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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION

Paper Code: UN439 (UPDATED)

Solutions for Class : 12_PCM

MATHEMATICS

1. **(B)** $\Delta = xyz\,(a^3 + b^3 + c^3) - abc\,(x^3 + y^3 + z^3)$

but $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$

Similarly, $x^3 + y^3 + z^3 = 3xyz$

Thus, $\Delta = xyz\,(3abc) - abc\,(3xyz) = 0$

2. **(D)** The function f is clearly continuous at each point in its domain except possibly at $x = 0$, as x, $\sin^{-1}x$ and $\tan^{-1}x$ are continuous functions near "0". So f to be continuous at $x = 0$, we have

$$f(0) = \underset{x\to 0}{\text{Lt}}\ f(x)$$

$$= \underset{x\to 0}{\text{Lt}} \frac{2x - \sin^{-1}x}{2x + \tan^{-1}x} = \frac{1}{3}$$

3. **(A)** $f(x) = \dfrac{x^2 - 1}{x^2 + 1} = 1 - \dfrac{2}{x^2 + 1}$

$f(x)$ will be minimum if $\dfrac{2}{x^2 + 1}$ is maximum.

i.e. If $x^2 + 1$ is least i.e. when $x = 0$.

Thus minimum value of $f(x)$ is $f(0) = -1$

4. **(A)** Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

So that $\vec{a}.\hat{i} = a_1$, $\vec{a}.\hat{j} = a_2$ and $\vec{a}.\hat{k} = a_3$

$\Rightarrow \vec{a} = (\vec{a}.\hat{i})\hat{i} + (\vec{a}.\hat{j})\hat{j} + (\vec{a}.\hat{k})\hat{k}$

5. **(A)** The given system has a unique solution if

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 5 & \lambda \end{vmatrix} \neq 0$$

$\Rightarrow \lambda \neq 8$

6. **(D)** A point on the curve $xy = -1$ is of the form $\left(t, \dfrac{-1}{t}\right)$. Now

$$y = \frac{-1}{x} \Rightarrow \frac{dy}{dx} = \frac{1}{x^2} \Rightarrow \left(\frac{dy}{dx}\right)_{\left(t, \frac{-1}{t}\right)} = \frac{1}{t^2}$$

$\therefore$ Equation of tangent to the curve $xy = -1$

at $\left(t, \dfrac{-1}{t}\right)$ is $y + \dfrac{1}{t} = \dfrac{1}{t^2}(x - t)$

$$\Rightarrow y = \frac{1}{t^2}x - \frac{2}{t}$$

This is a tangent to the parabola.

So this is of the form $y = mx + \dfrac{2}{m}$

So $m = \dfrac{1}{t^2}$ and $\dfrac{-2}{t} = \dfrac{2}{m} \Rightarrow m = -t$

$-t = \dfrac{1}{t^2} \Rightarrow -t^3 = 1 \Rightarrow t = -1$

$\therefore$ Equation of tangent is $y = x + 2$

7. **(C)** $2ax + x^2 = (x + a)^2 - a^2$

Put $x + a = a\sec\theta$, so that $dx = a\sec\theta\tan\theta\, d\theta$

$$\therefore I = \int \frac{a\sec\theta\tan\theta}{a^3\tan^3\theta}.d\theta$$

$$= \frac{1}{a^2}\int \frac{\cos\theta}{\sin^2\theta}.d\theta = \frac{-1}{a^2\sin\theta} + C$$

$$= \frac{-1}{a^2}\frac{\sec\theta}{\tan\theta} + C$$

$$= \frac{-1}{a^2}\frac{x+a}{\sqrt{2ax+x^2}} + C$$

8. **(A)** $\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} = \left(\frac{dy}{dx}\right)^{\frac{1}{2}} + 4$

Squaring on both sides,

$$\left(\frac{d^2y}{dx^2}\right)^3 = \frac{dy}{dx} + 16 + 8\left(\frac{dy}{dx}\right)^{\frac{1}{2}}$$

$$\Rightarrow \left[\left(\frac{d^2y}{dx^2}\right)^3 - \frac{dy}{dx} - 16\right] = 8\left(\frac{dy}{dx}\right)^{\frac{1}{2}}$$

Again squaring on both sides

$$\left[\left(\frac{d^2y}{dx^2}\right)^3 - \frac{dy}{dx} - 16\right]^2 = 64\frac{dy}{dx}$$

$\therefore$ Degree = 6

9. **(B)** $B = -A^{-1}BA$

$\Rightarrow AB = -BA$

$(A+B)^2 = (A+B)(A+B)$

$= A(A+B) + B(A+B)$

$= A^2 + AB + BA + B^2$

$= A^2 + B^2$ ($\because$ $AB = -BA$)

10. **(C)** $y = \log\left\{e^x\left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}\right\} = \log e^x + \log\left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}$

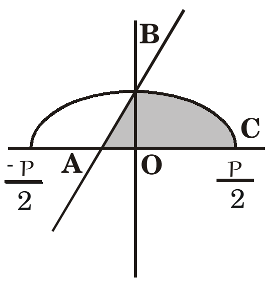
$$= x.\log e + \frac{3}{4}\log\left(\frac{x-2)}{x+2}\right)$$

$$= x + \frac{3}{4}[\log(x-2) - \log(x+2)]$$

$$\therefore \frac{dy}{dx} = 1 + \frac{3}{4}\left[\frac{1}{x-2} - \frac{1}{x+2}\right]$$

$$= \frac{x^2-1}{x^2-4}$$

11. **(A)**



Required area = area of ΔOAB + area of curve OBC

$$= \frac{1}{2} \times 1 \times 1 + 1$$

$$= \frac{3}{2}$$

12. **(B)** $\sin nx - \sin(n-2)x = 2\cos(n-1)x.\sin x$

$\Rightarrow \sin nx = \sin(n-2)x + 2\cos(n-1)x.\sin x$

$$\Rightarrow \frac{\sin nx}{\sin x} = \frac{\sin(n-2)x}{\sin x} + 2\cos(n-1)x$$

$$I_n = \int \frac{\sin nx}{\sin x}.dx = \int \frac{\sin(n-2)x}{\sin x}dx + 2\int \cos(n-1)x.dx$$

$$= I_{n-2} + \frac{2}{n-1}\sin(n-1)x$$

$$\Rightarrow I_n - I_{n-2} = \frac{2}{n-1}\sin(n-1)x$$

13. **(C)** Let T be the thickness of the sides, then that of the top will be $\frac{5}{4}$T.

$$\therefore S = (2\pi rh)T + \pi r^2.\frac{5}{4}T = 2\pi T.r.\frac{V}{\pi r^2} + \frac{5\pi}{4}.T.r^2$$

$$= 2TV.\frac{1}{r} + \frac{5\pi}{4}.T.r^2 = f(r)$$

$$f'(r) = \frac{-2TV}{r^2} + \frac{5\pi}{2}T.r = 0 \Rightarrow r^3 = \frac{4V}{5\pi}$$

$$\Rightarrow 5\pi r^3 = 4V = 4\pi r^2 h$$

$$\Rightarrow \frac{r}{h} = \frac{4}{5}$$

14. **(D)** If "a" is any real number, then every numbered of "a" contains infinitely many rational as well as infinitely irrational points.

$\therefore$ f(x) = 0 or 1 in every numbered of "a".

$\therefore$ $\lim_{x\to a}$ f(x) does not exist

$\therefore$ f(x) is discontinuous everywhere.

15. **(A)** $xA + B = \begin{vmatrix} x^3+x & x+1 & x-2 \\ 2x^3+3x-1 & 3x & 3x-3 \\ x^3+2x+3 & 2x-1 & 2x-1 \end{vmatrix}$

$$R_2 - R_1 - R_3 = \begin{vmatrix} x^3+x & x+1 & x-2 \\ -4 & 0 & 0 \\ x^3+2x+3 & 2x-1 & 2x-1 \end{vmatrix}$$

$$R_1+\frac{1}{4}x^3R_2\ \ R_3+\frac{1}{4}x^3R_2=\begin{vmatrix} x & x+1 & x-2 \\ -4 & 0 & 0 \\ 2x+2x+3 & 2x-1 & 2x-1\end{vmatrix}$$

$$R_3-2R_1=\begin{vmatrix} x & x+1 & x-2 \\ -4 & 0 & 0 \\ 3 & -3 & 3\end{vmatrix}$$

$$=\begin{vmatrix} x & x & x \\ -4 & 0 & 0 \\ 3 & -3 & 3\end{vmatrix}+=\begin{vmatrix} 0 & 1 & -2 \\ -4 & 0 & 0 \\ 3 & -3 & 3\end{vmatrix}$$

$$=x\begin{vmatrix} 1 & 1 & 1 \\ -4 & 0 & 0 \\ 3 & -3 & 3\end{vmatrix}+\begin{vmatrix} 0 & 1 & -2 \\ -4 & 0 & 0 \\ 3 & -3 & 3\end{vmatrix}$$

16. **(A)** $I=\int\frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}dx=\int\frac{4e^{2x}+6}{9e^{2x}-4}dx$

Put $e^{2x}=y\Rightarrow 2e^{2x}dx=dy\Rightarrow dx=dy/2y$

$$I=\int\frac{4y+6}{9y-4}\frac{dy}{2y}=\int\frac{2y+3}{y(9y-4)}dy\ \int\left[\frac{35}{4(9y-4)}-\frac{3}{4y}\right]dy$$

$$=\frac{35}{36}\log|9y-4|-\frac{3}{4}\log|y|+c$$

$$=\frac{35}{36}\log|9e^{2x}-4|-\frac{3}{4}\log e^{2x}+c$$

$$=\frac{35}{36}\log\left(9e^{2x}-4\right)-\frac{3}{2}x+c$$

$\therefore$ A = –3/2

17. **(C)** $S=\sin^{-1}\frac{1}{\sqrt{2}}+\sin^{-1}\frac{\sqrt{2}-1}{\sqrt{6}}+\sin^{-1}$

$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{12}}+......+\sin^{-1}\left(\frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n(n+1)}}\right)$$

Now $T_n=\sin^{-1}\left(\frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n(n+1)}}\right)$

$$=\sin^{-1}\left[\frac{1}{\sqrt{n}}\sqrt{1-\left(\frac{1}{\sqrt{n+1}}\right)^2}-\frac{1}{\sqrt{n+1}}\sqrt{1-\left(\frac{1}{\sqrt{n}}\right)^2}\right]$$

$$=\sin^{-1}\frac{1}{\sqrt{n}}-\sin^{-1}\frac{1}{\sqrt{n+1}}$$

$$\left[\therefore \sin^{-1}x-\sin^{-1}y=\sin^{-1}\left(x\sqrt{1-y^2}-y\sqrt{1-x^2}\right)\right]$$

$$\therefore S=\sin^{-1}\frac{1}{\sqrt{2}}+\left(\sin^{-1}\frac{1}{\sqrt{2}}-\sin\frac{1}{\sqrt{3}}\right)$$

$$+\left(\sin^{-1}\frac{1}{\sqrt{3}}-\sin^{-1}\frac{1}{\sqrt{4}}\right)+....+\infty$$

$$=2\sin^{-1}\frac{1}{\sqrt{2}}=2\left(\frac{\pi}{4}\right)=\frac{\pi}{2}$$

18. **(B)** Given equation is

$$\frac{dy}{dx}+\frac{y^2+y+1}{x^2+x+1}=0$$

$$\Rightarrow\frac{1}{x^2+x+1}dx+\frac{1}{y^2+y+1}dy=0$$

$$\Rightarrow\int\frac{1}{(x+1/2)^2+\left(\sqrt{3}/2\right)^2}dx$$

$$+\int\frac{1}{(y+1/2)^2+\left(\sqrt{3}/2\right)^2}dy=0$$

$$\Rightarrow\frac{2}{\sqrt{3}}\tan^{-1}\frac{x+1/2}{\sqrt{3}/2}+\frac{2}{\sqrt{3}}$$

$$\tan^{-1}\frac{y+1/2}{\sqrt{3}/2}=\frac{2}{\sqrt{3}}c$$

$$\Rightarrow\tan^{-1}\frac{2x+1}{\sqrt{3}}+\tan^{-1}\frac{2y+1}{\sqrt{3}}=c$$

19. **(C)** $k=f(0)=\underset{x\to 0}{Lt}f(x)$

$$=\underset{x\to 0}{Lt}\left(|x|\cos\frac{1}{x}+15x^2\right)=0+0=0$$

20. **(C)** Let $f(x)=x^{27}\cos x$. Then

$f(-x)=(-x)^{27}\cos(-x)=-x^{27}\cos x=-f(x)$

$\Rightarrow f(x)$ is an odd function

$$\Rightarrow\int_{-1}^{1}x^{27}\cos x\,dx=0$$

$$\therefore\int_{-1}^{1}\left(x^{27}\cos x+e^x\right)dx=$$

$$\int_{-1}^{1}x^{27}\cos x\,dx+\int_{-1}^{1}e^x\,dx=0+\left[e^x\right]_{-1}^{1}=e^1-e^{-1}=e-\frac{1}{e}$$

21. **(B)** Let $10-3\pi=\theta$, then $0<\theta<\frac{\pi}{2}$

$$\Rightarrow\theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$$

$\therefore\ \sin^{-1}(\sin 10)=\sin^{-1}(\sin(3\pi+\theta))$

$=\sin^{-1}(\sin(\pi+\theta))$

$= \sin^{-1}(-\sin\theta) = \sin^{-1}(\sin(-\theta)) = -\theta$

$= -(10 - 3\pi) = 3\pi - 10$.

22. **(C)** As $\sin(\cos^{-1}x) = \cos(\sin^{-1}x)$

$= \sqrt{1-x^2}$

for $|x| \le 1$, therefore,

given expression =

$$\sin^{-1}\left(\sqrt{1-x^2}\right) + \cos^{-1}\left(\sqrt{1-x^2}\right) = \frac{\pi}{2}$$

23. **(D)** $$\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} + 2\begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 21+4+10 \\ 27+8+5 \end{bmatrix} + \begin{bmatrix} 8 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 35 \\ 40 \end{bmatrix} + \begin{bmatrix} 8 \\ 4 \end{bmatrix} = \begin{bmatrix} 35+8 \\ 40+4 \end{bmatrix} = \begin{bmatrix} 43 \\ 44 \end{bmatrix}$$

24. **(C)** As A + B is defined, therefore, both A and B are of some order, say, m x n. Also AB is defined, therefore, number of columns in A should be equal to number of rows in B.

$\Rightarrow$ $n = m$.

25. **(C)** We have, $\begin{vmatrix} x-a & x-b & x-c \\ x-b & x-c & x-a \\ x-c & x-a & x-b \end{vmatrix} = 0$

$$\begin{vmatrix} 3x-(a+b+c) & x-b & x-c \\ 3x-(a+b+c) & x-c & x-a \\ 3x-(a+b+c) & x-a & x-b \end{vmatrix} = 0$$

Applying $C_1 \to C_1 + C_2 + C_3$

$$(3x-(a+b+c))\begin{vmatrix} 1 & x-b & x-c \\ 1 & x-c & x-a \\ 1 & x-a & x-b \end{vmatrix} = 0$$

$$\Rightarrow (3x-(a+b+c))\begin{vmatrix} 1 & x-b & x-c \\ 0 & b-c & c-a \\ 0 & b-a & c-b \end{vmatrix} = 0$$

$$\Rightarrow (3x-(a+b+c))\,(a^2+b^2+c^2-ab-bc-bc-ca) = 0$$

$$\Rightarrow x = \frac{1}{3}(a+b+c)$$

26. **(C)** Given, $\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$

Expanding the given determinant, we have

$2x^3 + 2x(ac - ab - bc) = 0$

$\therefore\ x = 0$

27. **(B)** Since $\phi(x) = 2x^3 - 5$ is an increasing function on (1, 2) such that $\phi(1) = -3$ and $\phi(2) = 11$. Clearly, between -3 and 11 there are thirteen points where $I(x) = [2x^2 - 5]$ is discontinuous.

28. **(B)** Since, $f(x) = \sin x - bx + c$ is decreasing on $(-\infty, \infty)$, therefore,

$f'(x) = \cos x - b \le 0$ for all $x \in R$

$\Rightarrow \cos x \le b$ for all $x \in R$

$\therefore\ b \ge 1 \quad (\square\ |\cos x| \le 1)$

29. **(C)** Here, $f(x)\ \sin x\cos x = \frac{1}{2}(\sin 2x) \le \frac{1}{2}(1)$

$= \frac{1}{2}\ \forall\ x \in R$

$\therefore$ Max $f(x) = \frac{1}{2}$ when $\sin 2x = 1$, i.e., when

$x = \frac{\pi}{4}$.

30. **(D)** Let the two positive numbers be x and y

$\therefore\quad x + y = k$

Let $\quad E = x^2 + y^2$

$\Rightarrow\quad E = x^2 + (k - x)^2$

for E to be minimum, we must have

$$\Rightarrow \frac{dE}{dx} = 2x - 2(k - x) = 0$$

$$\Rightarrow x = \frac{k}{2}$$

Now, $\frac{d^2E}{dx^2} = 4 > 0\ \forall x \in R$ throughout

$\therefore$ E attains its minimum at $x = \frac{k}{2}$

$$\Rightarrow y = \frac{k}{2}$$

31. **(A)** $\int e^{\tan^{-1}x}\left(1+\frac{x}{1+x^2}\right)dx =$

$\int e^{\tan^{-1}x}.1dx+\int e^{\tan^{-1}x}\frac{x}{1+x^2}dx$

Apply rule of integration by parts taking $e^{\tan^{-1}x}$ as the first function

$xe^{\tan^{-1}x}-\int e^{\tan^{-1}x}\frac{1}{1+x^2}.x\,dx+\int\frac{e^{\tan^{-1}x}x}{1+x^2}dx+C$

$xe^{\tan^{-1}x}+C$

32. **(Del)**

33. **(B)** The two curves $y = \sin x$ and $y = \cos x$ meet where $\sin x = \cos x$, i.e., where $\tan x = 1$

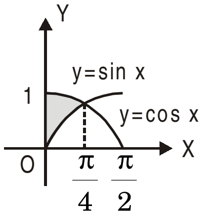
$\Rightarrow x = \pi/4.$

Required area (shown shaded)

$$= \int_0^{\pi/4}(\cos x - \sin x)\,dx$$

$$= [\sin x+\cos x]_0^{\pi/4} = \sin\frac{\pi}{4}+\cos\frac{\pi}{4}-(0+1)$$

$$= \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}-1=\frac{2}{\sqrt{2}}-1=\sqrt{2}-1.$$



Correct option is (B).

34. **(B)** Any circle with centre at origin is $x^2+y^2=r^2$ where r is the parameter.

Differentiating (1) w.r.t.x,

$2x+2y\frac{dy}{dx}=0$ or $x+yy_1=0$

35. **(B)** The given differential equation can be written as $\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^5=\left(\frac{d^2y}{dx^2}\right)^3$.

Here order is 2 and the degree is 3.

36. **(D)** As $|\vec{a}+\vec{b}|^2+|\vec{a}-\vec{b}|^2=2|\vec{a}|^2+2|\vec{b}|^2$

therefore,

$|\vec{a}-\vec{b}|^2=-|\vec{a}+\vec{b}|^2+2|\vec{a}|^2+2|\vec{b}|^2$

$=-1+2+2=3$

$|\vec{a}-\vec{b}|=\sqrt{3}$

37. **(C)** $(\hat{i}+\hat{j})\times(\hat{j}+\hat{k})\cdot(\hat{k}+\hat{i})=(\hat{i}\times\hat{j}+\hat{i}+\hat{k}+\hat{j}\times\hat{k})\cdot(\hat{k}+\hat{i})$

$=(\hat{k}-\hat{j}+\hat{i})\cdot(\hat{k}+\hat{i})=\hat{k}\bullet\hat{k}+\hat{i}\bullet\hat{i}=|\hat{k}|^2+|\hat{i}|^2$

$= 1 + 1 = 2.$

38. **(A)** $\bar{z}=\begin{vmatrix}3-2i & 5+i & 7+3i\\ -i & -2i & 3i\\ 3+2i & 5-i & 7-3i\end{vmatrix}$

Applying $(R_1\leftrightarrow R_3)$

$=\begin{vmatrix}3+2i & 5-i & 7-3i\\ i & 2i & -3i\\ 3-2i & 5+i & 7+3i\end{vmatrix}=z$

$\therefore\ z=\bar{z}$, z is purely real.

39. **(A)** $\frac{dy}{dx}-\frac{2xy}{1+x^2}=0$

$\Rightarrow \frac{1}{y}dy-\frac{2x}{1+x^2}dx=0$

$\Rightarrow \log y-\log(1+x^2)$

$=\log(1+x^2)=\log A$

$\Rightarrow y=A(1+x^2)$

40. **(C)** $r = 10$ cm $\delta(2r)=0.02\,cm \Rightarrow \delta r=0.01$ cm

$\frac{\delta V}{V}\times 100=3.\frac{\delta r}{r}\times 100=3\frac{(0.01)}{10}\times 100=0.3$

PHYSICS

41. **(A)** A parallel plate capacitor is charged and the battery used for charging is then disconnected. The voltage across the plates increases.

42. **(C)** The magnitude of the e.m.f. across the secondary of a transformer depends upon.

(a) the magnitude of the emf applied across the primary.

(b) the number of turns in the primary and the secondary.

$\therefore\ e_S = \frac{N_S}{N_P^2}.e^2p$

43. **(D)** The scattering of $\alpha-\text{particles}$ by a gold foil explains the experimental evidences of the existence of the nucleus in an atom.

44. **(A)** When the refractive index of both lens and transparent liquid are equal, then the lens becomes invisible when immersed in the liquid.

45. **(B)** There will be no loss of energy if the potential of spheres is the same i.e., if

$V = \frac{q}{4\pi\epsilon_0 r} = \frac{Q}{4\pi\epsilon_0 R}$ (or) $\frac{q}{r} = \frac{Q}{R}$, $qR = Qr$

46. **(B)** Acceptor impurity is group III like boron, gallium, indium. It forms p-type semi-conductor.

47. **(B)** We know that, $E = \alpha\theta + \frac{1}{2}\beta\theta^2$,

Given that $E = 14\theta - 0.02\theta^2$

Comparing the two equations,

$\alpha = 14\ \mu V/^\circ C,\ \beta = (-0.04)\ \mu V/^\circ C^2$

Neutral temperature =

$\theta_n = \frac{\alpha}{\beta} = \frac{14}{0.04} = 350^\circ C$

$\theta_0 = 10^\circ C,\ \theta_n = 350^\circ C, \theta_i = ?$

$\theta_0 = 2\theta_n - \theta_0 = 2\times350 - 10 = 690^\circ C.$

48. **(B)** Electromagnetic waves are transverse waves. They can produce interference and can travel through vacuum.

49. **(B)** The concentration decreases to half of its initial amount is from nearly 6000 to 3000.

50. **(C)** A current I flows along the length of an infinitely long, straight, thin walled pipe. The magnetic field at any point inside the pipe is zero.

51. **(B)** f_m = 20 kHz; f_c = 1MHz = 1,000 kHz

side bands are produced at,

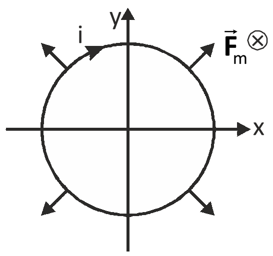
$f_{min} = f_c - f_m$ = 1,000 – 20 = 980 kHz

$f_{max} = f_c + f_m$ = 1,000 + 20 = 1,020 kHz

52 **(C)** $R = R_o A^{1/3}$

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{3}} = \left(\frac{1}{27}\right)^{\frac{1}{3}} = \left(\frac{1}{3^3}\right)^{\frac{1}{3}} = \frac{1}{3}$$

53. **(B)** Net force on a current carrying loop in uniform magnetic field is zero. Hence, the loop cannot translate. From fleming's left hand rule we can see that if magnetic field is perpendicular to paper inwards and current in the loop is clockwise (as shown) the magnetic force $\vec{F}_m$ on each element of the loop is radially outwards, or the loops will have a tendency to expand.



54. **(D)** According to the given information substance 'X' is ferromagnetic in nature which is gadolinium.

A paramagnetic substance is feebly attracted by a magnet. When a rod of paramagnetic substance is suspended in a magnetic field, it slowly sets itself parallel to the direction of the magnetic field. It also moves from a weaker part of the magnetic field to its stronger part, but it is feebly attracted by the magnet.

When a rod of diamagnetic substance is suspended in a magnetic field, it slowly sets itself at right angles to the direction of field. It moves from stronger part of the magnetic field to its weaker part i.e., it is feebly repelled by the magnet.

Aluminium and oxygen are paramagnetic. Gold is diamagnetic. Gadolinium is ferromagnetic. So, substance X is gadolinium.

55. **(A)** Given I = 20 A, n = 9×10^{30} m^{-3} ; A = 10^{-4} m^2 and e = 1.6×10^{-19} C

$$V_d = \frac{I}{neA} = \frac{20}{9\times10^{30}\times1.6\times10^{-19}\times10^{-4}}$$

= 0.138×10^{-6} m s^{-1}

56. **(B)** Here, λ = 600 nm = 600×10^{-9} m;

a = 2 mm = 2×10^{-3} m

Now, the required distance

$$Z_F = \frac{a^2}{\lambda} = \frac{(2\times10^{-3})^2}{600\times10^{-9}} = 6.67 \text{ m}$$

57. **(C)** We know, 1.6×10^{-19} J

Therefore, $1 \text{ J} = \frac{1}{1.6\times10^{-19}} = 6.25\times10^{18}$ eV

58. **(A)** Amplitude of the magnetic field

$$= B_o = \frac{E_o}{c} = \frac{48}{3\times10^8} = 1.6\times10^{-7}\text{T}$$

59. **(A)** When the cells are connected in series, the total e.m.f. is equal to sum of their e.m.fs.

Therefore, total e.m.f. of the cells, E = 2.0 × 6 = 12.0 V

Also, total internal resistance, r = 0.015 × 6 = 0.09 Ω

External resistance, R = 8.5 Ω

Therefore, the current drawn from the supply,

$$I = \frac{E}{R+r} = \frac{12.0}{8.5+0.09} = 1.397 \text{ A}$$

Terminal voltage, V = I R = 1.397 × 8.5

= 11.875 V

60. **(C)** l = 20.0 cm = 0.20 m

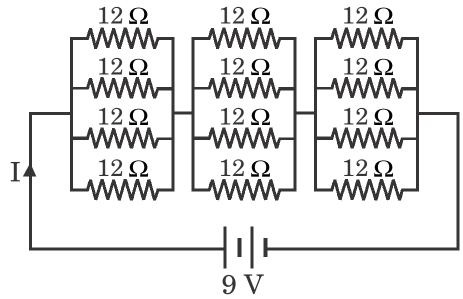
$A = 4.00\times10^{-4}$ m^2, $\rho = 2.82\times10^{-8}$ Ω-m

$$\text{Resistance} = R = \frac{\rho l}{A} = \frac{2.82\times10^{-8}\times0.20}{4.00\times10^{-4}}$$

= $1.41 \times 10^{-5}\,\Omega$

61. **(A)** The network of resistors connected to the battery of e.m.f. 9 V is shown below. Let I be the total current in the circuit. If R' is the effective resistance of the four resistors of 12 Ω each connected in parallel, then

$$\frac{1}{R'} = \frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12} = \frac{4}{12} \text{ or } R' = 3\ \Omega$$



Therefore, the effective resistance of the network of resistors,

R = $R'+R'+R'$ = 3 + 3 + 3 = 9 Ω

The current in the circuit

$$I = \frac{E}{R} = \frac{9}{9} = 1 \text{ A}$$

Since all the four resistors are of the same resistance, same current will pass through the each resistor, Therefore,

the current through each resistor =

$$\frac{1}{4}I = \frac{1}{4}\times1 = 0.25\text{ A}$$

62 **(C)** $e = B\times\pi r^2\times v = \frac{1}{2}B r^2 \omega$

Here, r = 200 cm = 2 m; B = 0.05 Wb m^{-2}

and ω = 60 rad s^{-1}

$$\therefore\ e = \frac{1}{2}\times0.05\times(2)^2\times60 = 6\text{ V}$$

63. **(D)** Here, h = 6.62×10^{-34} J s

de-Broglie wavelength of electron :

Here, $m_e = 9.1\times10^{-31}$ kg ; $v_e = 10^5$ m s^{-1}

$$\therefore\ \lambda_e = \frac{h}{m_e v_e} = \frac{6.62\times10^{-34}}{9.1\times10^{-31}\times10^5}$$

= 7.27×10^{-9} m

64. **(D)** $\text{K.E.} = \frac{e^4 m}{8\varepsilon_0^2 n^2 h^2} = 13.6\text{ eV},$

$\text{P.E.} = \frac{-Ze^2}{8\pi\varepsilon_0 r}$ = −2 × K.E. = −27.2 eV

65. **(B)** $\frac{1}{v}+\frac{1}{9} = \frac{1}{10}$

i.e., v = – 90 cm

Magnitude of magnification =

$\frac{90}{9}$ = 10 cm

CHEMISTRY

66. **(B)** In the addition of HCN to carbonyl group of a ketone, microphic nitrite or cyanide ion attacks the carbonyl atom with simultaneous transfer of π-electrons to the oxygen atom of the carbonyl group

67. **(C)** Radius of Hf^{4+} should been greater than that of Zr^{4+} by atleast 20 pm, but the lanthanoid contraction of about the same magnitude almost cancels the expected increase. As a result, Hf^{4+} and 4 Zr^{4+} have almost equal radii, being 80 pm and 81 pm respectively.

68. **(B)** $\Delta H = (E_a)_f - (E_a)_b$

$(E_a)_b = (E_a)_f - \Delta H$

$\therefore (E_a)_b > (E_a)_f$ and

Given E_{af} = 50 kcal (from options)

ΔH must be negative

$\therefore (E_a)_b$ = (50 + 10) kcal = 60 kcal

69. **(B)** $W = \frac{ItE}{96500}$

$= \frac{1.4 \times 31.75 \times 5\cancel{00}}{965\cancel{00}}$ = 0.23 g.

70. **(A)** Addition of HCl to acetylene in the presence of mercuric salt leads to formation of vinyl chloride which in the monomer for poly vinyl chloride.

71. **(A)** The ideal conditions for the manufacture of H_2SO_4 by contact process are low temperature, high pressure and high concentration of reactants.

72 **(B)** The process of heating the quenched steel to a temperature much below redness and cooling it slowly.

73. **(D)** $CH_3NC + 4H \xrightarrow[C_2H_5OH]{Na} CH_3NHCH_3$

74. **(B)** Benzoic acid in benzene exists as dimer.

75. **(B)** It reacts with KCl to give Cu_2Cl_2 is not correct regarding copper sulphate.

76. **(C)** Daniel cell is an electrochemical cell that converts chemical energy of a spontaneous redox reaction into electrical energy. But from the given redox reactions, the following can be concluded:

(i) A redox reaction is a combination of two half reactions whose addition gives the given overall reaction.

(ii) The reduction half reaction occurs on the copper electrode.

(iii) The oxidation half reaction occurs on the zinc electrode.

77. **(B)** On reduction, aldehydes give primary alcohols, while ketones give secondary alcohols. e.g.,

(i) $CH_3\ CHO + H_2 \xrightarrow{Ni} CH_3CH_2OH$

acetaldehyde (ethyl alcohol)

(ii) $CH_3.\ CO.\ CH_3 + H_2 \xrightarrow{Ni} CH_3\ CH\ OH.CH_3$

acetone (propan - 2 - ol)

78. **(A)** Molality is a preferred unit for measuring concentration because it is temperature independent.

$n_{solvent} = \frac{\text{Mass of solute(w)}}{\text{Molar mass of solute(M)}}$

Molality (m) = $\frac{w / M}{W}$

Molality (m) of a solution does not change with temperature.

79. **(B)** All (copper, lead and chromium) have higher standard reduction potentials than Mn except Mg, which has lower standard reduction potential than Mn.

$\therefore$ Mg will displace Mn from its salt solution ($MnSO_4$).

80. **(D)** $\Delta T_f = k_f m$

$k_f = \frac{\Delta T_f}{m} = \frac{4\,\text{deg}}{0.25\,\text{mol kg}^{-1}}$

= 16 deg kg mol^{-1}

81. **(D)** Secondary amine is formed by the replacement of two hydrogen atoms of NH_3 by alkyl groups i.e., $\begin{matrix} R \searrow \\ \quad NH \\ R \nearrow \end{matrix}$

82. **(C)** 150 mL C_2H_5OH = 150 $\times$ 0.78 g = 117.0 g

$= \frac{117}{46}$ mol

Water = 850 g

Molality = $\frac{117}{46} \times \frac{1}{850} \times 1000$ = 2.99

83. **(B)** All the given uses are of helium. (Refer) Pg: 205 of NCERT book XII chemistry, Part - I - Uses of helium).

84. **(B)** Potassium ethyl xanthate acts as a collector. KCN and NaOH depress the floatation property of ZnS and FeS_2 particles. Thus, only PbS particles go into the froth. Now copper sulphate is added to the tank (mixture) which activates floating character of ZnS and this time only ZnS comes along with froth. The remaining slurry is acidified and FeS_2 floats alongwith the froth.

85. **(C)** 100 g Zn = $\frac{100}{65}$ mole = 1.53 mole

1 L of 1 M $CuSO_4$ sol contains 1 mole of $CuSO_4$.

$\therefore$ In Zn + $CuSO_4 \rightarrow ZnSO_4$ + Cu, $CuSO_4$ is the limiting reagent.

To deposit completely 1 mole of Cu, electricity required = 2×96500 C

Time taken =

$$t = \frac{Q}{I} = \frac{2\times96500}{1}s = \frac{2\times96500}{3600}h$$

= 53.6 h

86. **(C)** Aldehydes are easily oxidised to the corresponding acids by Tollens' reagent while all others are strong oxidising agents and hence, cleave the molecule at the site of the double bond yielding a mixture of products.

$$CH_3CH = CHCHO \xrightarrow{\text{Tollens' reagent}}$$
$$CH_3CH = CHCOOH$$

87. **(D)** Order may or may not be equal to molecularity.

88. **(A)** Reactions I and II give 2-propanol, i.e.,

I. $CH_3CH = CH_2 + H_2O \xrightarrow[\text{Mark.add.}]{H^+} CH_3CHOHCH_3$
2-Propanol

II. $CH_3CHO \xrightarrow[\text{(ii) } H^+/H_2O]{\text{(i) } CH_3MgI} CH_3\text{–}CHOH\text{–}CH_3$
2-Propanol

In contrast, reaction III gives 1-propanol and IV gives 1, 2-propanediol.

III. $CH_2O \xrightarrow[\text{(ii) } H^+/H_2O]{\text{(i) } C_2H_5MgI} C_2H_5CH_2OH$
1-Propanol

IV. $CH_2 = CHCH_3 \xrightarrow[KMnO_4]{\text{Neutral}} \underset{OH}{CH_2} - \underset{OH}{CH} - CH_3$
1, 2 -Propanediol

89. **(D)** Copper ferrocyanide ppt. acts as a semipermeable membrane.

90. **(A)** $HCOOH \xrightarrow{P_2O_5} CO + H_2O$

GENERAL AWARENESS

91. **(C)** 92. **(A)** 93. **(B)** 94. **(C)**
95. **(C)** 96. **(D)** 97. **(B)** 98. **(D)**
99. **(A)** 100. **(A)**

The End