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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION (UPDATED)

CLASS - 11 (PCM)

Question Paper Code : UN446

KEY

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11. A	12. C	13. B	14. B	15. C	16. D	17. C	18. D	19. C	20. C
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SOLUTIONS

MATHEMATICS

1. **(A)** F_1 = The set of parallelograms

F_2 = The set of rectangles

F_3 = The set of rhombus

F_4 = The set of squares

By definition of parallelogram, opposite sides are equal and parallel

2. **(C)** Given,

$f(x) = |x - 1| + |x - 2| + |x - 3|$

Given $2 < x < 3$, then $f(x) = x$

$- 1 + x - 2 + 3 - x = x$

So, $f(x) = x$ is one-one and onto function.

$\therefore$ $f(x)$ is a bijective

3. **(A)** Given $f(x) = \sqrt{4-x} + \dfrac{1}{\sqrt{x^2-1}}$

For domain of $f(x)$,

$4 - x \geq 0$ and $x^2 - 1 > 0$

$\Rightarrow x \leq 4$ and $x^2 > 1$

$\Rightarrow x \leq 4$ and $x \in (-\infty, -1) \cup (1, \infty)$

$\therefore x \in (-\infty, -1) \cup (1, 4]$

4. **(B)** Given, $\dfrac{1}{6}$ $\sin\theta, \cos\theta, \tan\theta$ are in GP

$\therefore \cos^2\theta = \dfrac{1}{6} \sin\theta \times \tan\theta$

$\Rightarrow \cos^2\theta = \dfrac{\sin^2\theta}{6\cos\theta}$

$\Rightarrow 6\cos^3\theta - \sin^2\theta = 0$

$\Rightarrow 6\cos^3\theta - 1 + \cos^2\theta = 0$

$\Rightarrow 6\cos^3\theta + \cos^2\theta - 1 = 0$

$\Rightarrow (2\cos\theta - 1)(3\cos^2\theta + 2\cos\theta + 1) = 0$

For $3\cos^2\theta + 2\cos\theta + 1 = 0$ value of $\cos\theta$ is imaginary. Hence, consider only

$\Rightarrow 2\cos\theta - 1 = 0 \Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \cos\theta = \cos\frac{\pi}{3}$

$\therefore \theta = 2n\pi \pm \frac{\pi}{3}$

5. **(B)** tan 15° = tan (60° – 45°)

$= \frac{\tan 60° - \tan 45°}{1 + \tan 60°.\tan 45°}$

$\left[\because \tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x.\tan y}\right]$

$\Rightarrow \tan 15° = \frac{\sqrt{3}-1}{1+\sqrt{3}}$

and $\cot 15° = \frac{1}{\tan 15°} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$\therefore$ tan 15° + cot 15°

$= \frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$= \frac{\left(\sqrt{3}-1\right)^2 + \left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}$

$= \frac{3+1-2\sqrt{3}+3+1+2\sqrt{3}}{\left(\sqrt{3}\right)^2 - (1)^2}$

$= \frac{8}{3-1} = \frac{8}{2} = 4$

6. **(D)** Given, tan20° = λ

$\therefore \frac{\tan 160° - \tan 110°}{1 + (\tan 160°)(\tan 110°)}$

$= \frac{\tan(180° - 20°) - \tan(90° + 20°)}{1 + (\tan(180° - 20°)(\tan(90° + 20°))}$

$= \frac{-\tan 20° + \cot 20°}{1 + \tan 20° \cot 20°}$

[$\because \tan(180° - \theta) = -\tan\theta$; $\tan(90° + \theta) = -\cot\theta$]

$= \frac{-\lambda + 1/\lambda}{1+1}$

$= \frac{-\lambda^2 + 1}{2\lambda} = \frac{1-\lambda^2}{2\lambda}$

7. **(C)** Let $S_n = \frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + ...$ upto n terms

$= \frac{1}{3}\left[\frac{3}{2.5} + \frac{3}{5.8} + \frac{3}{8.11} + ... \text{ upto } n \text{ terms}\right]$

$= \frac{1}{3}\left[\frac{5-2}{2.5} + \frac{8-5}{5.8} + \frac{11-8}{8.11} + ... \text{ upto } n \text{ terms}\right]$

$= \frac{1}{3}\left[\left(\frac{1}{2} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{8}\right) + \left(\frac{1}{8} - \frac{1}{11}\right) + \left(\frac{1}{3n-1} - \frac{1}{3n+2}\right)\right]$

[$\because$ nth term of 2, 5, 8, ... = 2 + (n–1) 3 = 2 + 3n – 3 = 3n – 1]

$= \frac{1}{3}\left[\frac{1}{2} - \frac{1}{3n+2}\right] = \frac{1}{3}\left[\frac{3n+2-2}{2(3n+2)}\right]$

$= \frac{n}{2(3n+2}$

8. **(A)** We have,

$\frac{(1+i)^{2016}}{(1-i)^{2014}} = \frac{[(1+i)^2]^{1008}}{[(1-i)^2]^{1007}}$

$= \frac{(1+2i+i^2)^{1008}}{(1-2i+i^2)^{1007}}$

$= \frac{(2i)^{1008}}{(-1)(2i)^{1007}}$

$= -(2i)^{1008-1007} = -2i$

9. **(B)** We have, $a + ib = \frac{x+i}{x-i}$

$= \frac{x+i}{x-i} \times \frac{x+i}{x+i}$

[multiplying numerator and denominator by $x + i$]

$= \frac{x^2 + 2xi + i^2}{x^2 + i^2}$ (1)

$= \frac{x^2 - 1 + 2xi}{x^2 + 1}$ [$i^2 = -1$]

$\Rightarrow a + ib = \frac{x^2 - 1}{x^2 + 1} + \frac{2x}{x^2 + 1} i$

On comparing real and imaginary parts both sides, we get

$$a = \frac{x^2 - 1}{x^2 + 1} \text{ and } b = \frac{2x}{x^2 + 1} \qquad \dots \text{(i)}$$

Now, $a^2 + b^2$

$$= \left(\frac{x^2 - 1}{x^2 + 1}\right)^2 + \left(\frac{2x}{x^2 + 1}\right)^2 \quad \text{[from eq. (1)]}$$

$$= \frac{(x^2 - 1)^2 + 4x^2}{(x^2 + 1)^2} = \frac{x^4 + 1 - 2x^2 + 4x^2}{(x^2 + 1)^2}$$

$$= \frac{x^4 + 1 + 2x^2}{(x^2 + 1)^2} = \frac{(x^2 + 1)^2}{(x^2 + 1)^2} = 1$$

10. **(D)** We have, $\frac{4x}{3} - \frac{9}{4} < x + \frac{3}{4}$ ... (i)

and $\frac{7x - 1}{3} - \frac{7x + 2}{6} > x$ ...(ii)

From inequality (i), we get

$$\frac{4x}{3} - \frac{9}{4} < x + \frac{3}{4} \Rightarrow \frac{16x - 27}{12} < \frac{4x + 3}{4}$$

$\Rightarrow 16x - 27 < 12x + 9$

[multiplying both sides by 12]

$\Rightarrow 16x - 27 + 27 < 12x + 9 + 27$

[adding 27 on both sides]

$\Rightarrow 16x < 12x + 36$

$\Rightarrow 16x - 12x < 12x + 36 - 12x$

[subtracting $12x$ from both sides]

$\Rightarrow 4x < 36 \Rightarrow x < 9$

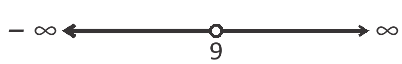
[dividing both sides by 4]

Thus, any value of x less than 9 satisfies the inequality

So, the solution of inequality (i) is given by

$x \in (-\infty, 9)$

$x < 9$



From inequality (ii), we get

$$\frac{7x - 1}{3} - \frac{7x + 2}{6} > x \Rightarrow \frac{14x - 2 - 7x - 2}{6} > x$$

$\Rightarrow 7x - 4 > 6x$

[multiplying by 6 on both sides]

$\Rightarrow 7x - 4 + 4 > 6x + 4$

[adding 4 on both sides]

$\Rightarrow 7x > 6x + 4$

$\Rightarrow 7x - 6x > 6x + 4 - 6x$

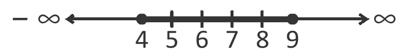
[subtracting $6x$ from both sides]

$\therefore x > 4$

Thus, any value of x greater than 4 satisfies the inequality. So the solution set is $x \in (4, \infty)$ ...(iv)

$x > 4$

The solution set of inequalities (i) and (ii) are represented graphically on number line as given below



Clearly, the common value of x satisfying inequalities (iii) and (iv) lie between 4 and 9

Hence, the solution of the given system is $4 < x < 9$ i.e., $x \in (4, 9)$

11. **(A)** We have 10 intermediate stations between A and B. There are 7 stations where the train does not stop and the three stations where the train stops should be any three of the 8 places

$\therefore$ Total number of ways = ${}^8C_3$

$$\frac{8 \times 7 \times 6}{1 \times 2 \times 3} = 56$$

12. **(C)** We have, $P_m = {}^mP_m = m!$

$\therefore 1 + 1.P_1 + 2.P_2 + 3.P_3 + \dots + n.P_n$

$= 1 + 1 + 2.2! + 3.3! + 4.4! + \dots + n.n!$

$$= 1 + \sum_{r=1}^{n} r.r! = 1 + \sum_{r=1}^{n} [(r+1) - 1]r!$$

$$= 1 + \sum_{r=1}^{n} [(r+1)r! - r!]$$

$$= 1 + \sum_{r=1}^{n} [(r+1)! - r!]$$

$= 1 + [(2! - 1!) + (3! - 2!) + (4! - 3!) + \dots + (n+1)! - n!]$

$= 1 + [(n+1)! - 1!] = (n+1)!$

Hence proved

13. **(B)** We have 6 distinct white roses and 5 distinct red roses

Total number of way making a garland such that no two red roses come together is

$\frac{1}{2} \times (6-1)!\,6!$

$\frac{6! \times 5!}{2} = \frac{720 \times 120}{2} = 43200$

14. **(B)** We have,

$x = \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + \frac{1.3.5.7}{3.6.9.12} + ...$

$\Rightarrow x = \frac{1.3}{3^2.(2!)} + \frac{1.3.5}{3^3\,(3!)} + \frac{1.3.5.7}{3^4\,(4!)}$

$\Rightarrow x = \frac{\frac{1}{2}\left(\frac{1}{1}+1\right)}{2!}\left(\frac{2}{3}\right)^2$

$+ \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right)\left(\frac{1}{2}+2\right)}{3!}\left(\frac{2}{3}\right)^2 + ..$

$\Rightarrow x = \left[1 + \frac{1}{2}\left(\frac{2}{3}\right) + \frac{\frac{1}{2}\left(\frac{1}{2}+1\right)}{2!}\left(\frac{2}{3}\right)^2\right.$

$\left. + \frac{\frac{1}{2}\left(\frac{1}{2}+1\right)\left(\frac{1}{2}+1\right)}{3!}\left(\frac{2}{3}\right)^3 ..\right]$

$\Rightarrow x = \left(1-\frac{2}{3}\right)^{-1/2} - \frac{4}{3} \Rightarrow x = \left(\frac{1}{3}\right)^{-1/2} - \frac{4}{3}$

$\Rightarrow x = \sqrt{3} - \frac{4}{3} \Rightarrow 3x + 4 = 3\sqrt{3}$

$\Rightarrow (3x+4)2 = \left(3\sqrt{3}\right)^2$

$\Rightarrow 9x^2 + 24x + 16 = 27$

$\Rightarrow 9x^2 + 24x = 11$

15. **(C)** We have, the binomial of $\left(\frac{x^3}{2} - \frac{2}{x^2}\right)^6$

Clearly, the expansion contains 7 terms, therefore 5th term from the end is (6 – 5 + 2) = 3rd term from the beginning

$\therefore T_3 = T_{2+1} = {}^6C_2 \left(\frac{x^3}{2}\right)^4 \left(\frac{(-2)^2}{x^2}\right)$

$= \frac{6!}{2!4!}.\frac{x^{12}}{2^4}.\frac{(2)^2}{x^4} = \frac{6\times5\times4!}{2\times1\times4!}.\frac{x^8}{2^2} = \frac{15}{4}.x^8$

16. **(D)** We have x_1, x_2, x_3 and y_1, y_2, y_3 are in GP with the same common ratio

Let r be the common ratio

$\therefore \quad x_1 = x,\ x_2 = xr$ and $x_3 = xr_2$

Similarly, $\quad y_1 = y$

$y_2 = y$r and $y_3 = yr^2$

$\therefore \quad$ Area of $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} x & y & 1 \\ xr & yr & 1 \\ xr^2 & yr^2 & 1 \end{vmatrix}$

$= \frac{1}{2}xy\begin{vmatrix} 1 & 1 & 1 \\ r & r & 1 \\ r^2 & r^2 & 1 \end{vmatrix} = \frac{1}{2} \times 0 = 0$

[$\because C_1, C_2$ are identical]

$\therefore$ The given points are collinear

17. **(C)** In the expansion of $(1 + x)^n$, we have

$T_5 = T_{4+1} = {}^nC_4x^4,$

$T_6 = T_{5+1} = {}^nC_5x^5$

and $T_7 = T_{6+1} = {}^nC_6x^6$

Given, coefficients of T_5, T_6 and T_7 and in AP.

$\therefore {}^nC_4, {}^nC_5$ and ${}^nC_6$ are in AP

$\Rightarrow \frac{n!}{4!(n-4)!}, \frac{n!}{5!(n-5)!}, \frac{n!}{6!(n-6)!}$

are in AP

$\Rightarrow \frac{n(n-1)(n-2)(n-3)}{4.3.2.1}, \frac{n(n-1)(n-2)(n-3)(n-4)}{5.4.3.2.1}$

$\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{6.5.4.3.2.1}$ are in AP

On multiplying each term by

$\frac{4.3.2.1}{n(n-1)(n-2)(n-3)}$, we get

$1, \frac{n-4}{5}, \frac{n^2-9n+20}{30}$ are in AP

As we know that if a, b and c are in AP then b – a = c – b

So, here we get

$$\frac{n-4}{5} - 1 = \frac{n^2-9n+20}{30} - \frac{n-4}{5}$$

$$\Rightarrow \frac{n-4-5}{5} = \frac{n^2-9n+20-6n+24}{30}$$

$$\Rightarrow \frac{n-9}{5} = \frac{n^2-15n+44}{30}$$

$$\Rightarrow (n-9) = \frac{n^2-15n+44}{6}$$

$\Rightarrow 6(n–9) = n^2 - 15n + 44$

$\Rightarrow 6n - 54 = n^2 - 15n + 44$

$\Rightarrow x^2 - 21n + 98 = 0$

$\Rightarrow (n - 7)(n - 14) = 0$

$\therefore$ n = 7 or 14

18. **(D)** Since, $x + 3y - 9 = 0$,

$4x + by - 2 = 0$ and $2x - y - 4 = 0$ are concurrent

$$\therefore \begin{vmatrix} 1 & 3 & -9 \\ 4 & b & -2 \\ 2 & -1 & -4 \end{vmatrix} = 0$$

$\Rightarrow 1(-4b-2)-3(-16+4)-9(-4-2b) = 0$

$\Rightarrow -4b - 2 + 36 + 36 + 18b = 0$

$\Rightarrow 14b = -70 \Rightarrow b = -5$

On solving equations $x + 3y - 9 = 0$ and $2x - y - 4 = 0$,

we het $x = 3$ and y =2

$\therefore$ Concurrency point of line is (3, 2)

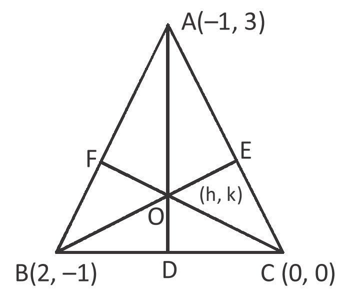
Equation of line passing through (–5, 0) and (3, 2) is

$$y - 0 = \frac{2-0}{3+5}(x+5)$$

$\Rightarrow y = \frac{1}{4}(x + 5) \Rightarrow 4y = x + 5$

$\therefore x - 4y + 5 = 0$

19. **(C)** Let the vertices of ΔABC be A (–1, 3), B(2, –1) and C (0, 0). The orthocentre is the point of intersection of the altitudes from the vertices to the opposite sides



Let AD, BE and CF be the altitudes and O (h, k) be the orthocentre of ΔABC

$\because$ AO $\perp$ BC

$\therefore$ Slope of line AO × Slope of line BC = – 1

$$\Rightarrow \frac{k-3}{h+1} \times \left(-\frac{1}{2}\right) = -1 \; [\because m_1 \times m_2 = -1]$$

$$\Rightarrow \frac{k-3}{h+1} = 2$$

$\Rightarrow 2h - k + 5 = 0$ ... (i)

Also, BO $\perp$ AC

$\therefore$ Slope of line BO × Slope of line AC = – 1

$$\frac{k+1}{h-2} = \times \frac{3-0}{-1-0} = -1 \; [\because m_1 m_2 = -1]$$

$$\Rightarrow \frac{k+1}{h-2} = \times (-3) = -1$$

$\Rightarrow h - 2 = 3k + 3$

$\Rightarrow h - 3h - 5 = 0$ ... (ii)

On solving Eqs. (i) and (ii), we get

$$\frac{h}{5+15} = \frac{k}{5+10} = \frac{1}{-6+1}$$

$\Rightarrow \frac{h}{20} = \frac{k}{15} = \frac{1}{-5}$

$\therefore$ $h = -4$, $k = -3$

Hence, orthocentre is (–4, –3)

20. **(C)** Slopes of lines 4X – y + 7 = 0 and kx – 5y – 9 = 0 are 4 and $\frac{k}{5}$, respectively.

Let $m_1 = 4$ and $m_2 = \frac{k}{5}$

Then, angle between two lines whose slopes are m_1 and m_2, is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\therefore \quad \tan 45° = \left|\frac{4 - \frac{k}{5}}{1 + 4.\frac{k}{5}}\right|$$

$$\Rightarrow 1 = \left|\frac{20 - k}{5 + 4k}\right|$$

$$\Rightarrow \frac{20 - k}{5 + 4k} = \pm 1$$

$$\Rightarrow \frac{20 - k}{5 + 4k} = 1 \text{ or } \frac{20 - k}{5 + 4k} = -1$$

$\Rightarrow$ 20 – k = 5 + 4k or 20 – k = – 5 – 4k

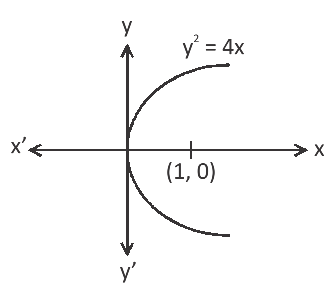
$$\Rightarrow k = 3 \text{ or } -\frac{25}{3}$$

But $k > 0$

$\therefore$ $k = 3$

21. **(B)** Given curve is $y^2 = 4x$.

Also, point (1, 0) is the focus of the parabola. It is clear from the graph that only one normal is possible



22. **(B)** Equation of line BC is

$$\frac{x}{2} = \frac{y + 11}{8} = \frac{z - 4}{-3} = \lambda \text{ (say)}$$

Thus, coordinates of any point on te line BC are $(2\lambda, 8\lambda - 11, -3\lambda + 4)$

$\therefore$ Coordinates of D are of form

$(2\lambda, 8\lambda - 11, -3\lambda + 4)$

Now, DR's of BC are 2, 8, – 3 and DR's of AD are $2\lambda, -1, 8\lambda - 19, -3\lambda$

$\because$ AD $\perp$ BC

$\therefore$ $2(2\lambda - 1) + 8(8\lambda - 19) + (-3)(-3\lambda) = 0$

$\Rightarrow$ $4\lambda - 2 + 64\lambda - 152 + 9\lambda = 0$

$\Rightarrow$ $77\lambda = 154$

$\Rightarrow$ $\lambda = 2$

Hence, the coordinates of D are (4, 5, –2)

23. **(D)** Consider,

$$\lim_{x\to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$$

$$= \lim_{x\to 0} \frac{2\sin^2 x\,(3 + \cos x)}{x \tan 4x}$$

$$= 2.\lim_{x\to 0} \frac{\sin^2 x}{x^2}.\lim_{x\to 0} \frac{x}{\tan 4x}.\lim_{x\to 0}(3 + \cos x)$$

$$= 2.1.\frac{1}{4}\lim_{x\to 0} \frac{4x}{\tan 4x}.(3 + 1)$$

$$= 2.1.\frac{1}{4}.1.4 = 2$$

24. **(A)** To prove, $x\frac{dy}{dx} = y(1 - y)$

We have, $y = \frac{x}{x + 5}$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x}{x + 5}\right)$$

$$= \frac{(x + 5)\frac{d}{dx}x - x\frac{d}{dx}(x + 5)}{(x + 5)^2}$$

$$\left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}\right]$$

$$= \frac{(x+5)(1) - x(1+0)}{(x+5)^2} = \frac{x+5-x}{(x+5)^2}$$

$$\frac{dy}{dx} = \frac{5}{(x+5)^2}$$

Now, LHS = $x\frac{dy}{dx} = \frac{5x}{(x+5)^2}$

RHS = $y(1-y) = \frac{x}{x+5}$

$$\left[1 - \frac{x}{x+5}\right] = \frac{x}{x+5}\left[\frac{x+5-x}{x+5}\right]$$

RHS = $\frac{5x}{(x+5)^2}$

From Eqs. (i) and (ii), we get

$x\frac{dy}{dx}$ $y(1-y)$ Hence proved

25. **(D)** We have,

$g(x) = (f(2f(x) + 2)^2$

$g'(x) = 2\,(f(2f(x)+2)).\, f'(2f(x)+2)f'(x).2$

$\Rightarrow g'(0) = 2\,(f(2f(0) + 2).\, f'(2f(0) + 2).2f'(0)$

$\Rightarrow g'(0) = 2(f(2(-1)+2).f'(2(-1)+2.2(1)$

$[\because f(0) = -1,\ f'(0) = 1]$

$\Rightarrow g'(0) = 2[f(-2+2).f'(-2+2)].2$

$\Rightarrow g'(0) = 4f(0).f'(0)$

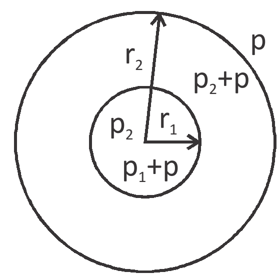
$\Rightarrow g'(0) = 4 \times (-1)\,(1)$

$= -4$

PHYSICS

26. **(C)** $\frac{dU}{dW} = \frac{C_v dT}{RdT} = \frac{C_v}{R} = \frac{3R/2}{R} = \frac{3}{2}$

27. **(C)** p = atmospheric pressure, p_1 = excess pressure inside smaller bubble of radius r_1 = 1 cm



p_2 = excess pressure inside bigger bubble of radius r_2 = 3 cm

Now, $p_2 = \frac{4\sigma}{r_2}; 111p_1 - p_2 = \frac{4\sigma}{r_1}$

$$\Rightarrow p_1 = p_2 + \frac{4\sigma}{r_1} = \frac{4\sigma}{r_2} + \frac{4\sigma}{r_1}$$

Also $p_1 = \frac{4\sigma}{r}$

Hence $\frac{4\sigma}{r} = \frac{4\sigma}{r_2} + \frac{4\sigma}{r_1}$

$$\Rightarrow \frac{1}{r} = \frac{1}{r_2} + \frac{1}{r_1}$$

$$\Rightarrow r = \frac{r_1 r_2}{r_1 + r_2} = \frac{1\times 3}{1+3} = \frac{3}{4}\text{ cm}$$

28. **(A)** Let R be the original radius of a planet. Then attraction on a body of mass m placed on its surface will be $F = \frac{GMm}{R^2}$

If size of the planet is made double i.e., R' = 2 R, then mass of the planet becomes

$$M' = \frac{4}{3}\pi(2R)^3\rho = 8\times\frac{4}{3}\pi R^3\rho = 8M$$

New force

$$F' = \frac{GM'm}{R'^2} = \frac{G\ 8\ M \times m}{(2R)^2} = 2F$$

i.e., force of attraction increases due to the increase in mass of the planet.

29. **(B)** The mass of the complete disc will be m = 2M and its moment of inertia about the x – axis will be $\frac{mR^2}{4}$

Therefore, the moment of inertia of the half disc about the x – axis will be

$$\frac{1}{2}\left(\frac{mR^2}{4}\right) = \frac{1}{2}\left(\frac{2MR^2}{4}\right) = \frac{MR^2}{4}$$

30. **(A)** Relative velocity = 5 + 7 = 12 m/s

Time required to meet

$$= \frac{\text{distance}}{\text{relative vel.}} = \frac{120}{12} = 10\text{ s}$$

Distance from 1st point = vel. of first × t = 5 × 10 = 50 m

31. **(C)** 1. Energy density $= \frac{\text{energy}}{\text{volume}}$

$$= \frac{ML^2T^{-2}}{L^3} = [M^1\ L^{-1}\ T^{-2}]$$

4. Young's modulus $= \frac{\text{stress}}{\text{strain}}$

$$= \frac{MLT^{-2}/L^2}{1} = [M^1\ L^{-1}\ T^{-2}]$$

32. **(A)** Mass of flywheel = m = 100 kg

Radius = r = 1 m

$I = mr^2 = 100 \times I^2 = 100$ kg m^2

Initial angular velocity

$$= \omega_0 = 2\pi\ n = 2 \times 3\cdot14 \times \frac{420}{60}$$

$$= 43\cdot96\text{ rad/s}^2$$

Final angular velocity $= \omega = 0$

Angular displacement in 14 revolutions = 14 × 2 π = 28 π radian

$$\alpha = \frac{\omega^2 = \omega_0^2}{2\theta} = \frac{0 - 43\cdot96^2}{2 \times 28\ \pi}$$

$$= -10\cdot99\text{ rad/s}^2$$

Torque required to stop the flywheel

$$= \tau = I\alpha = 100 \times 10\cdot99 = 1099\text{ Nm}$$

33. **(D)** [pc] = (MLT^{-1}) (LT^{-1}) = ML^2T^{-2} = [torque].

34. **(B)** Instantaneous velocity = υ = 2 bt

t = 4s, υ_1 = 2 × 1 × 4 = 8.00 m/s

t = 4.01 s, υ_2 = 2 × 1 × 4.01 = 8.02 m/s

Average velocity

$$= \frac{\upsilon_1 + \upsilon_2}{2} = \frac{8 + 8.02}{2} = 8.01\text{ m/s}$$

35. **(D)** Apply the law of conservation of momentum $V = \frac{100\upsilon}{1000} = \frac{\upsilon}{10}$

36. **(C)** The total work done is (40 J) + (–20 J) = 20 J. So, by the work-energy theorem, W_{total} = ΔK, we have 20 J = ΔK. Since ΔK = $K_f - K_i$, we find $K_f = K_i$ + ΔK = 10 J + 20 J = 30 J.

37. **(C)** When a metal wire elongates by hanging a load Mg on it, decrease in potential energy of the load = Mgl

(where l = elongation in metal wire)

Elastic potential energy stored in stretched wire

$$= \frac{1}{2} \times Mgl$$

Difference of Mgl and $\frac{1}{2}Mgl$ appears as heat energy in the stretched wire.

∴ Energy appearing as heat

$$= Mgl = -\frac{1}{2}Mgl = \frac{1}{2}Mgl$$

38. **(A)** The coefficient of expansion of iron is less than that of the water but its density is more than the liquids. The relative decrease in the density of water will be more than that or iron. As a result, the buoyant force will decrease and the apparent weight will increase.

39. **(B)** m = 0.15 kg

$$u = 54\text{ km} = \frac{54 \times 1000}{60 \times 60} = 15\text{ m s}^{-1}$$

$$\theta = 22 \cdot 5^{\circ}$$

Impulse imparted to the ball = – 2 mu cos θ

= – 2 × 0·15 × 15 × cos (22·5°)

= – 2 × 0·15 × 15 × 0·9239

= – 4·157 kg m s^{-1}

The impulse imparted to the ball is 4·157 kg m s^{-1} directed along the bisector of initial and final direction.

40. **(A)** x = elongation in spring due to mass 10 kg

$$= \frac{10 \times 10}{100} = 1\text{m}$$

$W_F = \frac{1}{2} \times 100 \times [(3)^2 - (1)^2] - 10 \times 10 \times 2 = 200$ J.

CHEMISTRY

41. **(C)** In B_2H_6, the B atoms are linked through hydrogen bridges. The structure is not similar to that of C_2H_6, there is no B–B bond and also all the atoms do not lie in the same plane.

42. **(D)** In the 1st oxide, oxygen = 27.6 parts,

metal = 100 – 27.6 = 72.4 parts.

In the 2nd oxide, oxygen = 30 parts,

metal = 100 – 30 = 70 parts.

As 1st oxide is M_3O_4, 72.4 parts of M = 3 atoms of M and 27.6 parts of O = 4 atoms of O.

$\therefore$ 70 parts of

$$M = \frac{3}{72.4} \times 70 \text{ atoms of M}$$

= 2.9 atoms of M

30 parts of $O = \frac{4}{27.6} \times 30$ atoms of O

= 4.35 atoms of O.

$\therefore$ Ratio of M : O in the 2nd oxide = 2.9 : 4.35 = 1 : 1.5 = 2 : 3.

Hence, the formula is M_2O_3.

43. **(B)** In H_2O_2 structure, two O–H bonds lie in different planes.

44. **(B)** Initial pressure: p atm

C(s) + CO_2 (g) f 2CO (g)

Equilibrium pressure : (p–.5p) p

As given: p –.5p + p = 12; p = 8 atm.

$\therefore$ At equilibrium: Pco = 8 atm

P_{CO_2} = 4 atm

$$K_p = \frac{p^2CO}{PCO_2} = \frac{8^2}{4} = 16\text{ atm.}$$

45. **(A)** $\text{Mole of P} = \frac{0.50}{60} = 0.0083.$

$$\text{Mole of Q} = \frac{0.20}{45} = 0.0044.$$

Total mole = 0.0127.

Total pressure = 750 mm.

Partial pressure of P

$$= \frac{\text{moles of P}}{\text{total moles}} \times \text{total pressure}$$

$$= \frac{0.0083}{0.0127} \times 750 = 490\text{ mm.}$$

Partial pressure of Q

$$= \frac{0.0044}{0.0127} \times 750 = 260\text{ mm.}$$

46. **(C)** The given ions of elements belong to 2nd period. The atomic radius decreases from left to right (Li, B, O, F) in the period.

Cations are smaller, whereas anions are larger than the corresponding atoms.

The anion carrying more negative charge is larger.

So, O^{2-} has the highest value of ionic radius.

47. **(B)** N_2 (g) + 3 H_2 (g) $\rightarrow$ 2 NH_3 (g)

1 L of N_2 reacts with 3 L of H_2 to form 2 L of NH_3.

Thus, N_2 is the limiting reactant.

10 L N_2 will react with 30 L H_2 to form 20 L NH_3.

As actual yield is 50% of the expected value, NH_3 formed = 10 L, N_2 reacted = 5 L,

H_2 reacted = 15 L

$\therefore$ Mixture will contain 10 L NH_3, 25 L N_2, 15 L H_2.

48. **(A)** Element with At. No. 19 will lose one electron and element with At.No. 17 will gain that electron to form an ionic water soluble compound.

49. **(A)** $KMnO_4$ +1 + x – 8 = 0 Ox. no. of Mn

x = + 7 +7

K_2MnO_4 + 2 + x – 8 = 0

x = +6 +6

MnO_2 x – 4 = 0

x = + 4 +4

Mn_2O_3 2x – 6 = 0

x = +3 +3

Thus, the highest oxidation number of + 7 for Mn is in $KMnO_4$.

50. **(C)** Given that,

(i) C (graphite) + O_2 (g) $\rightarrow$ CO_2 (g); ΔH = 94.05 kcal.

(ii) C (diamond) +O_2 (g) CO_2 (g); ΔH = – 94.50 kcal

Thus, applying the inspection method,

[Eqn. (i) – Eqn. (ii)] we get

C(graphite) + O_2(g) – C (diamond) – O_2 (g) $\rightarrow$ CO_2(g) – CO_2(g);

ΔH = – 94.05 – (–94.50)

or C(graphite) $\rightarrow$ C (diamond); ΔH= + 0.45 kcal

Since this enthalpy change is only for conversion of 1 mole, i.e., 12 g of C (graphite) to C (diamond), therefore, for the conversion of 10 g of C (graphite) to C (diamond)

$$\Delta H = 0.45 \times \frac{10}{12} = 0.375 \text{ kcal}$$

51. **(D)** C has six, N has seven and O has eight electrons. Thus, the total number of electrons and their distribution for each species are given below :

Species	Total no. of electrons
CN^-	14
NO^+	14
O_2^-	17

MO configuration									
	σ1s	σ*1s	σ2s	σ*2s	$\pi 2p_x$	$\pi 2p_y$	$\sigma 2p_z$	$\pi^* 2p_x$	$\pi^* 2p_y$
CN^-	2	2	2	2	2	2	2	–	–
NO^+	2	2	2	2	2	2	2	–	–
O_2^-	2	2	2	2	2	2	2	2	1

	N_b	N_a	Bond Order
CN^-	10	4	3
NO^+	10	4	3
O_2^-	10	7	3/2

So, the species CN^- and NO^+ have the same bond order.

52. **(A)** Lines cannot be assigned quantum numbers.

53. **(C)** 'a' is directly related to forces of attraction. Hence, greater the value of 'a', more easily the gas is liquefied.

54. **(B)** $H_2 + I_2 \rightleftharpoons 2HI$

Applying law of mass action,

$$K_c = \frac{[HI]^2}{[H_2][I_2]}$$

Given $[H_2]$ = 8.0 mole litre^{-1}

$[I_2]$ = 3.0 mole litre^{-1}

$[HI]$ = 28.0 mole litre^{-1}

So, $K_c = \frac{(28.0)^2}{(8.0)\times(3.0)} = 32.66$

55. **(D)** $2\ Mg + O_2 \rightarrow 2MgO$

2 × 24 g   2 × 16 g   2 × (24 + 16)g

Thus, 48 g of Mg requires 32 g of O_2 to form 80 g of MgO

Therefore, 30 g of Mg requires

$\frac{32}{48}\times 30$ g of O_2 to form $\frac{80}{48}\times 30$ g of MgO or 30 g of Mg requires 20 g of O_2 to form 50 g of MgO.

The residual mixture thus contains 50 g of MgO and 10 g 0f O_2.

CRITICAL THINKING

56. **(C)** 57. **(D)** 58. **(Del)**

59. **(A)** 60. **(A)**

THE END