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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION

Paper Code: UN 415

Solutions for Class : 12 (PCM)

Mathematics

1. **(C)** $\frac{d}{dx}(y^2) = \frac{d}{dx}(x)$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2y}$, at (1, 1), $m_1 = \frac{1}{2}$

$\frac{d}{dx}(x^2) = \frac{dy}{dx} = 2x$, $m_2 = 2$

$\therefore \tan\theta = \left|\frac{\frac{1}{2}-2}{1+\left(\frac{1}{2}\right)(2)}\right| = \frac{3}{4}$

$\Rightarrow \theta = \text{Tan}^{-1}\left(\frac{3}{4}\right)$.

2. **(D)** $\begin{vmatrix} x+2 & x+3 & x+5 \\ x+4 & x+6 & x+9 \\ x+8 & x+11 & x+15 \end{vmatrix}$ $R_2 - R_1$, $R_3 - R_2$

$= \begin{vmatrix} x+2 & x+3 & x+5 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$ $C_2 - C_1$, $C_1 - C_2$

$= \begin{vmatrix} x+2 & 1 & 2 \\ 2 & 1 & 1 \\ 4 & 1 & 1 \end{vmatrix}$

$= (x + 2)(1 - 1) - 1(2 - 4) + 2(2 - 4)$

$= 0 + 2 - 4 = -2.$

3. **(A)** $ax^3 + bx$ is an odd function for all values of a and b.

$\therefore \int_{-1}^{1}(ax^3 + bx)\,dx = 0$ for all values of a and b

4. **(A)** $f(5) = \underset{x \to 5}{\text{Lt}} f(x) = \underset{x \to 5}{\text{Lt}} \frac{x^2 - 10x + 25}{x^2 - 7x + 10}$

$= \underset{x \to 5}{\text{Lt}} \frac{(x-5)^2}{(x-2)(x-5)} = 0$.

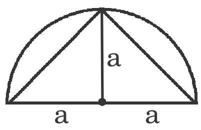
5. **(A)** $A^2 = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$

$A^3 = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 8 \end{bmatrix}$,

$A^3 - A^2 = \begin{bmatrix} -2 & 0 \\ 0 & 4 \end{bmatrix} = -2\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = 2A$

6. **(D)**



Area of triangle $= \frac{1}{2}(2a) \times a$

$= a^2$

7. **(B)** If $\text{Sin}^{-1} x = \alpha$, $\text{Cos}^{-1} x = \beta$ then $x = \sin\alpha$, $x = \cos = \beta$

$\text{Sin}^{-1} x - \text{Cos}^{-1} x = \frac{\pi}{6} \Rightarrow \alpha - \beta = \frac{\pi}{6}$

$\Rightarrow \sin(\alpha - \beta) = \sin\frac{\pi}{6}$

$\Rightarrow \sin\alpha\cos\beta - \cos\alpha\cos\beta = \frac{1}{2}$

$\Rightarrow x^2 - (1 - x^2) = \frac{1}{2}$

$\Rightarrow 2x^2 = \frac{3}{2}$

$\Rightarrow x = \frac{\sqrt{3}}{2}$.

8. **(B)** $\frac{d}{dx}\{\cos^2[\text{Tan}^{-1}(\sin \text{Cot}^{-1} x)]\}$

$$= \frac{d}{dx}\left\{\cos^2\left[\text{Tan}^{-1}\left(\sin\left(\text{Sin}^{-1}\frac{1}{\sqrt{1+x^2}}\right)\right)\right]\right\}$$

$$= \frac{d}{dx}\left\{\cos^2\left[\text{Tan}^{-1}\frac{1}{\sqrt{1+x^2}}\right]\right\}$$

$$= \frac{d}{dx}\left\{1-\frac{1}{2+x^2}\right\}$$

$$= \frac{1}{(2+x^2)^2}(2x)$$

$$= \frac{2x}{(2+x^2)^2}.$$

9. **(A)** Since $x \le x$ for all $x \in$ Z so R is reflexive but is not symmetric as $(1, 2) \in$ R and $(2, 1) \notin$ R. Also R is transitive as $x \le y$, $y \le z \Rightarrow x \le z$. R is antisymmetric for if $x \le y$ and $y \le x$ then $x = y$.

Hence R is partial order.

10. **(B)** $x = a\cos^3\theta$, $y = a\sin^3\theta$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin\theta}{\cos\theta}$$

Equation of the tangent is

$$y - a\sin^3\theta = \frac{-\sin\theta}{\cos\theta}(x - a\cos^3\theta)$$

$\Rightarrow x\sin\theta + y\cos\theta$

$= a\cos\theta\sin\theta(\cos^2\theta + \sin^2\theta)$

$= a\cos\theta\sin\theta$

Equation of the normal is

$$y - a\sin^2\theta = \frac{\cos\theta}{\sin\theta}(x - a\cos^2\theta)$$

$\Rightarrow x\cos\theta + y\sin\theta$

$= a(\cos^4\theta + \sin^4\theta)$

$= a \,.\, \cos 2\theta$

p = length of the perpendicular from origin to the tangent

$$= \frac{|a\cos\theta\sin\theta|}{\sqrt{\sin^2\theta+\cos^2\theta}} = \frac{1}{2}a.\sin 2\theta$$

q = length of the perpendicular from origin to the normal

$$= \frac{|a\cos 2\theta|}{\sqrt{\cos^2\theta+\sin^2\theta}} = a.\cos 2\theta$$

$\therefore\ 4p^2 + q^2 = 4.\frac{1}{4}a^2\sin^2 2\theta + a^2\cos^2 2\theta$

$= a^2$.

11. **(C)** $\begin{pmatrix} 1 & 2 & x \\ 4 & -1 & 7 \\ 2 & 4 & -6 \end{pmatrix}$ is singular

$$\Rightarrow \begin{vmatrix} 1 & 2 & x \\ 4 & -1 & 7 \\ 2 & 4 & -6 \end{vmatrix} = 0$$

$\Rightarrow 1(6 - 28) - 2(-24 - 14) + x(16 + 2) = 0$

$\Rightarrow -22 + 76 + 18x = 0$

$\Rightarrow 18x = -54$

$\Rightarrow x = -3.$

12. **(B)** $A_1 \int_0^{\pi/3} \cos x\, dx = \frac{\sqrt{3}}{2}$ and

$A_2 \int_0^{\pi/2} \cos 2x\, dx = \frac{\sqrt{3}}{4}\quad \therefore \frac{A_1}{A_2} = \frac{2}{1}.$

13. **(B)** $\log\left(\underset{x\to 0}{\text{Lt}}\ f(x)\right)$

$$= \underset{x\to 0}{\text{Lt}}\ \log(x+1)^{\cot x}$$

$$= \underset{x\to 0}{\text{Lt}}\ \frac{\log(x+1)}{\tan x}$$

$$= \underset{x\to 0}{\text{Lt}}\ \frac{1/(x+1)}{\sec^2 x} = \frac{1}{\sec^2 0} = 1$$

$$\Rightarrow \underset{x\to 0}{\text{Lt}}\ f(x) = e = f(0).$$

14. **(C)** $\begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$ = KA

$$= K\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix}$$

$\Rightarrow$ 3a = 2K, 2b = 3K, 24 = –4K

$\Rightarrow$ K = –6, a = –4, b = –9

15. **(C)** $\int\left[\frac{1}{\log x}-\frac{1}{(\log x)^2}\right]dx = \int\left[\frac{1}{t}-\frac{1}{t^2}\right]e^t$

dt[where $\log x = t$] $= \frac{e^t}{t}+c = \frac{x}{\log x}+c$

16. **(A)** $x^2 + y^2 + 2fy + c = 0$

$\Rightarrow 2x + 2yy_1 + 2fy_1 = 0$

$\Rightarrow x + yy_1 + fy_1 = 0$

$\Rightarrow 1 + y_1^2 + yy_2 + fy_2 = 0$

$\Rightarrow 1 + y_1^2 + yy_2 - y_2\left[\frac{-(x+yy_1)}{y_1}\right] = 0$

$\Rightarrow y_1 + y_1^3 + yy_1y_2 + xy_2 - yy_1y_2 = 0$

$\Rightarrow y_1^3 - xy_2 + y_1 = 0$

$\Rightarrow xy'' - (y')^3 - y' = 0.$

17. **(B)**

18. **(A)** $xy = 2,\ y^2 = 4x \Rightarrow x = 1,\ y = 2$

$xy = 2 \Rightarrow x\frac{dy}{dx} + y = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$

$\Rightarrow m_1 = \left(\frac{dy}{dx}\right)_{(1,2)} = -2$

$y^2 = 4x \Rightarrow 2y\frac{dy}{dx} = 4$

$\Rightarrow \frac{dy}{dx} = \frac{2}{y}$

$\Rightarrow m_2 = \left(\frac{dy}{dx}\right)_{(1.2)} = \frac{2}{2} = 1.$

19. **(C)** $|A| = 5(5\alpha) - 5\alpha(0) + \alpha(0) = 25\alpha.$

$|A^2| = |A.A| = |A||A| = |A^2| = 625\alpha^2.$

Given $|A^2| = 25$

$\Rightarrow 625\alpha^2 = 25$

$\Rightarrow \alpha^2 = 1/25$

$\Rightarrow \alpha = 1/5.$

20. **(C)** $0 \le x - [x] < 1$

$\Rightarrow -\frac{1}{2} \le x - [x] - \frac{1}{2} < \frac{1}{2}$

$\Rightarrow -\frac{1}{2} \le f(x) < \frac{1}{2}$

$\Rightarrow f(x) \ne \frac{1}{2}\ \forall x$

$\Rightarrow \{x \in R : f(x) = 1/2\} = \varnothing.$

21. **(B)** $a + b + c = 0 \Rightarrow (a + b + c)^2 = 0$

$\Rightarrow a^2 + b^2 + c^2 + 2(a.b + b.c + c.a) = 0$

$\Rightarrow 2(a.b + b.c + c.a) = -(1 + 4 + 9)$

$\Rightarrow a.b + b.c + c.a = -7.$

22. **(B)** $A^2 = \begin{bmatrix}1 & -2\\4 & 5\end{bmatrix}\begin{bmatrix}1 & -2\\4 & 5\end{bmatrix}$

$= \begin{bmatrix}1-8 & -2-10\\4+20 & -8+25\end{bmatrix} = \begin{bmatrix}-7 & -12\\24 & 17\end{bmatrix}$

$f(A) + \begin{bmatrix}3 & 6\\-12 & 9\end{bmatrix} = \begin{bmatrix}-7 & -12\\24 & 17\end{bmatrix} - 3\begin{bmatrix}1 & -2\\4 & 5\end{bmatrix} + 7\begin{bmatrix}0 & 0\\1 & 1\end{bmatrix} + \begin{bmatrix}3 & 6\\-12 & -9\end{bmatrix}$

$= \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

23. **(A)** $\int\frac{x\,e^x}{(x+1)^2}dx = \int\frac{(x+1-1)\,e^x}{(x+1)^2}$

$= \int e^x\left[\frac{1}{x+1}-\frac{1}{(x+1)^2}\right]dx$

$= e^x\left[\frac{1}{x+1}\right]+c.$

24. **(A)** Equation of family of parabolas with x-axis as axis is $y^2 = 4a(x+\alpha)$ where a, α are two arbitrary constants. So differential equation is of order 2 and degree 1.

25. **(B)** $f(x) = \frac{1}{1+1/x} = \frac{x}{x+1}$

$g(x) = \frac{1}{1+1/f(x)} = \frac{1}{1+(x+1)/x} = \frac{x}{2x+1}$

$g'(x) = \frac{(2x+1)1 - x(2)}{(2x+1)^2} = \frac{1}{(2x+1)^2}$

$\Rightarrow g'(2) = \frac{1}{25}.$

26. **(A)** Put $x = \tan\alpha$, $y = \tan\beta$, $z = \tan\gamma$

$\text{Tan}^{-1}x + \text{Tan}^{-1}y + \text{Tan}^{-1}z = \pi$

$\Rightarrow \alpha + \beta + r = \pi \Rightarrow \alpha + \beta = \pi - \gamma$

$\Rightarrow \text{Tan}(\alpha + \beta)\ \text{Tan}(\pi - \gamma)$

$$\Rightarrow \frac{\text{Tan}\alpha + \text{Tan}\beta}{1 - \text{Tan}\alpha\,\text{Tan}\beta} = -\text{Tan}\gamma$$

$$\Rightarrow \frac{x + y}{1 - xy} = -z$$

$\Rightarrow x + y = -z + xyz \Rightarrow x + y + z = xyz$

27. **(D)** (2, 4) (2, 3) $\in$ R $\Rightarrow$ 2 has two images $\Rightarrow$ R is not a function.

(1, 1) $\notin$ R $\Rightarrow$ R is not reflexive (2, 3) $\in$ R, (3, 2) $\notin$ R $\Rightarrow$ R is not symmetric.

(2, 3) $\in$ R and (3, 1) $\in$ R

$\Rightarrow$ (2, 1) $\notin$ R

Hence, R is not transitive.

28. **(D)** $$\frac{ds}{dt} = 8\pi r\frac{da}{dt}$$

$= 8\pi\ (10)\ (0.05) = 4\pi$

29. **(C)** $$\begin{vmatrix} a & 2b & 2c \\ 3 & b & c \\ 4 & a & b \end{vmatrix} = 0$$

$\Rightarrow a\ (b^2 - ac) - 2b\ (3b - 4c) + 2c\ (3a - 4b) = 0$

$\Rightarrow ab^2 - a^2c - 6b^2 + 8bc + 6ac - 8bc = 0$

$\Rightarrow a\ (b^2 - ac) - 6\ (b^2 - ac)$

$\Rightarrow (a - 6)\ (b^2 - ac) = 0$

$\Rightarrow b^2 - ac = 0$

$\Rightarrow b^2 - ac$

$\Rightarrow abc = b^3$.

30. **(B)** The function f : R $\rightarrow$ R is defined as

$f(x) = x^3 + 5x + 1$

Let y $\in$ R then $y = x^3 + 5x + 1$

$\Rightarrow x^3 + 5x + 1 - y = 0$.

As a polynomial of odd degree has always at least one real rot, corresponding to any y $\in$ co-domain. So $\exists$ some $x \in$ domain such that $f(x) = y$. Hence f is onto.

Also f is continuous on R, because it's a polynomial function. Now $f'(x) = 3x^2 + 5 > 0$.

$\therefore$ f is strictly increasing. Hence f is one-one also.

31. **(A)** Let c = ai + bj. Since b and c are perpendicular to each other, we have b.c = 0

$\Rightarrow 4a + 3b = 0 \Rightarrow a : b = 3 : -4$. Thus c = 3i – 4j. Let a = xi + yj be the required vector.

Component of a on b is 1

$\Rightarrow a\ .\ b/|b| = 1$

$\Rightarrow 4x + 3y = 5 \rightarrow (1)$

Component of a on c is 2

$\Rightarrow a\ .\ c/|c| = 2$

$\Rightarrow 3x - 4y = 10 \rightarrow (2)$

Solving (1) and (2), we get $x = 2$, $y = -1$.

$\therefore$ a = 2i – j

32. **(C)** Put $x + y + 1 = z$

$$\Rightarrow 1 + \frac{dy}{dx} = \frac{dz}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dz}{dx} - 1$$

$$(x + y + 1)\frac{dy}{dx} = 1 \Rightarrow z\left(\frac{dz}{dx} - 1\right) = 1$$

$$\Rightarrow \frac{dz}{dx} = 1 + \frac{1}{z} \Rightarrow \frac{z}{z+1}dz = dx$$

$$\Rightarrow \int\left(1 - \frac{1}{z+1}\right)dz = \int dx$$

$\Rightarrow z - \log(z + 1) = x + c \Rightarrow x + y + 1$

$= \log(x + y + 2) + x + c$

$\Rightarrow y = \log(x + y + 2) + \log c$

$\Rightarrow e^y = (x + y + z)c$

$\Rightarrow x + y + 2 = ce^y$.

33. **(C)** For f to be continuous at $x = 2$,

$$f(2) = \underset{x \to 2}{\text{Lt}}(x-1)^{1/(2-x)}$$

$$= \underset{x \to 2}{\text{Lt}}(1 + (x-2))^{-1/(x-2)} = e^{-1}$$

34. **(A)** Let $\alpha = \frac{1}{2}\tan 2A = \frac{\tan A}{1-\tan^2 A}$,

$\beta = \cot A = \frac{1}{\tan A}$, $\gamma = \cot^3 A = \frac{1}{\tan^3 A}$

$$\alpha+\beta+\gamma = \frac{\tan A}{1-\tan^2 A} + \frac{1}{\tan A} + \frac{1}{\tan^3 A}$$

$$= \frac{\tan^4 A + \tan^2 A - \tan^4 A + 1 - \tan^2 A}{\tan^3 A(1-\tan^2 A)}$$

$$= \frac{1}{\tan^3 A(1-\tan^2 A)}$$

$$\alpha.\beta.\gamma = \frac{\tan A}{1-\tan^2 A}\cdot\frac{1}{\tan A}\cdot\frac{1}{\tan^3 A}$$

$$= \frac{1}{\tan^3 A(1-\tan^2 A)}$$

$\therefore\ \alpha+\beta+\gamma = \alpha.\beta.\gamma \Rightarrow \mathrm{Tan}^{-1}\alpha + \mathrm{Tan}^{-1}\beta + \mathrm{Tan}^{-1}\gamma = 0$ or π

$$\frac{\pi}{4} < A < \frac{\pi}{2} \Rightarrow \frac{\pi}{2} < 2A < \pi$$

$\Rightarrow \mathrm{Tan}^{-1}\alpha < 0,\ \mathrm{Tan}^{-1}\beta > 0,\ \mathrm{Tan}^{-1}\gamma > 0$

$\Rightarrow \mathrm{Tan}^{-1}\alpha + \mathrm{Tan}^{-1}\beta + \mathrm{Tan}^{-1}\gamma = 0.$

35. **(C)** Required area = $\int_a^b x\,dx$

$$= \left(\frac{x^2}{2}\right)^3$$

$$\Rightarrow \frac{9}{2} - \frac{1}{2} = \frac{8}{2} = 4$$

36. **(A)** $f(x) = 2x^3 - 21x^2 + 36x + 20$

$\therefore\ f'(x) = 0 \Rightarrow 6x^2 - 42x + 36 = 0$

$\Rightarrow 6(x^2 - 7x + 6) = 0$

$\Rightarrow (x - 1)(x - 6) = 0 \Rightarrow x = 1$ or 6

$f''(x) = 12x - 42$

$f''(1) = 12 - 42 = -30 < 0$

$\therefore$ $f(x)$ has maximum when $x = 1$.

Maximum value = f(1)

= 2 – 21 + 36 + 20 = 37.

37. **(C)** Let $y = \sqrt{x} \rightarrow (1)$, $2y - x + 3 = 0 \rightarrow (2)$

Substituting (1) in (2), we get $2y - y^2 + 3 = 0$

$\Rightarrow y^2 - 2y - 3 = 0$

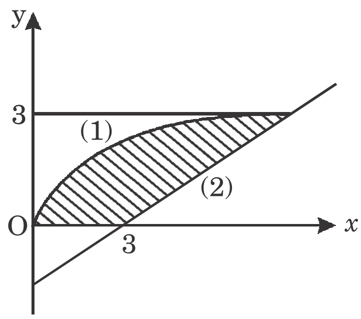
$\Rightarrow (y - 3)(y + 1) = 0$

$\Rightarrow y = -1$ or 3.

Required are = $\int_0^3 [(2y+3) - y^2]\,dy$

$$= \left[y^2 + 3y - \frac{y^3}{3}\right]_0^3 = 9 + 9 - 9 = 9$$

sq.unit



38. **(C)** $f(-1) = 0,\ f(2) = -1,\ f(3) = 1,\ f(4) = -2,\ f(5) = 2,\ f(6) = -3, \ldots$

$\therefore\ f : N \rightarrow Z$ is one one onto.

39. **(B)** If a, b, c are the sides of the triangle respectively then

$|a| = |2i + 3j - 6k|$

$= \sqrt{4+9+36} = \sqrt{49} = 7,$

$|b| = |i + 2j - 3k|$

$= \sqrt{36+4+9} = 7,$

$|c| = |3i + 6j - 2k|$

$= \sqrt{9+36+4} = \sqrt{49} = 7.$

$\therefore$ Perimeter = 7 + 7 + 7 = 21

40. **(B)** $(AB - BA)' = (AB)' - (BA)' = B'A' - A'B'$

$= BA - AB = -(AB - BA)$

Physics

41. **(A)** Here, $B = 2.52 \times 10^{-3}$ T;

$\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}

Length of the solenoid, $l = 0.5$ m;
Total number of turns in the solenoid, N = 500

Therefore, number of turns per unit length of the solenoid,

$$n = \frac{N}{l} = \frac{500}{0.5} = 1000\ \mathrm{m}^{-1}$$

If I is the current through the solenoid, then $B = \mu_0 n I$

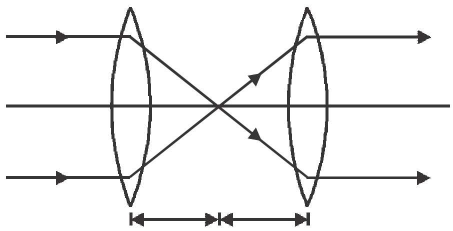
$$\text{or } I = \frac{B}{\mu_0 n} = \frac{2.52\times10^{-3}}{4\pi\times10^{-7}\times1000} = 2.0 \text{ A}$$

42. **(D)** Here, $r = 3.2\times10^{-15}$ m; charge on α-particles, $q_1 = q_2 = 2\times1.6\times10^{-19}$ C

$$\text{Now, } F = \frac{1}{4\pi\varepsilon_0}.\frac{q_1 q_2}{r^2}$$

$$= 9\times10^9\times\frac{(2\times1.6\times10^{-19})^2}{(3.2\times10^{-15})^2} = 90 \text{ N}$$

43. **(B)** As shown in the figure the distance between the lenses should be 30 cm.



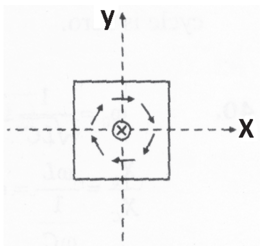
f_1 = 20 cm f_2 = 10 cm

44. **(B)** For meter bridge,

$$\text{Unknown resistance, } R = \frac{l_2}{l_1}\times X$$

$$= \frac{3}{2}\times5 = 7.5\ \Omega$$

45. **(D)** Magnetic lines are tangential to the coil as shown below. Thus, net magnetic flux passing through the coil is always zero or the induced current will be zero.



46. **(A)** $Z^{A\,A} \rightarrow Z-2^{B^{A-4}} + {}_2He^4$

$Z-2^{B^{A-4}} \quad {}_ZC^{A-4} + 2\,{}_{-1}e^0$

Thus, A and C have same atomic number, so these are isotopes.

47. **(D)** Finger prints on a piece of paper can be detected by sprinkling fluorescent powder on the paper and then looking it into ultra-violet light.

48. **(A)** $F = 6\times10^{-6}$ N

$$n = \frac{F}{3\times10^{-10}} = \frac{6\times10^{-6}}{3\times10^{-10}} = 2\times10^4$$

49. **(C)** Magnetic field due to a long current carrying conductor

$$B = \frac{\mu_0 I}{2\pi r} \text{ i.e., } B \ \alpha \ \frac{1}{r}$$

So, correct graph will be (c).

50. **(D)** Here, $v_0 = 2\times10^{14}$ Hz

$\omega = hv_0 = 6.62\times10^{-34}\times2\times10^{14}$

$= 1.324\times10^{-19}$ J

$$= \frac{1.324\times10^{-19}}{1.6\times10^{-19}} = 0.8275 \text{ eV}$$

51. **(A)** By Ohm's law $R = \frac{V}{I}$

The resistance of the heater is constant.

$$R = \frac{V_1}{I_1} = \frac{V_2}{I_2}$$

$$\text{or } I_2 = V_2\frac{I_1}{V_1} = \frac{120\times8}{240} = 4 \text{ A}$$

52. **(C)** Fringe width, $\beta \alpha \lambda$

$$\therefore \frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2}$$

$$\text{or } \frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_1/\mu}$$

$$\therefore \beta_2 = \frac{\beta_1}{\mu} = \frac{0.6}{1.5}$$

$= 0.4$ mm

53. **(B)** Here, rate of production of energy at the atomic power house,

$P = 400$ MW $= 400\times10^6$ J s^{-1}

Therefore, total energy produced in a day i.e., $24\times60\times60$s,

$E = P\times24\times60\times60 = 400\times10^6\times24\times60\times60 = 3.456\times10^{13}$ J

If mass of U^{235} consumed per day is m (in kg) so as to produce the required amount of energy, then

$E = m c^2$

or $3.456\times10^{13} = m c^2$

$$\text{or } m = \frac{3.456\times10^{13}}{c^2} = \frac{3.456\times10^{13}}{(3\times10^8)^2}$$

$= 0.384\times10^{-3}$ kg $= 0.384$ g

54. **(D)** Here, wavelength $\lambda = 5.5\times 10^{-5}$ cm

$= 5.5\times 10^{-7}$ m

Velocity of light $= 3\times 10^{8}$ m s^{-1}

If v is the frequency, then

$$v = \frac{c}{\lambda} = \frac{3\times10^{8}}{5.5\times10^{-7}} = 5.45\times 10^{14} \text{ Hz}$$

$= 5.45\times 10^{8}$ MHz

55. **(A)** Incident wavelength

$= \lambda = 1000$ Å $= 1000\times 10^{-10}$ m

$= 10^{-7}$m

$hv = W + \frac{1}{2}mv^2$ Given, Work function, $W = 0$

Kinetic energy of photoelectrons $= \frac{1}{2}$

$$mv^2 = hv = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{10^{-7}}$$

$$= 19.86\times 10^{-19} \text{ J} = \frac{19.86\times10^{-19}}{1.6\times10^{-19}}$$

$= 12.41$ eV

56. **(B)** Polarity of emf will be opposite in the two cases while entering and while leaving the coil. Only in option (B) polarity is changing.

57. **(B)** In case of transistors, constant α is current gain in common-base configuration and constant β is current gain in common-emitter configuration. Also α is always less than 1 while β is always greater than 1.

58. **(B)** Let B_1, B_2 and B_3 be the magnetic fields produced at the point O by the straight part AB, circular part BC and straight part CD of the current carrying conductor ABCD. Since the point O lies on the straight parts AB and CD,

$B_1 = B_3 = 0$

Further, as the circular segment BC subtends an angle θ at the point O,

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2\pi I}{r}\times\frac{\theta}{2\pi} = \frac{\mu_0}{4\pi}\cdot\frac{I\theta}{r}$$

$$= \frac{10^{-7}\times6\times\pi/3}{0.1} = 6.28\times 10^{-6} \text{ T}$$

Hence,
total magnetic field at the point O,

$B = B_1 + B_2 + B_3 = 0 + 6.28\times 10^{-6} + 0$

$= 6.28\times 10^{-6}$ T

59. **(D)** The work function of lithium is 2.5 eV. The threshold wavelength is

$\lambda = hc/\varphi$.

$$= \frac{(4.14\times10^{-15}\text{ eV}-\text{s})\times(3\times10^{8}\text{ m/s})}{2\times5\text{ eV}}$$

$$= \frac{1242\text{ eV}-\text{nm}}{2.5\text{ eV}} = 497 \text{ nm}.$$

This is the required maximum wavelength.

60. **(C)** The capacitance C of a capacitor depends only on the geometrical configuration (shape, size, separation) of the system of two conductors.

61. **(B)** When x-rays fall on a neutral metallic block, then they eject electrons from it, so block becomes positively charged.

62. **(B)** Energy released

$= (E_A + E_B) - E_X$

$= (110\times8.2 + 90\times8.2) - 200\times7.4$

$= 1640 - 1480$

$= 160$ MeV

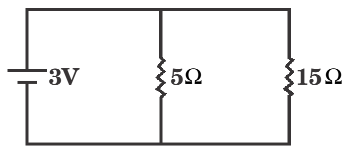
63. **(B)** $P_R = i_{rms}^2 R$ or $P_R \propto i_{rms}^2$

Impedance of the circuit will increase.

Therefore, i_{rms} in the circuit will decrease or average power absorbed by the resistance will decrease.

64. **(B)** Equivalent resistance in series is sum of individual resistances.

In the given figure 3 resistors of 5Ω are connected in series.



$\therefore$ $R' = 5 + 5 + 5 = 15\Omega$

This 15 Ω resistor is connected with the 5 Ω resistor in parallel hence, equivalent resistance now is

$$\frac{1}{R''} = \frac{1}{15} + \frac{1}{5}$$

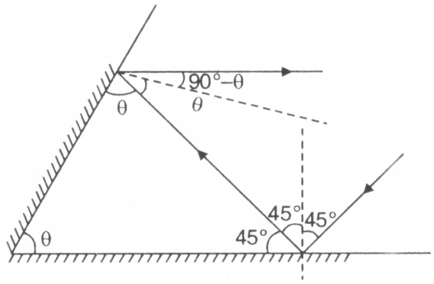
$$\frac{1}{R''} = \frac{5+15}{5\times15}$$

$\Rightarrow R'' = 3.75\Omega$

From Ohm's law, $V = IR$

$$\therefore \quad I = \frac{V}{R} = \frac{3}{3.75} = 0.8\text{A}$$

65. **(C)** The situation is shown in the figure.



$\therefore\ \theta + \theta + 45^\circ = 180^\circ$

or $2\theta = 180^\circ - 45^\circ$

or $2\theta = 135^\circ$

$\therefore\ \theta = \frac{135}{2}$

$= 67.5^\circ$

$= 67^\circ\ 30'$

Chemistry

66. **(C)** Rutile (magnetic) is separated from chlorapatite (non-magnetic) by magnetic separation method.

67. **(B)** Geometrical isomerism is shown by square planar and octahedral complexes.

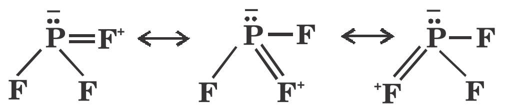
68. **(C)** Ketones are less reactive than aldehydes.

The aromatic aldehydes and ketones are less reactive than the aliphatic aldehydes and ketones. So

$CH_3CHO > CH_3COCH_3 > PhCOPh$

II I III

69. **(C)** The molecule PF_3 is expected to acquire partial double bond character due to back donation of a pair of electrons from F to P which results in the formation of $p\pi - d\pi$ bond. Due to the resonance forms



the bond pair-bond pair repulsion increases to give a higher bond angle.

70. **(C)** Doping of Si with P gives extra electrons while doping with Al gives rise to holes.

71. **(D)** Only 1° amides undergo Hofmann bromamide reaction. Since $CH_3CONHCH_3$ is a 2° amide, therefore, it does not undergo Hofmann bromamide reaction.

72. **(B)** $C_6H_5Cl \xrightarrow{Mg} C_6H_5MgBr$

$\xrightarrow{CH_3CH_2OH} C_6H_6 + CH_3CH_2OMgBr$
Benzene

73. **(C)** The sequence of ribonucleotides in mRNA molecule called codon is a group of three nucleotides.

74. **(A)** To lower the freezing point of water.

75. **(D)** Ionisation energies of d-block elements vary slightly from one another because when we move from left to right, the nuclear charge increases but at the same time screening effect also goes up. These two factors tend to neutralise the effect of each other.

76. **(C)** Vapour pressure of pure, $P_A^o = 40$ mm Hg

Vapour pressure of A in solution,
$P_A = 32$ mm Hg

According to the Raoult's law,
$P_A = P_A^o X_A$

Then, $X_A = \frac{P_A}{P_A^o} = \frac{32 \text{ mm Hg}}{40 \text{ mm Hg}} = 0.8$

77. **(B)** Blasting of TNT is done by mixing it with NH_4NO_3. This mixture of 1 : 5 ratio is called amatol.

78. **(B)** Decomposition of NH_3 on the surface of finely divided platinum is zero order at high concentration but first order at low concentration.

79. **(C)** $C_3H_8 \xrightarrow[+2\,Cl]{-2\,H} C_3H_6Cl_2$. The following four structural isomers are possible $CH_3CH_2CHCl_2$ (I), $CH_3CCl_2CH_3$ (II), $ClCH_2CH_2Cl$ (III), CH_3—CHCl—CH_2Cl (IV). Since (IV) has a chiral carbon, therefore, it has two optical isomers. Therefore, in all, five isomers are possible.

80. **(D)** Bragg's equation

$n\lambda = 2d \sin\theta$

$2\times 1 = 2\times d\times \sin 60^\circ$

$2\times 1 = 2\times d\times \frac{\sqrt{3}}{2}$

$d = \frac{2}{\sqrt{3}} = \frac{2}{1.7} = 1.17\,\text{Å}$

81. **(B)** HCOOH reduces $HgCl_2$ to Hg_2Cl_2 but CH_3COOH does not.

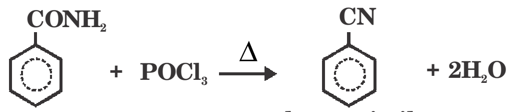
82. **(C)** $PH_3(g) + 4Cl_2(g) \rightarrow PCl_5 + 3HCl(g)$
Phosphine

83. **(D)** Alcohols show extensive association due to intermolecular hydrogen bonding. Ethers do not show intermolecular hydrogen bonding.

84. **(D)** Zeigler-Natta catalyst used for polymerisation of ethylene is a mixture of $TiCl_4$ + trialkyl aluminium.

85. **(D)** CN^- ion is at the extreme right but before CO in the spectrochemical series. So it can cause maximum splitting of d-orbitals.

86. **(B)**



benzonitrile

87. **(A)** Heating steel to 825—875 K in presence of NH_3 is called nitriding.

88. **(A)** NaCl being a salt of strong acid and strong base does not hydrolyse, therefore it will remain as such in the mixture.

89. **(C)**

$$Cl_2 + H_2O \rightarrow HCl + HClO$$
$$HCl \downarrow H^+ + Cl^-$$

$$AgNO_3 + Cl^- \rightarrow \underset{\text{whiteppt.}}{AgCl} + NO_3^-$$

$$\underset{\text{aqueous solution}}{2HCl} + Mg \rightarrow MgCl_2 + H_2(g)$$

90. **(C)** Cell potential

$= E^o_{Red}\,(RHS) - E^o_{Red}\,(LHS)$

$= 0.34 - (-0.76) = +\ 1.10$ V