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# NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION (UPDATED)

***CLASS - 11 (PCM)***

***Question Paper Code : UN449***

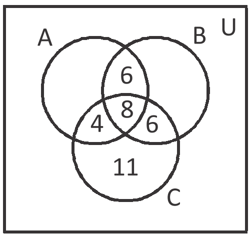
## KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. D | 2. D | 3. B | 4. D | 5. C | 6. C | 7. B | 8. B | 9. A | 10. A |
| 11. C | 12. C | 13. A | 14. C | 15. B | 16. A | 17. D | 18. C | 19. A | 20. A |
| 21. D | 22. C | 23. B | 24. D | 25. A | 26. D | 27. B | 28. A | 29. D | 30. A |
| 31. A | 32. B | 33. C | 34. D | 35. A | 36. A | 37. C | 38. B | 39. D | 40. C |
| 41. C | 42. C | 43. B | 44. A | 45. A | 46. A | 47. C | 48. B | 49. B | 50. B |
| 51. D | 52. C | 53. A | 54. B | 55. C | 56. D | 57. D | 58. D | 59. C | 60. B |

## SOLUTIONS

### MATHEMATICS

1. **(D)**




Hence, number of people who like product *C* only is 11.

2. **(D)** $f(x) = (P - X^n)^{1/n}$

$$\Rightarrow f(f(x)) = \left(p - \left(\left(P - x^n\right)^{1/n}\right)^n\right)^{1/n} = x$$

3. **(B)** Let $f(x) = \dfrac{x}{x^2 - 3x + 2}$

By the definition of rational function, we know that, the function f(x) is not defined, if $x^2 - 3x + 2 = 0$.

Now, let x2 – 3x + 2 = 0

$\therefore$ x2 – 2x – x + 2 = 0

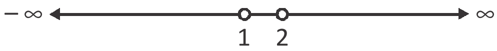
$\Rightarrow$ x(x–2) – 1(x–2) = 0

$\Rightarrow$ (x–1) (x–2) = 0

$\therefore$ x = 1, 2

So, f(x) is not defined for x = 1, 2

Hence, its domain is R – {1, 2}.




4. **(D)** We have,

$$\text{cosec}\theta - \cot\theta = 2018 \qquad \text{.... (i)}$$

$$\therefore \text{cosec}\theta + \cot\theta = \frac{1}{2018} \qquad \text{.... (ii)}$$

$$\left[\begin{array}{l} Q\ \text{cosec}^2\theta - \cot^2\theta = 1 \\ \Rightarrow \text{cosec}\theta - \cot\theta = \dfrac{1}{\text{cosec}\theta + \cot\theta} \end{array}\right]$$

Adding Eqs. (i) and (ii), we get

$$2\ \cos e\theta = 2018 + \frac{1}{2018}$$

$$\Rightarrow \cos e\theta = \frac{1}{2}\left[2018 + \frac{1}{2018}\right] > 0$$

$\theta$ lie in I or II quadrant.

Subtracting Eq. (i) from Eq. (ii), we get

$$2\ \cot\theta = \frac{1}{2018} - 2018$$

$$\cot\theta = \frac{1}{2}\left(\frac{1}{2018} - 2018\right) < 0$$

$\therefore$ $\theta$ lie in II and IV quadrant.

Hence, $\theta$ lies in II quadrant.

5. **(C)** LHS = $\cos\frac{2\pi}{15}.\cos\frac{4\pi}{15}.\cos\frac{8\pi}{15}.\cos\frac{16\pi}{15}$

$$= \cos\frac{2\pi}{15}\cos 2\left(\frac{2\pi}{15}\right)\cos 4\left(\frac{2\pi}{15}\right)\cos 8\left(\frac{2\pi}{15}\right)$$

Put $\frac{2\pi}{15} = \alpha$

$\therefore$ Expression becomes

LHS = $\cos\alpha.\cos 2\alpha.\cos 4\alpha.\cos 8\alpha$

$$= \frac{2\sin\alpha[\cos\alpha.\cos 2\alpha.\cos 4\alpha.\cos 8\alpha]}{2\sin\alpha}$$

[multiplying numerator and denominator by 2 $\sin\alpha$]

$$= \frac{(2\sin\alpha.\cos\alpha).\cos 2\alpha.\cos 4\alpha.\cos 8\alpha}{2\sin\alpha}$$

$$= \frac{2(\sin 2\alpha.\cos 2\alpha.\cos 4\alpha.\cos 8\alpha}{2(2\sin\alpha)}$$

[$\because$ $2\sin\alpha\cos\alpha = \sin 2\alpha$ and multiplying numerator and denominator by 2]

$$= \frac{(2\sin 2\alpha.\cos 2\alpha).\cos 4\alpha.\cos 8\alpha}{4\sin\alpha}$$

$$= \frac{2(\sin 4\alpha.\cos 4\alpha)\cos 8\alpha}{2(4\sin\alpha)}$$

[$\because$ $2\sin\alpha\cos\alpha = \sin 2\alpha$ andmultiplying numerator and denominator by 2]

$$= \frac{2(\sin 8\alpha.\cos 8\alpha)}{2(8\sin\alpha)}$$

$$= \frac{\sin 16\alpha}{8\sin\alpha} = \frac{\sin(15\alpha + \alpha)}{16\sin\alpha}$$

[$\because$ $2\sin\alpha\cos\alpha = \sin 2\alpha$ and multiplying numerator and denominator by 2]

$$= \frac{\sin(2\pi + \alpha)}{16\sin\alpha} = \frac{\sin\alpha}{16\sin\alpha} = \frac{1}{16} = \text{RHS}$$

$\therefore$ LHS = RHS

6. **(C)** Given, $\frac{\cos 13° - \sin 13°}{\cos 13° + \sin 13°} + \frac{1}{\cot 148°}$

$$= \frac{1 - \tan 13°}{1 + \tan 13°} + \tan 148°$$

$$= \frac{\tan 45° - \tan 13°}{1 + \tan 45°\tan 13°} + \tan 148°$$

$$= \tan(45° - 13°) + \tan(180° - 32°)$$

$$= \tan 32° - \tan 32° = 0$$

7. **(B)** We have, $7^n = (1+6)^n$

$= {}^nC_0 + {}^nC_1 6^1 + {}^nC_2\ 6^2 + {}^nC_3 6^3 + .... + {}^nC_n 6^n$

$= 1 + 6n + 6^2\ [{}^nC_2 + {}^nC_3\ 6 + .... + {}^nC_n\ 6^{n-2}]$

$= 1 + 6n + 36\lambda$ [where, ${}^nC_2 + .... + {}^nC_n\ 6^{n-2} = \lambda$ ]

$\Rightarrow$ $7^n - 6n = 36\lambda + 1$

$\Rightarrow$ $7^n - 6n - 50 = 36\lambda - 49$

$\Rightarrow$ $7^n - 6n - 50 = 36\lambda - 72 + 23$

$\Rightarrow$ $7^n - 6n - 50 = 36(\lambda - 2) + 23$

$\Rightarrow$ $7^n - 6n - 50 = 36\mu + 23$

[where $\lambda - 2 = \mu$ ]

$\therefore$ When $7^n - 6n - 50$ is divided by 36, then remainder will be equal to 23.

8. **(B)** We have,

$(n + 16)(n + 17)(n + 18)(n + 19)$

These numbers are the product of four consecutive natural numbers.

$\therefore$ This number divides by $4! = 24$.

9. **(A)** We have,

$$(x - 1)^3 + 64 = 0$$

$\Rightarrow (x - 1)^3 = -64$

$\Rightarrow (x - 1)^3 = (-4)^3$

$\Rightarrow x - 1 = -4, -4w, -4w^2$

$\Rightarrow x = -3, -4w + 1, -4w^2 + 1$

Complex roots of the equations are

$-4w + 1, -4w^2 + 1$

Sum of complex roots are

$-4w + 1 - 4w^2 + 1 = -4(w + w^2) + 2$

$= -4(-1) + 2 = 4 + 2 = 6$

$[\because 1 + w + w^2 = 0]$

10. **(A)** We have,

$$x^2 - 5ix - 6 = 0$$

$\Rightarrow \quad x^2 - 5ix + 6i^2 = 0$

$\Rightarrow \quad x^2 - 3ix - 2ix + 6i^2 = 0$

$\Rightarrow \quad x(x-3i) - 2i(x-3i) = 0$

$\Rightarrow \quad (x-2i)(x-3i) = 0$

Either $x - 2i = 0$ or $x - 3i = 0$

$\Rightarrow \quad x = 2i$ or $x = 3i$

Hence, roots are 3 i and 2i.

11. **(C)** We have, $|x+1| + |x| > 3$

Put $x+1 = 0$ and $x = 0 \Rightarrow x = -1$ and $x = 0$.

$\therefore$ $x = -1, 0$ are critical point. So, we will consider three intervals $(-\infty, -1)$, $[-1, 0)$, $[0, \infty)$.

**Case I** When $-\infty < x < -1$, then $|x+1| = -(x+1)$ and $|x| = -x$

$\therefore \quad |x+1| + |x| > 3$

$\Rightarrow -x - 1 - x > 3$

$\Rightarrow -2x - 1 > 3$

$\Rightarrow -2x > 4$

$\Rightarrow x < -2$

**Case II** When $-1 \leq x < 0$, then $|x+1| = x + 1$ and

$|x| = -x$

$\therefore \quad |x+1| + |x| > 3$

$\Rightarrow x+1 - x > 3 \Rightarrow 1 > 3$ [not possible]

**Case III** When $0 \leq x < \infty$, then $|x+1| = x + 1$ and $|x| = x$

$\therefore |x+1| + |x| > 3$

$\Rightarrow x + 1 + x > 3$

$\Rightarrow 2x + 1 > 3 \Rightarrow 2x > 2$

$\therefore x > 1$

On combining the results of case I, II and III, we get

$x < -2$ and $x > 1$

$\therefore x \in (-\infty, -2) \cup (1, \infty)$

12. **(C)** Total number of ways $= 8^4$

$\therefore$ Required number of ways of 4 letter words that can be formed by the word 'EQUATION' when atleast one letter repeated is

$= 8^4 - {}^8P_4 = 4096 - 1680$

$= 2416$

13. **(A)** Given, $\dfrac{{}^{56}P_{r+6}}{{}^{54}P_{r+3}} = \dfrac{30800}{1}$

$$\therefore \frac{\dfrac{56!}{[56-(r+6)]!}}{\dfrac{54!}{[54-(r+3)]!}} = \frac{30800}{1}$$

$$\left[\because {}^nP_r = \frac{n!}{(n-r)!}\right]$$

$$\Rightarrow \frac{\dfrac{56!}{(50-r)!}}{\dfrac{54!}{(51-r)!}} = \frac{30800}{1}$$

$$\Rightarrow \frac{56 \times 55 \times 54! \times (51-r)(50-r)!}{(50-r)! \times 54!} = \frac{30800}{1}$$

$\Rightarrow 56 \times 55 \times (51 - r) = 30800$

$\Rightarrow \quad (51 - r) = 10$

$\Rightarrow \quad r = 41$

14. **(C)** ${}^{2n}C_3 : {}^{n}C_2 = 12 : 1$

$$\Rightarrow \frac{(2n)!}{3!(2n-3)!} : \frac{n!}{2!(n-2)!} = \frac{12}{1}$$

$$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{2!(n-2)!}{n!} = 12$$

$$\Rightarrow \frac{2n(2n-1)(2n-2)(2n-3)!}{3\times2\times(2n-3)!} \times \frac{2\times1\times(n-2)!}{n(n-1)(n-2)!} = 12$$

$$\Rightarrow \frac{2n(2n-1)(2n-2)}{3} \times \frac{1}{n(n-1)} = 12$$

$$\Rightarrow \frac{2n(2n-1)\,2\,(n-1)}{3} \times \frac{1}{n(n-1)} = 12$$

$$\Rightarrow \frac{4(2n-1)}{3} = 12$$

$$\Rightarrow \quad 2n-1 = 12 \times \frac{3}{4}$$

$$\Rightarrow \quad 2n - 1 = 9 \Rightarrow 2n = 10$$

$$\therefore \quad n = 5$$

15. **(B)** We have, $(1 + x)^{42}$

$T_{2r+1} = {}^{42}C_{2r}X^{2r}$

$T_{r+1} = {}^{42}C_rX^r$

Coefficient of $(2r + 1)^{th}$ and $(r + 1)^{th}$ term are equal

$\therefore\ {}^{42}C_{2r} = {}^{42}C_r$

$\therefore\ 2r + r = 42$ [$\because {}^nC_x = {}^nC_y \Rightarrow x = y = n$]

$\Rightarrow r = 14$

16. **(A)** Here, value of index n is 10 (even), therefore therer is only one middle term given by $T_{\frac{10}{2}+1}$ i.e., $T_6$.

$$\text{Now, } T_6 = T_{5+1} = {}^{10}C_5\left(\frac{2x^2}{3}\right)^{10-5}\left(\frac{3}{2x^2}\right)^5$$

$$= {}^{10}C_5\left(\frac{2x^2}{3}\right)^5\left(\frac{3}{2x^2}\right)^5 = \frac{10.9.8.7.6}{5.4.3.2.1}.\frac{2^5.x^{10}}{3^5}.\frac{3^5}{2^5.x^{10}}$$

$$\left[Q\ {}^{10}C_5 = \frac{10!}{5!(10-5)!} = \frac{10!}{5!5!}\right]$$

= 252

Also, it is independent of x.

17. **(D)** Let Sn = $(3^3 - 2^3) + (5^3 - 4^3) + (7^3 - 6^3)$ + .... to n terms .... (i)

Now, nth term of the sequence

3, 5, 7, ... = 3 + (n–1) 2 = 2n + 1

$\therefore$ nth term of the series

$3^3 + 5^3 + \ldots$ is $(2n + 1)^3$

nth term of the sequence

2, 4, 6, .... is 2 + (n – 1) 2 = 2n

$\therefore$ nth term of the series

$2^3 + 4^3 + \ldots$ is $(2n)^3$ i.e., $8n^3$

Now, nth term of the given series

$T_n = (2n+1)^3 - 8n^3$

$= 8n^3 + 3.4n^2\ 1 + 3.2n.1^2 + 1^3 - 8n^3$

$$\left[Q\ (a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2\right]$$

$= 12n^2 + 6n+1$

On taking summation both sides, we get

$$S_n = \sum T_n = \sum(12n^2 + 6n + 1)$$

$$= 12\sum n^2 + 6\sum n + n$$

$$= 12\,\frac{n(n+1)(2n+1)}{6} + \frac{6n(n+1)}{2} + n$$

= 2n (n+1) (2n+1) + 3n (n+1) +n

= n[2(2n² + 3n+1) + 3n+3+1]

$= n\,(4n^2 + 9n + 6)$

$= 4n^3 + 9n^2 + 6n$

On putting n = 10 in Eq. (1) we get

$S_{10} = 4.10^3 + 9.10^2 + 6.10$

= 4000 + 900 + 60 = 4960

18. **(C)**

$$\text{Given, } \sum_{k=1}^{n_1} k(k+2) = \sum_{k=1}^{n_1}(k^2 + 2k)$$

$$= \sum_{k=1}^{n_1} K^2 + 2\sum_{k=1}^{n_1} k$$

$$= \frac{n(n+1)(2n+1)}{6} + 2\,\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{2n+1}{3} + 2\right]$$

$$= \frac{n(n+1)(2n+7)}{6}$$

19. **(A)** Given lines,

$y - 3kx + 4 = 0$ .... (i)

and $(2k - 1)\, x - 1)\, y - 6 = 0$ .... (ii)

Slope of Eq. (i), $m_1 = \frac{-(-3k)}{1} = 3k$

and slope Eq. (ii) $m_2 = \frac{-(2k-1)}{-(8k-1)} = \frac{2k-1}{8k-1}$

Since, lines are perpendicular.

$\therefore \quad m_1\, m_2 = -1$

$\Rightarrow \quad 3k \times \frac{2k-1}{8k-1} = -1$

$\Rightarrow \quad 3k \times (2k - 1) = -(8k - 1)$

$\Rightarrow \quad 6k^2 - 3k = -8k + 1$

$\Rightarrow \quad 6k^2 - 3k + 8k - 1 = 0$

$\Rightarrow \quad 6k^2 + 5k - 1 = 0$

$\Rightarrow \quad 6k^2 + 6k - k - 1 = 0$

$\Rightarrow \quad 6k\,(k + 1) - 1\,(k + 1) = 0$

$\Rightarrow \quad (6k - 1)\,(k + 1) = 0$

$\therefore \quad k = \frac{1}{6}, -1$

20. **(A)** Given parallel lines are $3x - 8y - 15 = 0$.

and $6x - 8y - 15 = 0$

On putting $y = 0$ in Eq. (1), we get

$3x - 4\,(0) + 9 = 0$

$\Rightarrow 3x = -9 \Rightarrow x = \frac{-9}{3} = -3$

Thus, $(-3, 0)$ is a point on line $3x - 4y + 9 = 0$.

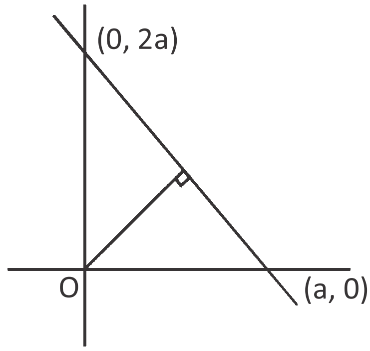
Now, perpendicular distance from $(-3, 0)$ to the $6x - 8y - 15 = 0$ is given by

$$d = \left|\frac{6(-3) - 8(0) - 15}{\sqrt{(6)^2 + (-8)^2}}\right| = \left|\frac{-18-15}{\sqrt{36+64}}\right|$$

$$= \left|\frac{33}{\sqrt{100}}\right|$$

21. **(D)** Let equation of line be

$$\frac{x}{a} + \frac{y}{2a} = 1$$




$\Rightarrow 2x + y = 2a$

Distance from origin is

$$\left|\frac{2a}{\sqrt{(2)^2 + (1)^2}}\right| = \left|\frac{2a}{\sqrt{5}}\right|$$

$$\Rightarrow \left|\frac{2a}{\sqrt{5}}\right| = 1$$

$\Rightarrow 2a = \pm\sqrt{5}$

Hence, equation of line is $2x + y = \pm\sqrt{5}$

22. **(C)** Given, foci are on X-axis. So, the major axis will be along the X-axis. So, the equation of the ellipse is of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Also, we have $a = 4$ and $c = 3$

Q $b^2 = a^2 - c^2$

$\Rightarrow \quad b = \sqrt{a^2 - c^2}$

$= \sqrt{16-9} = \sqrt{7}$

Hence, equation of the ellipse is

$$\frac{x^2}{16} + \frac{y^2}{7} = 1$$

23. **(B)** Equation of plane passes through the points (1, –1, 6), (0, 0,7) and perpendicular to the plane $x - 2y + z = 6$ is

$$\begin{vmatrix} x-1 & y+1 & z-6 \\ 0-1 & 0+1 & 7-6 \\ 1 & -2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y+1 & z-6 \\ -1 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ (x – 1) (1 + 2) – (y + 1) (–1 –1) + (Z – 6) (2 – 1) = 0

$\Rightarrow$ 3x – 3 + 2y + 2 + z – 6 = 0

$\Rightarrow$ 3x + 2y + z – 7 = 0

This plane is passes through (1, 1, 2).

24. **(D)** Given, $\lim_{x\to\infty}\left[\frac{x^2+x+3}{x^2-x+2}\right]^x$

$$= \lim_{x\to\infty}\left[1-1+\frac{x^2+x+3}{x^2-x+2}\right]^x$$

$$= \lim_{x\to\infty}\left[1+\frac{x^2+x+3-x^2+x-2}{x^2-x+2}\right]^x$$

$$= \lim_{x\to\infty}\left[1+\frac{2x+1}{x^2-x+2}\right]^x$$

$$= e^{\lim_{x\to\infty}\frac{2x+1}{x^2-x+2}.(x)}$$

$$\left[\because \lim_{x\to\infty}[1+f(x)]^{g(x)} = e^{\lim_{x\to\infty} f(x).g(x)}\right]$$

$$= e^{\lim_{x\to\infty}\frac{2x^2+x}{x^2-x+2}.(x)} = e^2$$

25. **(A)** We have,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Let x = $a\cos\theta$, y = b $\sin\theta$

$$\therefore \quad \frac{dx}{d\theta} = -a\sin\theta, \frac{dy}{d\theta} = b\cos\theta$$

$$\frac{dy}{dx} = -\frac{b}{a}\cot\theta$$

On differentiating w.r.t. $\theta$, we get

$$\frac{d^2y}{dx^2} = -\frac{b}{a}(-\text{cosec}^2\theta)\frac{d\theta}{dx}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{b\,\text{cosec}^2\theta}{-a^2\sin\theta}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{b}{a^2\sin^3\theta}$$

$$\Rightarrow \quad \frac{d^2y}{dx} = \frac{-b^4}{a^2y^3}$$

$$\left[\because \sin\theta = \frac{y}{b}\right]$$

**PHYSICS**

26. **(D)** P.E. of water at a height h = mgh

= m × 9.8 × 200 = 1960 mJ

Using this P.E. let the temperature of water rise by DT

DQ = mc DT

1960 m = m × 4200 × DT

$$\Delta T = \frac{1960}{4200} = 0.4667\text{ K}$$

27. **(B)** There will be no over flowing of liquid in a tube of insufficient height but there will be adjustment of the radius of curvature of meniscus so that hR = *a* finite constant.

28. **(A)** Escape velocity from the surface of Mars.

$$v = \sqrt{\frac{2GM_m}{R_m}}$$

Mass of Mars = $M_m$ = 6.42 × $10^{23}$ kg

Radius of Mars = $R_m$ = 3.375 × $10^6$ m

$$v = \sqrt{\frac{2GM_m}{R_m}} = \sqrt{\frac{2\times6.67\times10^{-11}\times6.42\times10^{23}}{3.375\times10^6}}$$

= 5.037 × $10^3$ m/s

29. **(D)** According to the definition of centre of mass, we can imagine one particle of mass (1 + 2 + 3) kg at (1, 2, 3); another particle of mass (2 + 3) kg at (–1, 3, –2).

Let the third particle of mass 5 kg be put at ($x_3$, $y_3$, $z_3$) i.e.,

$m_1$ = 6 kg, ($x_1$, $y_1$, $z_1$) = (1, 2, 3)

$m_2$ = 5 kg, ($x_2$, $y_2$, $z_2$) = (–1, 3, –2)

$m_3$ = 5kg, ($x_3$, $y_3$, $z_3$) = ?

Given ($x_{cm}$, $y_{cm}$, $z_{cm}$) = (1, 2, 3)

Using = $x_{cm} = \frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3}$

$$1=\frac{6\times1+5\times(-1)+5x_3}{6+5+5}$$

$5x_3 = 16 - 1 = 15$, $x_3 = 3$

Similarly, $y_3 = 1$ and $z_3 = 8$

30. **(A)** $R=\frac{u^2 \sin 2\theta}{g}$. Here R = 0.5. Therefore, $u^2/g$=1 km. When q = 45°, we have R = $u^2/g$ = 1 km.

31. **(A)** As area = length$^2$,

$$\therefore \frac{\Delta A}{A}\times100=2\frac{\Delta l}{l}\times100=2\times2\%=4\%$$

32. **(B)** m = 20 kg, angular speed = $\omega$= 100 $s^{-1}$, r = 0.25 m

M.I. of the cylinder about its own axis $= I = \frac{1}{2}mr^2$ Rotational K.E.

$$=\frac{1}{2}I\omega^2=\frac{1}{2}\times\frac{1}{2}mr^2\times\omega^2$$

$$=\frac{1}{4}\times20\times(0.25)^2\times(100)^2=3125\text{ J}$$

Angular momentum L = I $\omega$

$$=\frac{1}{2}mr^2\times\omega^2=\frac{1}{2}\times20\times(0.25)^2\times100=62.5\text{ J s}$$

33. **(C)** [Planck's constant] = [Moment of momentum]

= [Momentum × distance] = $ML^2T^{-1}$.

34. **(D)** s = 4t + $\frac{1}{2}$(1) $t^2$ = 2t + $\frac{1}{2}$ (2) $t^2$

or 4t + 0.5$t^2$ = 2t + $t^2$

or $\frac{t^2}{2}=2t$

or t = 0 and t = 4 s

$\therefore$ s = (4) (4) + $\frac{1}{2}$ (1) $(4)^2$ = 16 + 8 = 24 m

35. **(A)** Impulse = change in momentum

= m n cos 60° – (– m n cos 60°)

= 2 m n cos 60° = 2 m n (1 / 2) = m n.

36. **(A)** If there were no friction, velocity at the bottom of quarter circle = $\sqrt{2gR}$

$$\text{K.E.}=\frac{1}{2}mv^2=\frac{1}{2}m\times2gR=mgR$$

= 2 × 10 × 1 = 20 J

$$\text{Actual K.E.}=\frac{1}{2}\times2\times4^2=16\text{ J.}$$

$\therefore$ Work done by friction = 16 – 20 = – 4 J

37. **(C)** Bulk modulus $=K=\frac{F/A}{\Delta V/V}=\frac{PV}{\Delta V}$

Change in pressure = P = $10^6$ Pa

Original volume = V = $l^3$ = 1 $m^3$

Change in volume = DV = 1.5 × $10^{-5}$ $m^3$

$$=K=\frac{10^6\times1}{1.5\times10^{-5}}=6.67\times10^{10}\text{ N/m}^2.$$

38. **(B)** As all the four spheres have same radius, their volumes are equal. As mass = volume × density, the ratio of their masses = ratio of their densities.

$\therefore$ $m_1 : m_2 : m_3 : m_4 = 2 : 3 : 5 : 1$

Also, $c_1 : c_2 : c_3 : c_4 = 3 : 6 : 2 : 4$

As heat capacity = mass × sp. Heat heat capacity = 6 : 18 : 10 : 4

As the sphere with largest heat capacity has the fastest rate of cooling, therefore, sphere Q will exhibit the fastest cooling rate.

39. **(D)** Apparent weight = m (g + a)

= 80 (10 + 5) = 1200 N.

40. **(C)** As energy = work = force × distance, therefore, unit of energy becomes 3 × 3 times. Hence, energy in new units $=\frac{81}{9}=9$.

## CHEMISTRY

41. **(C)** In graphite, each carbon atom is covalently bonded with only three other carbon atoms out of four. The fourth valence electron of each carbon atom is free to move that enables it to

conduct electricity.

42. **(C)** $Ca_3P_2 + 6\ H_2O \rightarrow 3\ Ca(OH)_2 + 2\ PH_3$

43. **(B)** The equation is

$BaO_2 + H_2SO_4 \rightarrow BaSO_4(s) + H_2O_2$

The most electronegative atom in the products is oxygen. The oxidation state of O in $H_2O_2$ is –1, whereas that in $BaSO_4$ is –2.

44. **(A)** As forward reaction is accompanied with decrease of randomness, it must be exothermic. Further $n_p \neq n_r$. Hence, equilibrium will be affected by temperature and pressure.

45. **(A)** Total volume after mixing = 3 L,

$300 \times 1 = 3 \times p(H_2)$, i.e., $pH_2 = 100$ mm

$600 \times 2 = 3 \times p(O_2)$, $p(O_2) = 400$ mm,

P (total) = 100 + 400 = 500 mm

Alternatively, $P_1V_1 + P_2V_2 = P_3(V_1 + V_2)$

$300 \times 1 + 600 \times 2 = P_3(1 + 2)$

or $3\ P_3 = 1500$ or $P_3 = 500$ mm.

46. **(A)** $NO_3^-$ and $CO_3^{2-}$ both have same number of electrons (equal to 32) and central atom in each being $sp^2$ – hybridized are isostructural too.

47. **(C)** 100 g of crystalline salt contains $H_2O$ = 55.9 g

∴ Anhydrous salt = 100 – 55.9 = 44.1 g

Molecular mass of anhydrous $Na_2SO_4$

$= 2 \times 23 + 32 + 4 \times 16 = 142$

44.1 g of anhydrous salt combines with $H_2O$ = 55.9 g

∴ 142 g of anhydrous salt combine

with $H_2O = \frac{55.9}{44.1} \times 142\ g = 180\ g$

180 amu of $H_2O = \frac{180}{18} = 10$ molecules.

48. **(B)** Calcined gypsum is $CaSO_4$

Marble and sea shells contain $CaCO_3$

Dolomite is $CaCO_3 . MgCO_3$.

49. **(B)** The reaction is

$4NH_3\ (g) + 5O_2\ (g) \rightarrow 4NO(g) + 6H_2O\ (g)$

4 × 17 g   5 × 32 g   4 × (14 + 16 ) g

= 68 g = 160 g = 120 g

10.0 g   20.0 g   ?

As per the given data, it shows that in the given reaction, oxygen is the limiting reagent.

Therefore,

Maximum mass of nitric oxide formed

$= \frac{120\ g \times 20\ g}{160\ g} = 15\ g$

50. **(B)** Boron has three electrons in its valence shell ($2s^2\ 2p^1$), and three monovalent atoms for bonding. So, it shows $sp^2$ hybridization and trigonal planar geometry.

51. **(D)** H, E and V are all extensive.

52. **(C)** $Cu^+ = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}$.

Shells occupied = 3, sub-shells occupied = 6, filled orbitals = 14. Unpaired $e^- = 0$

53. **(A)** Pressure on mercury in the open arm = 743 mm = 74.3 cm. The mercury level in the arm connected to the bulb is lower than that in the open arm. So, pressure of the gas in the bulb is higher than the barometric pressure.

Therefore,

$P_{gas} = P_{barometric} + h$

= 74.3 cm + (43.7 cm – 15.6 cm)

$P_{gas}$ = 102.4 cm

$= \frac{102.4\ cm}{76\ cm} = 1.347$ bar

54. **(B)** A salt of strong acid and weak base on hydrolysis produces a strong acidic solution as they produce more $H^+$ ions and less $OH^-$ ions when dissolved in water. Similarly, when a salt of strong base and weak acid on hydrolysis

produces a strong basic solution as they produce more $OH^-$ ions less $H^+$ ions when dissolved water.

If the salt is $NH_4Cl$, it hydrolyses as $NH_4Cl + H_2O \rightleftharpoons NH_4OH + HCl$.

or $NH_4^+ + Cl^- + H_2O$ $NH_4OH + H^+ + Cl^-$ or $+ H_2O$ $NH_4OH + H^+$.

As it produces $H^+$ ions, the solution of such a salt is acidic.

55. **(C)** Be > Mg > Ca > Sr > Ba. The size of the $SO_4^{2-}$ ion is very big (approx. 3Å). Therefore, as the size of cation increases, their hydration energies decrease more rapidly than their lattice energies. Consequently the solubilities of sulphates decrease down the group.

**<u>CRITICAL THINKING</u>**

56. **(D)** Minute hand covers 480 / 60 = 80

57. **(D)** In fig. (X), one of the dots lies in the region common to the circle and the square only, another dot lies in the region common to the square, the triangle and the rectangle only and the third dot lies in the region common to the triangle and the rectangle only. In each of the figures (1), (2) and (3) there is no region common to the square, the triangle and the rectangle only. Only fig. (4) consists of all the three types of regions.

58. **(D)** The prices of petrol and diesel being stagnant in the domestic market and the increase in the same in the international market must be backed by independent causes.

59. **(C)**

60. **(B)** Seeing four girls surrounding another girl, while in possession of her backpack, is the most suspicious of the incidents described.

**THE END**