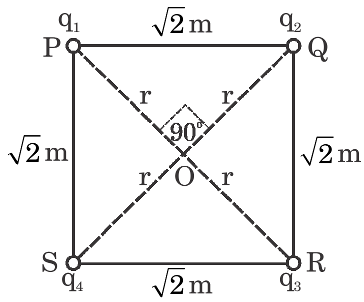
or u = –15 cm

Again, $m = \frac{f-v}{f}$ $\quad \therefore 4 = \frac{20-v}{20}$

or v = 20 – 80 = – 60 cm

65. **(A)** Four charges q_1, q_2, q_3 and q_4 are placed at the four corners of the square PQRS as shown below.

Here,



$q_1 = 2\,\mu C = 2 \times 10^{-6}$ C;

$q_2 = -2\,\mu C = -2 \times 10^{-6}$ C;

$q_3 = -3\,\mu C = -3 \times 10^{-6}$ C;

$q_4 = 6\,\mu C = 6 \times 10^{-6}$ C;

and PQ = QR = RS = PS = $\sqrt{2}$ m

Let r be the distance of each charge from the centre O of the square.

Then, $\sqrt{r^2 + r^2} = \sqrt{2}$ or r = 1 m

Potential at point O due to charges at the four corners,

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right)$$

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{1}{r}(q_1 + q_2 + q_3 + q_4)$$

$$\frac{9\times10^9}{1}\left(2\times10^{-6} + \left(-2\times10^{-6}\right) + \left(-3\times10^{-6}\right) + 6\times10^{-6}\right)$$

$= 2.7 \times 10^4$ V

CHEMISTRY

66. (A) Two 'α' Amino acids on condensation between COOH and NH_2 groups form dipeptide which further reacts with other amino acids forming polypeptides which are called proteins when there are 100 or more amino acids.

67. (A) From the given structure, it is p - p[1] dichlorodiphenyl trichloroethane (DDT) and it was the first chlorinated insecticide.

68. (A) Metals liberated readily react with active carbon under hot conditions to form their carbides.

69. (A) Acetone and carbon disulphide pair shows positive deviation from Raoult's law.

70. (B) In ZnS, S^{-2} ions are present in FCC packing while Zn^{+2} ions occupy alternate tetrahedral voids.

71. (A) Lowering of temperature, increases the rate of physisorption.

72. (A) Let π be the osmotic pressure exerted by the two solutions. Then

$\pi = 0.01 \times RT$ (Glucose)

$\pi = i \times (0.004)\,RT$ (i is the vant-Hoff's factor) (Na_2SO_4)

This gives $i = \frac{0.01}{0.004} = 2.5$

The degree of dissociation (α) is related to i, by the relation,

$$\alpha = \frac{i-1}{n-1} = \frac{2.5-1}{3-1} \text{ (for } Na_2SO_4\text{, n = 3)}$$

$$\alpha = \frac{1.5}{2} = 0.75$$

or % α = 75%

73. (D) X must be an ketone.

$$CH_3 - CH_2 - \overset{O}{\overset{\|}{C}} - CH_3 \xrightarrow[\text{(ii) } H^{\oplus}]{\text{(i) } KMnO_4 / \overset{\ominus}{O}H / \Delta}$$

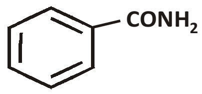
(C_4H_8O)

$CH_3 - CH_2 - COOH$
$(C_3H_6O_2)$
$+ CH_3 - COOH$

When ketones get oxidised, we get carboxylic acids with lesser number of carbon atoms than the original ketone.

74. (A) A catalyst does not change the value of equilibrium constant.

75. (D) The following represents an amide



76. (C) Amorphous solids do not melt. They simply soften on heating, and gradually begin to flow on further heating. These solids are, therefore considered as super cooled liquids.

77. (D) $[Fe(CN)_6]^{3-}$ is an octahedral complex ion and is paramagnetic in nature. Secondly it is an inner orbital complex ion with the presence of only one unpaired electron in it.

78. (D) (i) **Conversion of glucose into ethyl alcohol:** The zymasc enzyme converts glucose into ethyl alcohol and CO_2.

$$\underset{\text{Glucose}}{C_6H_{12}O_6} \xrightarrow{\text{Zymase}} \underset{\text{Ethyl alcohol}}{2C_2H_5OH} + 2CO_2$$

(ii) **Conversion of milk into curd:** It is an enzymatic reaction brought about by lacto bacilli enzyme present in curd.

(iii) **Decomposition of urea into ammonia and CO_2:** The enzyme urease catalyses this decomposition.

$$NH_2\,CONH_2 + H_2O \xrightarrow{\text{Urease}} 2NH_3 + CO_2$$

79. (D) In the complex ion, $[Co(NH_3)_3Cl_3]$, both the NH_3 molecule and Cl^- ions are the ligands.

80. (B) The given cubic solid belongs to the body centred cubic lattice. Since, each corner atom Y is shared by 8 cubes, hence no. of Y atoms per cube = $8 \times \frac{1}{8} = 1$.

Atom X is at the body centre. So, there is only one X atom per cube. Therefore, formula of the compound is XY.

81. **(C)** Acetylation of salicylic acid produces aspirin.

82. **(A)** (i) $CH_3CH_2COONa + NaOH \xrightarrow{CaO, 630K} \underset{\text{Ethane}}{CH_3CH_3} + Na_2CO_3$

(ii) $CH_3CH_2COOH + 6\ HI \xrightarrow[\Delta]{\text{Red P}} \underset{\text{Propane}}{CH_3CH_2CH_3} + 3\ I_2 + 2\ H_2O$

(iii) $CH_3CH_2COONa \xrightarrow[\text{electrolysis}]{\text{Kolbe's}} \underset{\text{Butane}}{CH_3CH_2CH_2CH_3}$

Hence, methane cannot be obtained by any of the above reactions.

83. **(C)** Sucrose on hydrolysis gives equimolar mixture of D (+) glucose and D(–) fructose.

$$C_{12}H_{22}O_{11} + H_2O \rightarrow \underset{\text{D(+)Glucose}}{C_6H_{12}O_6} + \underset{\text{D(–)Fructose}}{C_6H_{12}O_6}$$

84. **(C)** The colour of the complexes of any metal ion depends on the nature of ligands, oxidation state of the metal and the geometry of the complex.

85. **(D)** AgBr shows both Schottky defect and Frenkel defect.

86. **(D)** When o- or p-phenolsulphonic acid is treated with HNO_3, nitration occurs at o, p-positions with simultaneous replacement of SO_3H group by NO_2 group to give ultimately 2, 4, 6-trinitrophenol.

87. **(C)** Since, the compound is optically active and does not rotate the plane of polarized light, therefore, the compound must be a racemic mixture.

88. **(A)** Van-Arkel method involves converting the metal to a volatile stable compound.

$\underset{\text{Impure}}{Ti} + 2\ I_2 \xrightarrow{500\,K} TiI_4$ (volatile stable) ;

$TiI_4 \xrightarrow{1700\,K} \underset{\text{Pure}}{Ti} + 2\ I_2$

89. **(C)** Reducing character of hydrides increases down the group.

90. **(C)** $Al^{3+} + 3e^- \rightarrow Al$

$Cu^{2+} + 2e^- \rightarrow Cu$

$Na^+ + e^- \rightarrow Na$

Thus, 1 F will deposit $\frac{1}{3}$ mol, Al, $\frac{1}{2}$ mol, Cu and 1 mol Na, i.e., moles deposited are in the ratio

$\frac{1}{3} : \frac{1}{2} : 1$ i.e., 2 : 3 : 6 or 1 : 1.5 : 3.

GENERAL AWARENESS

91.	**(D)**	92.	**(D)**	93.	**(D)**
94.	**(A)**	95.	**(D)**	96.	**(A)**
97.	**(D)**	98.	**(C)**	99.	**(C)**
100.	**(A)**				

The End