UNIFIED CYBER OLYMPIAD - UC 329

Solutions for class : 10

Mental Ability

1. **(A)** $2x + 3y = 2$

$3x + 2y = 2$

$3(2x + 3y = 2) \Rightarrow 6x + 9y = 4$

$2(3x + 2y = 2) \Rightarrow 6x + 4y = 4$

$\quad - \quad - \quad -$

$5y = 2$

$\Rightarrow y = \frac{2}{5}$ in $2x + 3y = 2$

$\Rightarrow 2x + 3\left(\frac{2}{5}\right) = 2$

$\Rightarrow 2x + \frac{6}{5} = 2 \Rightarrow 2x = \frac{2}{1} - \frac{6}{5}$

$\Rightarrow 2x = \frac{10-6}{5} = \frac{4}{5}$

$\Rightarrow x = \frac{4}{5 \times 2} = \frac{2}{5}$

$\therefore$ solution $(x, y) = \left(\frac{2}{5}, \frac{2}{5}\right)$ lies in first quadrant.

2. **(C)** When a polynomial is divided by another polynomial, the degree of divisor is always greater than the degree of remainder or the remainder may be equal to zero (if divisor is a factor of dividend).

Hence the answer is option (C).

3. **(C)** Given quadratic equation $x^2 + 5px + 16 < 0$

if there are no real roots, $b^2 - 4ac < 0$

$\Rightarrow (5p)^2 - 4(1)(16) < 0$

$\Rightarrow (5p)^2 - (8)^2 < 0$

$\Rightarrow (5p + 8) - (5p - 8) < 0$

$(5p + 8) < 0$ and $(5p - 8) > 0$ or

$(5p + 8) > 0$ and $(5p - 8) < 0$

$p < -\frac{8}{5}$ and $p > \frac{8}{5}$ or $p > -\frac{8}{5}$ and $p < \frac{8}{5}$

Hence p lies between $-\frac{8}{5}$ and $\frac{8}{5}$

4. **(D)** Given DE ▯ BC

AB = 6 cm

AC = 9 cm

AD = 2 cm

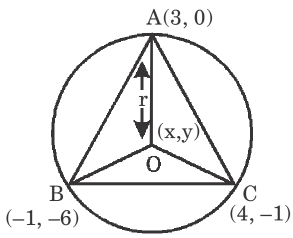
As per Thales theorem, in a triangle if a line is parallel one side then it divides the other two sides in the same ratio.

$\therefore \frac{AC}{AE} = \frac{AB}{AD} \Rightarrow \frac{9}{AE} = \frac{6}{2}$

$\Rightarrow AE = \frac{{}^{3}\cancel{9} \times \cancel{2}}{\cancel{6}_{2}} = 3$

$\therefore$ AE = 3 cm

5. **(A)** let O(x, y) be the circumcentre and r be its circumradius.



Then, OA = OB = OC

consider, OB = OC

$\Rightarrow \quad (x + 1)^2 + (y + 6)^2 = (x - 4)^2 + (y + 1)^2$

$\Rightarrow \quad x^2 + 1\ 2x + y^2 + 36 + 12y$

$x^2 + 16 - 8x + y^2 + 1 + 2y$

$\Rightarrow \quad 2x + 12y + 37 = -8x + 2y + 17$

$\Rightarrow \quad 10x + 10y = -20$

$\Rightarrow \quad x + y = -2 \quad \ldots (i)$

Now, Consider OA = OB

$(x-3)^2 + y^2 = (x+1)^2 + (y+6)^2$

$\Rightarrow x^2 + 9 - 6x + y^2 = x^2 + 1 + 2x + y^2 + 36 + 12y$

$\Rightarrow -6x + 9 = 2x + 12y + 37$

$\Rightarrow -8x - 12y = 28$

$\Rightarrow 2x + 3y = -7$ ... (ii)

On solving Eqs. (i) and (ii), we get

$$\begin{array}{r} 2x + 2y = -4 \\ 2x + 3y = -4 \\ - \quad - \quad + \\ \hline -y = 3 \end{array}$$

$\Rightarrow$ $y = -3$

$\therefore$ $x = 1$

Now, circumradius, $r = \sqrt{(x-3)^2 + y^2}$

$= \sqrt{(1-3)^2 + (-3)^2} = \sqrt{(2)^2 + (-3)^2}$

$= \sqrt{4+9} = \sqrt{13}$ units

6. **(B)** $\underline{17 \times 41 \times 43 \times 61} + (43)$

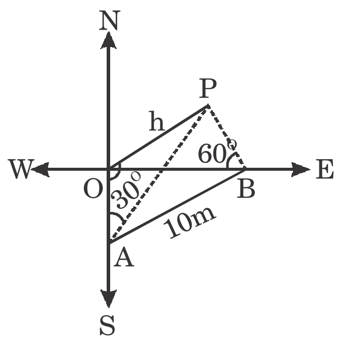
Four odd numbers are multiplied. So the product is also an odd number.

= odd number + odd number

= even number which is greater than 2.

It is clear to say that all even numbers greater than 2 are composite numbers.

7. **(B)**



Let OP be the tower and A and B to two points at a distance of 10 m.

Such that, $\angle OAP = 30^o$

and $\angle OBP = 60^o$ and OP = h

In ΔOAP,

$$\tan 30^o = \frac{h}{Oa} \Rightarrow OB = \frac{h}{\sqrt{3}}$$

In ΔOAB, right angled at O,

$$AB^2 = OA^2 + OB^2$$

$$\Rightarrow \quad 10^2 = (\sqrt{3}\,h)^2 + \left(\frac{h}{\sqrt{3}}\right)^2$$

$$\Rightarrow \quad 100 = 3h^2 + \frac{h^2}{3}$$

$$\Rightarrow \quad 100 = h^2\left(\frac{9+1}{3}\right)$$

$$\Rightarrow \quad 100 = h^2\left(\frac{10}{3}\right)$$

$$\Rightarrow \quad h^2 = 30 = \sqrt{30} \text{ m}$$

8. **(B)** Given TC is the tangent at C.

$\angle$ATC = 36° and $\angle$ACT = 48°

by alternate segment theorem

$\angle$ABC = $\angle$ACT

$\therefore$ $\angle$ABC = 48°

$\angle$CAB = $\angle$ATC + $\angle$ACT = 36° + 48° = 84°

(▯ exterior angle of a triangle is equal to sum of its two interior opposite angles)

now in Δ ABC, we have

$\angle$ABC = 48° and $\angle$CAB = 84°

$\therefore$ $\angle$ACB = 180° – (48° + 84°)

= 180° – 132°

= 48°

now required angle

$\angle$AOB = 2$\angle$ACB (by a theorem)

= 2 (48°)

= 96°

9. **(A)** Let the two numbers be x, y

$x - y = 5$ and $x^2 - y^2 = 65$

we know $(x+y)(x-y) = x^2 - y^2$

$(x+y)(5) = 65$

$x + y = \frac{65}{5}$

$x + y = 13$

$$\begin{array}{r} x - y = 5 \\ x + y = 13 \\ \hline 2x \quad = 18 \end{array}$$

x = 9 in x + y = 13

9 + y = 13

$\Rightarrow$ y = 13 – 9 = 4

$\therefore$ two numbers are 9, 4

larger number = 9.

10. **(C)** Given, p, a, b, q are in AP.

$\therefore \quad 2a = p + b \quad$ ... (i)

and $\quad 2b = a + q \quad$ ... (ii)

From Eqs. (i) and (ii), we get

$2(b - a) = q + a - p - b$

$\Rightarrow 2(b - a) = a - b + q - p$

$\Rightarrow 2(b - a) - (a - b) = q - p$

$\Rightarrow 2(b - a) + (b - a) = q - p$

$\Rightarrow 3(b - a) - = q - p$

$\Rightarrow b - a = \dfrac{q-p}{3} \quad$ ... (iii)

Now, consider p, m, n, q are in AP.

Similarly, we have

$2m = n + p \quad$ ... (iv)

and $\quad 2n = m + q \quad$ ... (v)

From Eqs. (iv) and (v), we get

$2(n - m) = m - n + q - p$

$\Rightarrow 3(n - m) - = q - p$

$\Rightarrow n - m = \dfrac{q-p}{3} \quad$ ... (vi)

From Eqs. (iii) and (vi),

$$\frac{b-a}{n-m} = \frac{\frac{q-p}{3}}{\frac{q-p}{3}} = 1$$

11. **(B)** Breadth of each rectangle = h cm

Lengths of the rectangles are b cm, (b+1) cm, (b+2)cm .... (b+6)cm

Perimeter of 7 rectangles

= 2(b + h) + 2(b + 1 + h) + 2(b + 2 + h) + ... + 2(b + 6 + h)

= 2[b + h + b +1 + h + b + 2 + h+....+ b + 6 + h]

= 2[b + b + ... 7 times + h + h + h + ... 7 times + 1 + 2 + 3 + 4 + 5 + 6]

=2[7b + 7h + 21]

= 2 × 7[b + h + 3]

= 14[b + h + 3]

12. **(A)** Let the age of father be 'x' years.

Let the age of son be 'y' years.

Given, x + y = 65 ......(1)

and 2(x – y) = 50

$\Rightarrow x - y = 25 \quad$ ..... (2)

Adding eq. (1) and eq. (2), we get

$\Rightarrow x = 45$

Hence, the age of father = 45 years.

13. **(D)** C(B, r) = C(B, 8)

area of C(B, 8) = $\pi (8)^2 = 64\pi$

Given area of Q = $\dfrac{1}{8}$ area of C(B, 8)

$= \dfrac{1}{8} \times 64\pi = 8\pi$.

Given area of P = Area of Q = 5 : 4

Area of P = 8π = 5 : 4

$\therefore$ Area of P = $\dfrac{8\pi \times 5}{4} = \dfrac{40\pi}{4} = 10\pi$

C(A, r) = C(A, 10)

Area of C(A, 10) = $\pi (10)^2 = 100\pi$

Area of unshanded part

$= (100\pi - 10\pi) - (64\pi - 18\pi) + (49 - 8\pi)$

$= 90\pi + 46\pi - 8\pi + 49$

$= 128\pi + 49 = 128(3.14) + 49 = 401.92 + 49$

= 450.92

14. **(B)** Three vertices of the triangle

Let $(x_1, y_1) = (a, 0)$

$(x_2, y_2) = (0, b)$

$(x_3, y_3) = (1, 1)$

Centroid = G = $\left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$

$= \left(\dfrac{a + 0 + 1}{3}, \dfrac{0 + b + 1}{3}\right)$

$= \left(\dfrac{a+1}{3}, \dfrac{b+1}{3}\right)$

15. **(B)** From the figure, we can notice that

ϕ, θ are complementary angles.

So, $\cos\theta = \sin\phi \quad$ ..... (1)

from Δ ABC, $\cos\theta = \dfrac{1}{\sqrt{2}}$

$\therefore \sin\phi$ is also $\dfrac{1}{\sqrt{2}} \quad$ [from (1)]

now $\sin^2\phi + \cos^2\theta$

$= \left(\dfrac{1}{\sqrt{2}}\right)^2 + \left(\dfrac{1}{\sqrt{2}}\right)^2$

$= \dfrac{1}{2} + \dfrac{1}{2}$

= 1

Reasoning

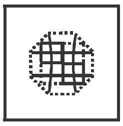
16. **(C)** Here, the black square in the top right corner is moving right to left one space at a time in each figure and the other black square is moving from the bottom to top.

On the following this pattern, we see that option figure (C) will complete the pairs.

Hence, option (D) is correct.

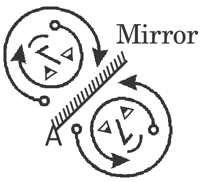
17. **(A)** Except option (C) all other have two + , • one and an arrow symbol.

18. **(A)** Figure (X) can be traced out in option figure (D) as shown below.



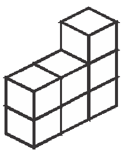
Hence, option (C) is correct.

19. **(A)** The mirror image can be obtained as

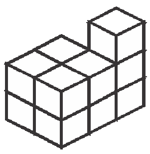


Hence the option (A) is correct

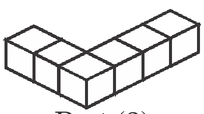
20. **(D)** Following will be the parts after fragmentation



Part (1)



Part (2)



Part (3)

Number of cubes in

Part 1 $\rightarrow$ 7, Part 2 $\rightarrow$ 11,

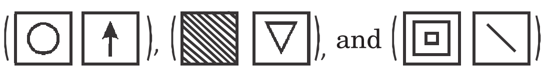
Part 3 $\rightarrow$ 6,

All sum up as 7 + 11 + 6 + 2 = 4

Hence option (D) is correct

21. **(A)** On folding each of the given nets, we find the net in option (A) makes the given cube.

Paries of opposite face



None of the two opposite faces appear as adjacent faces in the given cube.

Hence, option (A) is correct.

22. **(D)** Here, the First and third rows are similar and the second and fourth rows are similar.

On following this pattern, we see that option figure (D) will complete the pairs.

Hence, option (D) is correct.

23. **(A)** Here, the coded series can be decoded as

□ → D, △ → Q, ○ → C, and

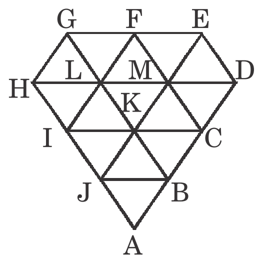
for inner/small figures.

□ → M, ■ → P, △ → R, ○ → E

From the above codes, we see that code for the last figure is CP, where C specifies outer figure and P specifies inner figure.

Hence, option (A) is correct.

24. **(C)** On labelling the given figure, we get



From the above figure, we see that

Number of single triangles

ΔHLG, ΔLIK, ΔLKM, ΔFLM, ΔFME,

ΔEMD, Δ HIL, ΔLIK, ΔLKM, ΔMKC,

ΔMCD, ΔIJK, ΔKBL, ΔKBC and ΔJAB = 14

Number of triangles formed from 4 small triangles =

= ΔACI, ΔHMJ, ΔLDB, ΔGKE, ΔICF = 5

Triangle formed from 9 small triangles

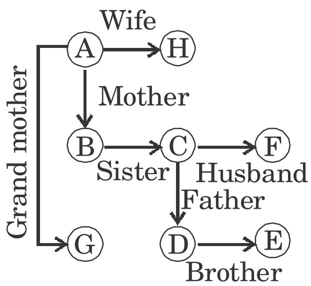
= ΔADH = 1

$\therefore$ Total number of triangles

= 14 + 5 + 1 = 20 triangles

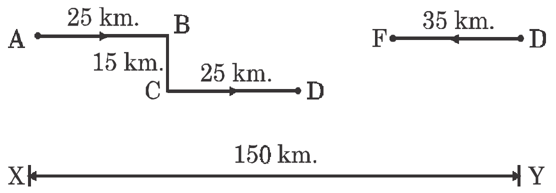
Hence, option (C) is correct.

25. **(A)**



From the above diagram. It is clear that C is ather of E.

26. **(A)**



Requried distance $= 150 - (25 + 25 + 35)$

$= (150 - 85)$km

$= 65$ km

27. **(D)** The words are LAP and COPY.

28. **(B)** $\sqrt{60+40} = 10, \sqrt{20+29} = 7$

The missing number is $\sqrt{55+66} = 11$

29. **(C)** Clearly, the answer is (C). This is the only group in which one letter has bee repeated.

30. **(A)** Begin by looking at the first group SHOUT and SHOT. Work out the positions that the letters in the second word occupy in the first word, S is position 1, H is position 2, O is position 3 and T is position 5. The order is 1235.

Next look at the second group SOLDER and SOLE. Are the positions that the letters in the second word occupy in the first word the same, S is position 1, O is position 2, L is position 3 and E is position 5. The order is 1235.

We have now found the relationship between the letters of the first and second words. 1, 2, 3, and 5.

Now we can use this relationship to work out the second word of the third group. The letters in position 1, 2, 3 and 5 of the first word FLUTED and F, L, U and E which make up the word FLUE.

Computers

31.	**(C)**	32.	**(D)**	33.	**(A)**
34.	**(C)**	35.	**(A)**	36.	**(A)**
37.	**(C)**	38.	**(C)**	39.	**(B)**
40.	**(C)**	41.	**(C)**	42.	**(D)**
43.	**(C)**	44.	**(D)**	45.	**(D)**

English

46.	**(B)**	47.	**(C)**	48.	**(A)**
49.	**(D)**	50.	**(A)**		