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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION (UPDATED)

CLASS - 12 (PCM)

Question Paper Code : UN444

KEY

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11. A	12. D	13. B	14. A	15. B	16. D	17. A	18. B	19. B	20. A
21. D	22. C	23. C	24. C	25. C	26. D	27. B	28. B	29. C	30. C
31. A	32. C	33. D	34. A	35. D	36. A	37. D	38. A	39. C	40. D
41. D	42. A	43. C	44. D	45. C	46. C	47. D	48. A	49. C	50. B
51. A	52. D	53. A	54. A	55. A	56. D	57. D	58. B	59. A	60. B

SOLUTIONS

MATHEMATICS

1. **(D)** Given that, f(x) = e^{2ix} and f : R $\to$ C. Function f(x) is not one-one, because after some values of x (ie, π) it will give the same values.

   Also, f(x) is not onto, because it has minimum and maximum values –1 –i and 1 + i respectively.

   Hence, option (d) is correct.

2. **(B)** $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$

$$\Rightarrow (\tan^{-1} x + \tan^{-1} y) + \tan^{-1} z = \frac{\pi}{2}$$

$$\Rightarrow \tan^{-1}\left(\frac{x+y}{1-xy}\right) + \tan^{-1} z = \frac{\pi}{2}$$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{(x+y)}{1-xy}+z}{1-\left(\frac{x+y}{1-xy}\right)z}\right] = \frac{\pi}{2}$$

$$\Rightarrow \frac{(x+y)+z(1-xy)}{(1-xy)-z(x+y)} = \tan\frac{\pi}{2} = \infty$$

ie, $(1 - xy) - z\,(x + y) = 0$

$$\Rightarrow 1 - xy - zx - yz = 0$$

3. **(D)** Let $A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

Now, $|A| = 7(1-0) + 3(-1-0) - 3(0+1)$ = 1

Cofactors of matrix A are

$C_{11} = 1,\ C_{12} = 1,\ C_{13} = 1$

$C_{21} = 3,\ C_{22} = 4,\ C_{23} = 3$

$C_{31} = 3,\ C_{32} = 3,\ C_{33} = 4$

$$\therefore\ adj\,(A) = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 3 \\ 3 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

$$\therefore\ A^{-1} = \frac{adj\,(A)}{|A|} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

4. **(B)** $f(x) = \begin{vmatrix} 2\cos x & 1 & 0 \\ x - \frac{\pi}{2} & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix}$

$$f'(x) = \begin{vmatrix} -2\sin x & 1 & 0 \\ 1 & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix} + \begin{vmatrix} 2\cos x & 0 & 0 \\ x - \frac{\pi}{2} & -2\sin x & 1 \\ 0 & 0 & 2\cos x \end{vmatrix} + \begin{vmatrix} 2\cos x & 1 & 0 \\ x - \frac{\pi}{2} & 2\cos x & 0 \\ 0 & 1 & -2\sin x \end{vmatrix}$$

$$f'(\pi) = \begin{vmatrix} -2\sin\pi & 1 & 0 \\ 1 & 2\cos\pi & 1 \\ 0 & 1 & 2\cos\pi \end{vmatrix} + \begin{vmatrix} 2\cos\pi & 0 & 0 \\ x - \frac{\pi}{2} & -2\sin\pi & 1 \\ 0 & 0 & 2\cos\pi \end{vmatrix} + \begin{vmatrix} 2\cos\pi & 1 & 0 \\ x - \frac{\pi}{2} & 2\cos x & 0 \\ 0 & 0 & -2\sin\pi \end{vmatrix}$$

$$f'(\pi) = \begin{vmatrix} 0 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{vmatrix} + \begin{vmatrix} -2 & 0 & 0 \\ \pi/2 & 0 & 1 \\ 0 & 0 & -2 \end{vmatrix} + \begin{vmatrix} -2 & 1 & 0 \\ \pi/2 & -2 & 0 \\ 0 & 1 & 0 \end{vmatrix}$$

$$\begin{bmatrix} \because \sin\pi = 0 \\ \cos\pi = -1 \end{bmatrix}$$

$f'(\pi) = -(-2 + 0) = 2$

5. **(B)** $f(x) = \begin{cases} \dfrac{1+3x^2-\cos 2x}{x^2}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$

RHL

$$f(0+h) = \lim_{h\to 0} \frac{1+3(0+h)^2-\cos 2(0+h)}{(0+h)^2}$$

$$= \lim_{h\to 0} \frac{1+3h^2-\cos 2h}{h^2}$$

$$= \lim_{h\to 0} \frac{1+3h^2-(1-2\sin^2 h)}{h^2}$$

$$= \lim_{h\to 0} \frac{1+3h^2-1+2\sin^2 h}{h^2}$$

$$= \lim_{h\to 0} \left\{3+2\left(\frac{\sin^2 h}{h^2}\right)\right\}$$

$$= 3 + 2.\lim_{h\to 0}\left(\frac{\sin h}{h}\right)^2$$

$$= 3+2.(1)^2 \qquad \left\{\because \lim_{h\to 0}\frac{\sin x}{x} = 1\right\}$$

= 3 + 2 = 5

$$f(0-h)=\lim_{h\to 0}\frac{1+3(0-h)^2-\cos 2(0-h)}{(0-h)^2}$$

$$=\lim_{h\to 0}\frac{1+3h^2-\cos 2h}{h^2}$$

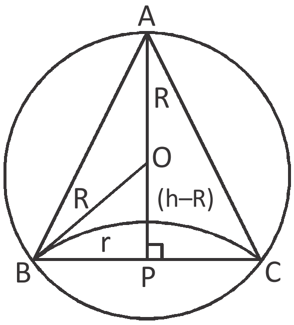
= 5

6. **(C)** Let the height of the cone = h

and the radius of the cone = r

Given, radius of the sphere = R

Now, In ΔOPB



$\Rightarrow$ $R^2 = r^2 + (h-R)^2$

$\Rightarrow$ $r^2 = R^2 - (h-R)^2$

$= (R + h - R)\ (R - h + R)$

$\Rightarrow$ $r^2 = h\ (2R - h)$

The volume of the cone is

$$V=\frac{1}{3}\pi r^2 h$$

$$\Rightarrow \quad V=\frac{1}{3}\pi\, h\,(2R-h)h$$

$$\Rightarrow \quad V=\frac{\pi}{3}(2Rh^2-h^3)$$

Differentiating with r to h

$$\frac{dV}{dh}=\frac{\pi}{3}(4Rh-3h^2)$$

For maximum or minimum value of volume

$$\frac{dV}{dh}=0$$

$$\Rightarrow \quad \frac{\pi}{3}(4Rh-3h^2)=0$$

$\Rightarrow$ $h(4R - 3h) = 0$

$\Rightarrow$ $h = 0$, (Not possible)

$$h=\frac{4R}{3}$$

Now, $$\frac{d^2V}{dh^2}=\frac{\pi}{3}(4R-6h)$$

$$\left(\frac{d^2V}{dh^2}\right)_{\left(\text{at } h=\frac{4R}{3}\right)}=\frac{\pi}{3}\left(4R-6.\frac{4R}{3}\right)$$

$$=\frac{\pi}{3}(4R-8R)=-\frac{4\pi}{3}R \Rightarrow \text{Negative}$$

ie., Maximum

Hence, the height of the cone of maximum volume is $\left(\dfrac{4R}{3}\right)$

7. **(C)** $$I_n=\int_0^{\pi/4}\tan^n x\,dx$$

We have, $$I_{r+2}=\int_0^{\pi/4}\tan^{r+2} x\,dx$$

$$=\int_0^{\pi/4}\tan^r x.\tan^2 x\,dx$$

and $$I_r=\int_0^{\pi/4}\tan^r x\,dx$$

Then, $$I_r+I_{r+2}=\int_0^{\pi/4}\tan^r x\,dx$$

$$+\int_0^{\pi/4}\tan^r x.\tan^2 x\,dx$$

$$=\int_0^{\pi/4}\tan^r x\,(1+\text{than}^2 x)\,dx$$

$$=\int_0^{\pi/4}\tan^r x.\sec^2 x\,dx$$

[Put $t = \tan x \Rightarrow dt = \sec^2 x\ dx$]

$$=\int_0^1 t^r\,dt \Rightarrow \left[\frac{t^{r+1}}{r+1}\right]_0^1$$

$$=\left(\frac{1}{r+1}\right)$$

So, $$I_r+I_{r+2}=\frac{1}{r+1}$$

ie., $$I_2+I_4=\frac{1}{3}$$

$$I_3 + I_5 = \frac{1}{4}$$

$$I_4 + I_6 = \frac{1}{5}$$

which are clearly in HP.

8. **(C)** We know that, if

$I_n = \int \sin^n x \, dx$, then

$$I_n = -\frac{\sin(n-1)x \cos x}{n} + \frac{n-1}{n} I_{n-2}$$

where n is a positive integer.

$\Rightarrow nI_n - (n-1) I_{n-2} = - \sin^{n-1} x \cos x$

9. **(B)** $\tan y \frac{dy}{dx} = \sin (x + y) + \sin (x - y)$

$$\tan y \frac{dy}{dx} = 2. \sin \left(\frac{2x}{2}\right).\cos\left(\frac{2y}{2}\right)$$

$$\left[Q \sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)\right]$$

$$\Rightarrow \tan y \frac{dy}{dx} = 2 \sin x \cos y$$

$$\Rightarrow \frac{\sin y}{\cos y} \frac{dy}{dx} = 2 \sin x \cos y$$

On integrating

$$\Rightarrow \int \frac{\sin y}{\cos^2 y} dy = \int 2 \sin x \, dx$$

$$\left[\begin{array}{l} \text{Let } t = \cos y \\ \frac{dt}{dy} = -\sin y \\ -dt = \sin y \, dy \end{array}\right]$$

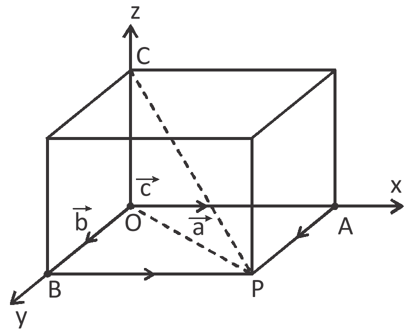
$$\Rightarrow -\int \frac{dt}{t^2} = 2 (-\cos x) + c$$

$$\Rightarrow -\left(-\frac{1}{t}\right) = -2 \cos x + c$$

$$\Rightarrow \frac{1}{\cos y} = -2 \cos x + c$$

$$\Rightarrow \sec y = -2 \cos x + c$$

10. **(A)** Here, OA, OB, OC are the co-terminal edges of a rectangular parallelopiped of volume V.



Also, we know that the volume of rectangular parallelopiped = $[\vec{a}\,\vec{b}\,\vec{c}]$

ie, $V = [\overrightarrow{OA}\ \overrightarrow{OB}\ \overrightarrow{OC}]$ .... (i)

Let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$

then from figure

$\overrightarrow{AP} = \vec{a} + \vec{b}$, $\overrightarrow{BP} = \vec{b} + \vec{c}$, $\overrightarrow{CP} = \vec{c} + \vec{a}$

$\because$ (By vector addition)

Now, we find

$[\overrightarrow{AP}\ \overrightarrow{BP}\ \overrightarrow{CP}] = [(\vec{a} + \vec{b})\ (\vec{b} + \vec{c})\ (\vec{c} + \vec{a})]$

$= (\vec{a} + \vec{b}) . [(\vec{b} + \vec{c}) \times (\vec{c} + \vec{a})]$

$= (\vec{a} + \vec{b}) . [\vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{c} + \vec{c} \times \vec{a}]$

$[\because \vec{c} \times \vec{c} = 0]$

$$\left\{\begin{array}{l} \vec{a} \times \vec{b} = \vec{c} \\ \vec{b} \times \vec{c} = \vec{a} \\ \vec{c} \times \vec{a} = \vec{b} \end{array}\right.$$

$= (\vec{a} + \vec{b}) . [\vec{a} - \vec{a} \times \vec{b} + 0 + \vec{b}]$

$= (\vec{a} + \vec{b}) . [\vec{a} - \vec{c} + \vec{b}]$

$= (\vec{a} + \vec{b}) . \vec{a} - (\vec{a} + \vec{b}) . \vec{c} + (\vec{a} + \vec{b}) . \vec{b}$

$= \vec{a} . \vec{a} + \vec{b} . \vec{a} - \vec{a} . \vec{c} - \vec{b} . \vec{c} + \vec{a} . \vec{b} + \vec{b} . \vec{b}$

$= 1 + 0 - 0 - 0 + 0 + 1$

$= 2.1$

$\because [\vec{a}\,\vec{b}\,\vec{c}] = \vec{a} . (\vec{b} \times \vec{c}) = \vec{a} . \vec{a} = 1$

$= 2\{\vec{a} . (\vec{b} \times \vec{c})\}$

$= 2[\vec{a}\,\vec{b}\,\vec{c}]$

$= 2[\overrightarrow{OA}\ \overrightarrow{OB}\ \overrightarrow{OC}]$

$= 2V$ [from Eq. (i)]

11. **(A)** Given, $f(x) = x^2 - 3$

Now, $f(-1) = (-1)^2 - 3 = 1 - 3 = -2$

$\Rightarrow$ $fof(-1) = f(-2) = (-2)^2 - 3 = 1$

$\Rightarrow$ $fofof(-1) = f(1) = 1^2 - 3 = -2$

Now, $f(0) = 0^2 - 3 = -3$

$\Rightarrow$ $fof(0) = f(-3) = (-3)^2 - 3 = 6$

$\Rightarrow$ $fofof(0) = f(6) = 6^2 - 3 = 33$

Again,

$f(1) = 1^2 - 3 = -2$

$\Rightarrow$ $fof(1) = f(-2) = (-2)^2 - 3 = 1$

$\Rightarrow$ $fofof(1) = (1) = (1)^2 - 3 = -2$

$\therefore$ $fofof(-1) + fofof(0) + fofof(1)$

$= -2 + 33 - 2 = 29$

Now,

$f\left(4\sqrt{2}\right) = \left(4\sqrt{2}\right)^2 - 3 = 32 - 3$

$= 29$

Hence, option (a) is correct.

12. **(D)** $\cos^{-1}\left(-\frac{1}{2}\right) - 2\sin^{-1}\left(\frac{1}{2}\right) + 3\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$- 4\tan^{-1}(-1)$

$= \pi - \cos^{-1}\left(\frac{1}{2}\right) - 2\left(\frac{\pi}{6}\right) + 3\left(\pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)$

$+ 4\tan^{-1}(1)$

$= \pi - \frac{\pi}{3} - \frac{\pi}{3} + 3\left(\pi - \frac{\pi}{4}\right) + 4.\frac{\pi}{4}$

$= \frac{\pi}{3} + 3.\frac{3\pi}{4} + \pi$

$= \frac{43\pi}{12}$

13. **(B)** Given that,

$A = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}$ and $f(t) = t^2 - 3t + 7$

Now, $A^2 = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}$

$= \begin{bmatrix} -7 & -12 \\ 24 & 17 \end{bmatrix}$

Now, $f(A) = A^2 - 3A + 7$

$= \begin{bmatrix} -7 & -12 \\ 24 & 17 \end{bmatrix} - 3\begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -3 & -6 \\ 12 & 9 \end{bmatrix}$

$\therefore f(A) + \begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix} = \begin{bmatrix} -3 & -6 \\ 12 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

14. **(A)** $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0, x \neq y \neq z$

$\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$

$\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} xyz = 0$

$C_1 \leftrightarrow C_2$ and $C_2 \leftrightarrow C_3$

$\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} xyz = 0$

$(1 + xyz)\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} = 0$

Q $R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - R_1$

$(1 + xyz)\begin{vmatrix} x & x^2 & 1 \\ y-x & y^2-x^2 & 0 \\ z-x & z^2-x^2 & 0 \end{vmatrix} = 0$

Expand with r to C_3

$(1 + xyz)$

$\{(y - x)(z - x)(z + x) - (z - x)(y - x)(y + x)\} = 0$

$(1 + xyz)(y - x)(z - x)(z + x - y - x) = 0$

$(1 + xyz)(x - y)(y - z)(z - x) = 0$

$\because \quad x \neq y \neq z \Rightarrow xyz + 1 = 0$

15. **(B)** Given, $\frac{x}{1} = \frac{1-\sqrt{y}}{1+\sqrt{y}}$

Applying componendo and dividendo, we get

$$\frac{1+x}{1-x} = \frac{(1+\sqrt{y})+(1-\sqrt{y})}{(1+\sqrt{y})-(1-\sqrt{y})}$$

$$\Rightarrow \frac{1+x}{1-x} = \frac{2}{2\sqrt{y}}$$

$$\Rightarrow y = \left(\frac{1-x}{1+x}\right)^2$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{-2(1+x)^2(1-x)-(1-x)^2.2(1+x)}{(1+x)^4}$$

$$= \frac{(1-x)(1+x)(-2-2x-2+2x)}{(1+x)^4}$$

$$= \frac{4(x-1)}{(x+1)^3}$$

16. **(D)** Given, error in diameter = $\pm$ 0.04

$\therefore$ Error in radius, (dr) = $\pm$ 0.02

$\therefore$ Per cent error in the volume of sphere

$$= \frac{dV}{V} \times 100 = \frac{d\left(\frac{4}{3}\pi r^3\right)}{\frac{4}{3}\pi r^3} \times 100 = \frac{3dr}{r} \times 100$$

$$= \frac{3 \times (\pm 0.02)}{10} \times 100 = \pm 0.6$$

17. **(A)** $\int (1-\cos x)\,\text{cosec}^2\, x\, dx$

$$= \int \left(2\sin^2 \frac{x}{2}\right).\frac{1}{\sin^2 x}\, dx$$

$$\left[\because \cos x = 1 - 2\sin^2 \frac{x}{2}\right]$$

$$= 2\int \frac{\sin^2 \frac{x}{2}}{\left(2\sin\frac{x}{2}.\cos\frac{x}{2}\right)^2}\, dx$$

$$\left[\because \sin x = 2\sin\frac{x}{2}.\cos\frac{x}{2}\right]$$

$$= 2\int \frac{\sin\frac{x}{2}.\sin\frac{x}{2}}{2.2\sin^2\frac{x}{2}.\cos^2\frac{x}{2}}\, dx$$

$$= \frac{1}{2}\int \sec^2 \frac{x}{2}\, dx = \frac{1}{2}\tan\frac{x}{2}.2 + c$$

$$= \tan\frac{x}{2} + c = f(x) + c$$

$$\Rightarrow \quad f(x) = \tan\frac{x}{2}$$

18. **(B)** Let I $= \int_0^{\pi} \frac{1}{1+\sin x}\, dx$

$$= \int_0^{\pi} \frac{1}{1+\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}\, dx$$

$$= \int_0^{\pi} \frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}\, dx$$

Put $\tan\frac{x}{2} = t \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}\, dx = dt$

$$\therefore I = \int_0^{\infty} \frac{2dt}{(1+t)^2} = \left[-\frac{2}{1+t}\right]_0^{\infty} = 2$$

19. **(B)** Given, $\frac{dy}{dx} = \sin(x+y)\tan(x+y) - 1$

Put $x + y = z \Rightarrow 1 + \frac{dy}{dx} = \frac{dz}{dx}$

$$\therefore \quad \frac{dz}{dx} - 1 = \sin z \tan z - 1$$

$$\Rightarrow \int \frac{\cos z}{\sin^2 z}\, dz = \int dx$$

Put $\sin z = t$

$\Rightarrow \cos z\, dz = dt$

$\therefore \int \frac{1}{t^2}\, dt = x - c \Rightarrow -\frac{1}{t} = x - c$

$\Rightarrow -\operatorname{cosec} z = x - c$

$\Rightarrow x + \operatorname{cosec}(x + y) = c$

20. **(A)** Given that, $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$

and $\vec{c} = \lambda\hat{i} + \hat{j} + (2\lambda - 1)\hat{k}$

Also, since $\vec{a}$ and $\vec{b}$ lies in the same plane, then $(\vec{a} + \vec{b})$ is perpendicular vector to this plane. Given that vector $\vec{c}$ is parallel to the plane containing $\vec{a}$ and $\vec{b}$, so vector $(\vec{a} + \vec{b})$ also perpendicular to the vector $\vec{c}$ *ie*, $(\theta = 90°)$

So, $(\vec{a} \times \vec{b}).\vec{c}$ should be equal to zero

or $(\vec{a} \times \vec{b}).\vec{c} = 0$ .... (i)

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 3 & -1 \end{vmatrix}$$

$= (2 - 9)\hat{i} + (6 + 1)\hat{j} + (3 + 4)\hat{k}$

$= -7\hat{i} + 7\hat{j} + 7\hat{k}$

Then from Eq. (i)

$(-7\hat{i} + 7\hat{j} + 7\hat{k}).(\lambda\hat{i} + \hat{j} + (2\lambda - 1\hat{k})$ $= 0$

$\Rightarrow -7\lambda + 7 + 7(2\lambda - 1) = 0$

$\Rightarrow -7\lambda + 7 + 14\lambda - 7 = 0$

$\Rightarrow 7\lambda = 0$

$\Rightarrow \lambda = 0$

Hence, the value of λ is 0.

21. **(D)** If matrix has no inverse it means the value of determinant should be zero.

$$\therefore \begin{vmatrix} 1 & -1 & x \\ 1 & x & 1 \\ x & -1 & 1 \end{vmatrix} = 0$$

If we put $x = 1$, then column Ist and IIIrd are identical.

Hence, option (d) is correct.

22. **(C)** $y = \cos^{-1}\left(\frac{a^2 - x^2}{a^2 + x^2}\right) + \sin^{-1}\left(\frac{2ax}{a^2 + x^2}\right)$

Put $x = a \tan\theta$

$\Rightarrow \theta = \tan^{-1}\left(\frac{x}{a}\right)$

$\Rightarrow y = \cos^{-1}\left(\frac{a^2 - a^2\tan^2\theta}{a^2 + a^2\tan^2\theta}\right) + \sin^{-1}\left(\frac{2a^2\tan\theta}{a^2 + a^2\tan^2\theta}\right)$

$\Rightarrow y = \cos^{-1}\left(\frac{1 - \tan^2\theta}{1 + \tan^2\theta}\right) + \sin^{-1}\left(\frac{2\tan\theta}{1 + \tan^2\theta}\right)$

$\Rightarrow y = \cos^{-1}(\cos 2\theta) + \sin^{-1}(\sin 2\theta)$

$$\left[\because \cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta} \qquad \sin^2\theta = \frac{2\tan\theta}{1 + \tan^2\theta}\right]$$

$\Rightarrow y = 2\theta + 2\theta$

$\Rightarrow y = 4\theta$

$\Rightarrow y = 4\tan^{-1}\left(\frac{x}{a}\right)$

$\Rightarrow \frac{dy}{dx} = 4.\frac{1}{1 + \frac{x^2}{a^2}}.\frac{1}{a} = 4.\frac{a^2}{a^2 + x^2}.\frac{1}{a}$

$\Rightarrow \frac{dy}{dx} = \frac{4a}{a^2 + x^2}$

23. **(C)** Let A be the area and x be the side of an equilateral triangle.

$\therefore \quad A = \frac{\sqrt{3}}{4} x^2$

on differentiating both sides w.r.t. x, we get

$\Rightarrow \quad \frac{dA}{dx} = \frac{\sqrt{3}}{4}(2x)$

$\Rightarrow \quad dA = \frac{\sqrt{3}}{2} x.dx$ or $\Delta A = \frac{\sqrt{3}}{2} \times \Delta x$

$\therefore$ Percentage error in area = $\frac{\Delta A}{A} \times 100$

$$= \frac{\frac{\sqrt{3}}{2} \times \Delta x \times 100}{\frac{\sqrt{3}}{4} x^2} = \frac{2\Delta x}{x} \times 100$$

$= \frac{2 \times 0.05}{10} \times 100 = 1\%$ $[\because \Delta x = 0.05, \text{given}]$

24. **(C)** $I = \int \frac{dx}{(x+1)\sqrt{4x+3}}$

Put $4x + 3 = t^2 \Rightarrow 4\, dx = 2t\, dt$

$\therefore \ I = \frac{1}{2}\int \frac{t\,dt}{\left(\frac{t^2-3}{4}+1\right)t} = 2\int \frac{dt}{1+t^2}$

$= 2\tan^{-1} t + c = 2\tan^{-1}\sqrt{4x+3} + c$

25. **(C)** $(\vec{a} + 2\vec{b} - \vec{c}) . (\vec{a} - \vec{b}) \times (\vec{a} - \vec{b} - \vec{c})$

$= (\vec{a} + 2\vec{b} - \vec{c}) . [\vec{a} \times \vec{a} - \vec{b} \times \vec{a} - \vec{a} \times \vec{b} + \vec{b} \times \vec{b}$
$- \vec{a} \times \vec{c} + \vec{b} \times \vec{c}]$

$= (\vec{a} + 2\vec{b} - \vec{c}) . [0 - (-\vec{c}) - (\vec{c}) + 0 - (-\vec{b}) + \vec{a}]$

$[\because \vec{a} \times \vec{a} = \vec{b} \times \vec{b} = 0]$

$= (\vec{a} + 2\vec{b} - \vec{c}) . (\vec{c} - \vec{c} + \vec{b} + \vec{a})$

$\left\{\begin{array}{l} \because \vec{a} \times \vec{b} = \vec{c} \\ \vec{b} \times \vec{c} = \vec{a} \\ \vec{c} \times \vec{a} = \vec{b} \end{array}\right.$

$= (\vec{a} + 2\vec{b} - \vec{c}) . (\vec{b} + \vec{a})$

$= \vec{a}.\vec{b} + 2\vec{b}.\vec{b} - \vec{c}.\vec{b} + \vec{a}.\vec{a} + 2\vec{b}.\vec{a} - \vec{c}.\vec{a}$

$= 0 + 2(1) - 0 + (1)\ 2(0) - (0)$

$\left\{\begin{array}{l} \because \vec{a}.\vec{b} = 0 \\ \vec{b}.\vec{c} = 0 \\ \vec{c}.\vec{a} = 0 \end{array}\right.$

$= 2 + 1 = 3.1$

$= 3.\{\vec{a}.(\vec{b} \times \vec{c})\}$ $\left\{\begin{array}{l} \because \vec{a}.\vec{b} = 0 \\ \vec{b}.\vec{c} = 0 \\ \vec{c}.\vec{a} = 0 \end{array}\right.$

$= 3[\vec{a}\,\vec{b}\,\vec{c}]$ $\quad \because [\vec{a}\,\vec{b}\,\vec{c}] = \vec{a}.(\vec{a} \times \vec{c})$

PHYSICS

26. **(D)** $j = nev_d$

or, $v_d = \frac{j}{ne} = \frac{i}{A\,ne}$

$$= \frac{1\text{ A}}{(2 \times 10^{-6}\text{m}^2)(8.5 \times 10^{22} \times 10^6\text{m}^{-3})(1.6 \times 10^{-19}\text{C})}$$

= 0.036 mm/s.

We see that the drift speed is indeed small.

27. **(B)** Here, B = 0.3 T; $\theta = 30^{\circ}$; $\tau = 0.06$ N m

Now, $\tau = MB\sin\theta$

$\therefore M = \frac{\tau}{B\sin\theta} = \frac{0.06}{0.3 \times \sin 30^{\circ}} = 0.4 \text{ A m}^2$

28. **(B)** $B = 8 \times 10^{-7}$ T, $c = 3 \times 10^8$ m/s, E = ?

$E = c \times B = 3 \times 10^8 \times 8 \times 10^{-7} = 240$ V/m

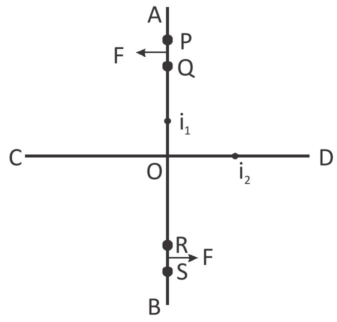
29. **(C)** The de Broglie wavelength of a particle whose momentum is p is $\lambda = h/p$. For this proton, we find that

$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}\text{ Js}}{3.3 \times 10^{-23}\text{ kgm/s}}$

$= 2.0 \times 10^{-11}$ m = 0.02 nm

30. **(C)** Due to the property of self-induction, a coil opposes the time variations in the own current.

31. **(A)** Applying Ampere's law to the rectangle shown below.



$(2Bl) = \mu_0(\lambda l)$

$\therefore \quad B = \dfrac{\mu_0 \lambda}{2}$

32. **(C)** Here, $p = 4 \times 10^{-9}$ C m, $E = 5 \times 10^4$ N C^{-1}; $\theta = 30^\circ$

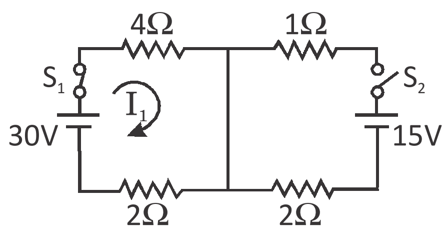
Now, magnitude of torque acting on the dipole,

$\tau = p E \sin \theta = 4 \times 10^{-9} \times 5 \times 10^4 \sin 30^\circ$

$= 2 \times 10^{-4} \times \dfrac{1}{2} = 10^{-4}$ N m

33. **(D)** When the switch S_1 is closed and S_2 is opened, the current flows only in the first loop.

$\therefore \quad I_1 = \dfrac{30}{4+2} = 5$ A



34. **(A)** At points where the two interfering waves meet in the same phase, the resultant intensity is maximum.

The resulting intensity at any point depends upon the phase difference (ϕ) between the two waves at that point

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

For maximum intensity or constructive interference $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$

$$= \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = K(a_1 + a_2)^2$$

(Since, intensity $\propto$ amplitude square.)

For destuctive interference or minimum intensity,

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

$$= \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = K(a_1 - a_2)^2$$

Given, $\dfrac{I_1}{I_2} = \dfrac{9}{16} = \dfrac{a_1^2}{a_2^2}$

$\Rightarrow \dfrac{a_1}{a_2} = \dfrac{3}{4}$

$\therefore \dfrac{I_{max}}{I_{min}} = \dfrac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \dfrac{(3+4)^2}{(3-4)^2} = \dfrac{49}{1}$

35. **(D)** When the key is open, 120 V is divided among C_1 and C_2 in the inverse ratio of their capacitances.

$\therefore V_1 = \dfrac{120}{2+3} \times 3 = 72$ V

$V_2 = \dfrac{120}{2+3} \times 2 = 48$ V

$\therefore \quad q_1 = 72 \times 2 = 144\ \mu C$

and $q_2 = 48 \times 3 = 144\ \mu C$

When the key is closed let q_1 and q_2 be the steady charge on C_1 and C_2. Then by the loop rule

$60 - \dfrac{q_1}{2 \times 10^{-6}} = 0 \Rightarrow q_1 = 120\ \mu C$

and $\quad 60 - \dfrac{q_2}{3 \times 10^{-6}} = 0 \Rightarrow q_2 = 180\ \mu C$

$\therefore$ Charge that flows through section 1 = 144 – 120 = 24 μC

Charge that flows through section 2 = 180 – 144 = 36 μC

Charge that flows through section 3 = 24 + 36 = 60 μC

36. **(A)** Angular magnification

$$= \frac{15}{0.01} = 1500$$

37. **(D)** Voltage lags the current.

Hence, $X_C > X_L$

$$\text{Further } \cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_C - X_L)^2}}$$

$$= \frac{1}{\sqrt{1 + \left(\frac{X_C - X_L}{R}\right)^2}}$$

To raise the power factor, the denominator should decrease. For this either R should increase or $X_C - X_L$ should decrease. If a resistance is added in series, the denominator will decrease $X_C - X_L$ or $\frac{1}{\omega C} = \omega L$ can be decreased either by increasing C or L. If we put a capacitor in series the equivalent capacitance $\left(= \frac{C_1 C_2}{C_1 + C_2}\right)$ of the circuit will decrease. Hence, either an inductor or a resistance should be placed in series to raise the power factor.

38. **(A)** The equivalent resistance of 6 Ω and 3 Ω resistors is $\frac{(6\Omega) \times (3\Omega)}{6\Omega + 3\Omega} = 2\ \Omega$.

This is connected in series with the 1 Ω resistor. The equivalent resistance of the circuit is R = 2 Ω + 1 Ω = 3 Ω.

The current through the battery is

$$i = \frac{9\text{ V}}{3\Omega} = 3\text{ A}.$$

(a) The current through the 1 Ω resistor is, therefore, 3 A.

The heat developed in this resistor is

$H = i^2 Rt$

$= (3\text{ A})^2 \times (1\ \Omega) \times (60\text{ s}) = 540\text{ J}.$

(b) The current through the 6 Ω resistor is

$$(3\text{ A}) = \frac{3\ \Omega}{6\ \Omega + 3\ \Omega} = 1\text{ A}.$$

The heat developed in it

$= (1\text{ A})^2 \times (6\ \Omega) \times (60\text{ s}) = 360\text{ J}.$

(c) The current through the 3 Ω resistor is 3 A – 1 A = 2 A.

The heat developed in it

$= (2\text{ A})^2 \times (3\ \Omega) \times (60\text{ s}) = 720\text{ J}.$

39. **(C)** Here, V = 18 k V = 18×10^3 V

if v_{max} is the velocity of the fastest electron, then

$$\frac{1}{2} v_{max} 2 = e\text{ V} \text{ or}$$

$$v_{max} = \sqrt{\frac{2e\text{ V}}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 18 \times 10^8}{9 \times 10^{-31}}}$$

$= 8 \times 10^7\text{ m s}^{-1}$

40. **(D)** Initially the capacity of the capacitor increases and then decreases. So, positive charge on plate A first increases and then decreases i.e., current in the outer circuit first flows from B to A and then from A to B.

CHEMISTRY

41. **(D)** The reaction at high temperature in the blast furnace is

$2CuFeS_2 + O_2 \rightarrow Cu_2S + 2FeS + SO_2$.

42. **(A)**

$$\underset{0.01\text{ mol/L}}{\left[Co(SO_4)(NH_3)_5\right]Br} + \underset{\text{from } AgNO_3}{Ag^+(\text{excess})} \rightarrow$$

$$\left[Co(SO_4)(NH_3)_5\right]^+ + \underset{0.01\text{ mol}}{AgBr(s)}$$

$$\underset{0.01\text{ mol/L}}{\left[CoBr(NH_3)_5\right]SO_4} + \underset{\text{from } BaCl_2}{Ba^{2+}(aq)} \rightarrow$$

$$\left[CoBr(NH_3)_5\right]^{2+} + \underset{0.01\text{ mol}}{BaSO_4(s)}$$

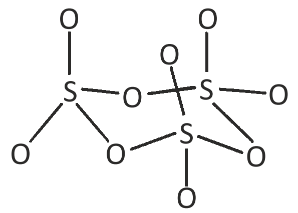
43. **(C)** Ethyl alcohol forms stronger H-bonds than ethylamine or ammonia due to greater electronegativity of oxygen than nitrogen atom. Diethyl ether, however, does not form H-bonds since it does not have a H-atom attached to O-atom.

44. **(D)** Ag does not react with $Cu(NO_3)_2$ solution as e.m.f. of cell reaction is negative.

45. **(C)** Reformatsky reaction is used to prepare β-hydroxy esters.

46. **(C)** Yellow colour on heating NaCl in the presence of Na is due to the presence of electrons in anion vacancies (F-centres)

47. **(D)** There are no S–S bonds in sulphur trioxide trimer (S_3O_9) as shown below.



48. **(A)** Mass of 5 L solution

= 5 L × 0.981 kg L^{-1} = 4.905 kg = 4905 g

Mass of 2 m solution

= 1000 g + 2 moles of methanol

= 1000 + 2 × 32

= 1000 + 64 = 1064 g

Now 1064 g of solution contains methanol = 2 mol

4905 g of solution contains methanol

$\frac{2}{1064} \times 4905 = 9.22$ mol.

49. **(C)** Colloidal solution of liquid-in-liquid is emulsion and not a gel.

50. **(B)** Co^{2+} and Fe^{2+} ions are coloured.

51. **(A)** Aromatic primary amines i.e., aniline gives dye test.

52. **(D)** For the same alkyl group, boiling point increases as the size of halogen increases. Thus, C_2H_5I has the highest boiling point.

53. **(A)** For a first order reaction, $t_{1/2}$ is independent of the initial concentration of the reactants.

From the given data, $t_{1/2}$ = 15 min

So, $0.1\text{ M} \xrightarrow{t_{1/2}} 0.05\text{ M} \xrightarrow{t_{1/2}} 0.025\text{ M}$

So, time required for concentration to change from 0.1 M to 0.025 M = $2t_{1/2}$

= 2 × 15 minutes = 30 minutes

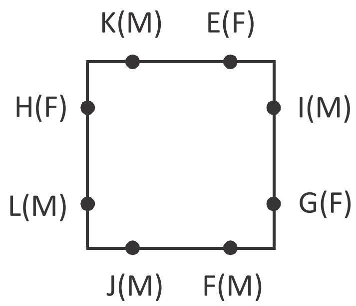
54. **(A)** Cl_2 is a weaker oxidising agent than F_2 and hence cannot displace F_2 from NaF.

55. **(A)** $(C_6H_5COO)_2Ca + Ca(OOCCH_3)_2 \xrightarrow{\text{Heat}} \underset{\text{Acetophenone}}{2C_6H_5COCH_3} + 2CaCO_3$

CRITICAL THINKING

56. **(D)**

57. **(D)**



Here M = Male, F = Female

Both are male.

58. **(B)** The government must have seen the unawareness of the people as a strong factor in the primary education programme being not successful. The step indicated in I must, thus, have been sought for as a remedy for the same.

59. **(A)** A is the mother of B, B is the brother of C and C is the daughter of D. Hence, D is the father.

A (Parents) D
| |
| |
B – is – Brother – of – C

60. **(B)**

THE END