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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION (UPDATED)

CLASS - 12 (PCM)

Question Paper Code : UN446

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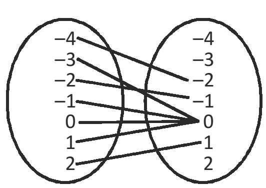
SOLUTIONS

MATHEMATICS

1. **(A)** $f(x) = \dfrac{\cos^2 x + \sin^4 x}{\sin^2 x + \cos^4 x}$

$$= \frac{1 - \frac{1}{4}\sin^2 2x}{1 - \frac{1}{4}\sin^2 2x} = 1 \Rightarrow f(2018) = 1$$

2. **(A)** It is onto but not one-one



3. **(A)** Let x = Tan α, y = Tan β, z = Tan γ

Given $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$

$\alpha + \beta + \gamma = \pi$

$\alpha + \beta = \pi - \gamma$

$\text{Tan}(\alpha + \beta) = -\text{Tan}\,\gamma$

$\dfrac{\text{Tan}\alpha + \text{Tan}\beta}{1 - \text{Tan}\alpha\,\text{Tan}\beta} = -\text{Tan}\gamma \Rightarrow \dfrac{x+y}{1-xy} = -z$

$\Rightarrow x + y + z = xyz$

4. **(B)** $\tan\left(\sec^{-1}\dfrac{1}{x}\right) = \sin(\tan^{-1} 2)$

$$\tan\left(\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right) = \sin\left(\sin^{-1}\frac{2}{\sqrt{5}}\right)$$

$$\Rightarrow \frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}} \Rightarrow x = \frac{\sqrt{5}}{3}$$

5. **(C)** $A^2 = \begin{bmatrix}1&2&2\\2&1&2\\2&2&1\end{bmatrix}\begin{bmatrix}1&2&2\\2&1&2\\2&2&1\end{bmatrix} = \begin{bmatrix}9&8&8\\8&9&8\\8&8&9\end{bmatrix}$

$A^2 . A = \begin{bmatrix}9&8&8\\8&9&8\\8&8&9\end{bmatrix}\begin{bmatrix}1&2&2\\2&1&2\\2&2&1\end{bmatrix}$

$= \begin{bmatrix}41&42&42\\42&41&42\\42&42&41\end{bmatrix}$; $A^3 - 4A^2 - 6A =$

$\begin{bmatrix}41&42&42\\42&41&42\\42&42&41\end{bmatrix} - 4\begin{bmatrix}9&8&8\\8&9&8\\8&8&9\end{bmatrix} - 6\begin{bmatrix}1&2&2\\2&1&2\\2&2&1\end{bmatrix}$

$\begin{bmatrix}-1&-2&-2\\-2&-1&-2\\-2&-2&-1\end{bmatrix} = -A$

6. **(B)** Given that $A = \begin{bmatrix}1&2\\3&4\end{bmatrix}$ and $B = \begin{bmatrix}a&0\\0&b\end{bmatrix}$

Now, $AB = \begin{bmatrix}1&2\\3&4\end{bmatrix}\begin{bmatrix}a&0\\0&b\end{bmatrix} = \begin{bmatrix}a&2b\\3a&4b\end{bmatrix}$

And $BA = \begin{bmatrix}a&0\\0&b\end{bmatrix}\begin{bmatrix}1&2\\3&4\end{bmatrix} = \begin{bmatrix}a&2a\\3b&4b\end{bmatrix}$

If AB = BA, then a = b

Hence, AB = BA is possible for infinitely many values of B's

7. **(A)** $\cos^2(A+B) + \sin^2(A+B) + \cos 2B = 0$

$1+\cos 2B = 0$; $2\cos^2 B = 0$

$\cos B = 0 \Rightarrow B = (2n+1)\frac{\pi}{2}$, $n \in Z$

8. **(D)** Let $x = AR^{p-1}$, $y = AR^{q-1}$, $Z = AR^{r-1}$

∴ $\text{Log}x = \text{Log}A + (p-1)\text{Log}R$

$\text{Log}y = \text{Log}A + (q-1)\text{Log}R$

$\text{Log}z = \text{Log}A + (r-1)\text{Log}R$

$$\begin{vmatrix}\log A + (p-1)\log R & p & 1\\ \log A + (q-1)\log R & q & 1\\ \log A + (r-1)\log R & r & 1\end{vmatrix} = 0$$

9. **(A)** Given, f(x)

$$= \begin{vmatrix}1 & x & x+1\\ 2x & x(x-1) & x(x+1)\\ 3x(x-1) & x(x-1)(x-2) & (x-1)(x)(x+1)\end{vmatrix}$$

Taking x and $x(x-1)$ common from R_2 and R_3 and $(x+1)$ common from C_3

$$= x \times x(x-1) \times (x+1)\begin{vmatrix}1 & x & 1\\ 2 & (x-1) & 1\\ 3 & (x-2) & 1\end{vmatrix}$$

(applying $C_2 \to C_1 + C_2$)

$$= x^2(x^2-1)\begin{vmatrix}1 & x+1 & 1\\ 2 & x+1 & 1\\ 3 & x+1 & 1\end{vmatrix}$$

$$= x^2(x^2-1)(x+1)\begin{vmatrix}1&1&1\\2&1&1\\3&1&1\end{vmatrix} = 0$$

$\Rightarrow f(x) = 0$

$\therefore f(2018) = 0$

10. **(B)** f is continuous at

$x = 0 \Rightarrow \underset{x\to 0}{\text{Lt}} f(x) = f(0)$

$$\Rightarrow \lambda = \underset{x\to 0}{\text{Lt}} \frac{\cos 3x - \cos x}{x^2} =$$

$$\underset{x\to 0}{\text{Lt}} \frac{-3\sin 3x + \sin x}{2x}$$

$$= \underset{x\to 0}{\text{Lt}} \frac{-9\cos 3x + \cos x}{2} = \frac{-9+1}{2} = -4$$

11. **(C)** $\sqrt{\frac{y}{x}} + \sqrt{\frac{x}{y}} = 2 \Rightarrow x + y = 2\sqrt{xy}$

$\Rightarrow \left(\sqrt{x} - \sqrt{y}\right)^2$

$\Rightarrow \sqrt{x} = \sqrt{y}$

$\Rightarrow x = y$

12. **(B)** Using componendo and dividendo, then find y and $\frac{dy}{dx}$

13. **(B)** Given $\frac{dv}{dt} = 2\pi$; $V = 288\pi$

$\Rightarrow r = 6 \therefore \frac{dr}{dt} = \frac{1}{72}$

14. **(C)** S = 490t – 4.9t2 ; v = $\frac{ds}{dt}$ = 490 – 9.8t

at maximum height v = 0

∴ t = 50 ∴ S = 12250

15. **(C)** Let y = f(x) = x^{3000}

here x = 1, δ = 0.0002

$\delta y = f'(x)\delta x = 3000x^{2999}\delta x$

= (3000) (0.0002) = 0.6

∴ f(x+δx) = y + δx = 1 + 0.6 = 1.6

16. **(C)** $\int \frac{dx}{x^2\sqrt{4+x^2}} = \int \frac{dx}{x^3\left(\frac{4}{x^2}+1\right)^{1/2}}$

$= -\frac{1}{4x}\sqrt{4+x^2}+c$ (assuming $\frac{4}{x^2}+1=t$)

17. **(B)** $\int \frac{(a+x)+(a-x)}{\sqrt{a^2-x^2}}dx = 2a\sin^{-1}\left(\frac{x}{a}\right)+c$

18. **(B)** $= \int \frac{\left(\sqrt{1+x}\right)^2+\sqrt{x}.\sqrt{1+x}}{\sqrt{x}+\sqrt{1+x}}dx$

$= \int \frac{\sqrt{1+x}\left[\sqrt{1+x}+\sqrt{x}\right]}{\sqrt{x}+\sqrt{1+x}}dx$

$= \int \sqrt{1+x}\,dx = \frac{2(1+x)^{3/2}}{3}+c$

19. **(B)** Put x = cosθ l.l : 0 = cosθ

$\Rightarrow \theta = \pi/2$ dx = – sinθdθ

U.L : 1 = cosθ $\Rightarrow \theta = 0°$

$= \int_{\pi/2}^{0} \sin\left[2\tan^{-1}\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}\right](-\sin\theta)\,d\theta$

$= \int_0^{\pi/2} \sin(\pi-\theta)\sin\theta d\theta = \int_0^{\pi/2}\sin^2\theta d\theta$

$= \frac{1}{2}\times\frac{\pi}{2} = \frac{\pi}{4}$

20. **(A)** $\int_1^2 \left(x^3-x^2\right)dx = \frac{17}{12}$

21. **(B)** $\tan y \frac{dy}{dx} = 2\sin x \cos y$

$\Rightarrow \int \sec y \tan y dy = 2\int \sin x dx$

secy = – 2cosx + c

22. **(B)** Let x + y = v

$1 + \frac{dy}{dx} = \frac{dv}{dx}$ we have,

$\frac{dv}{dx} - 1 = \sin v \tan v - 1$

$\frac{dv}{dx} = \sin v \tan v = \int \frac{\cos v}{\sin^2 v}dv = \int dx$

$-\frac{1}{\sin v} = x + c$

∴ cosec (x+y) + x = c

23. **(A)** $A = \bar{a}, B = \bar{b}, C = \bar{c}, D = \bar{d}$

$\therefore P = \frac{\bar{c}+2\bar{d}}{3}, Q = \frac{\bar{a}+\bar{c}}{2}$

∴ $\overrightarrow{AB} + 2\overrightarrow{AD} + \overrightarrow{BC} - 2\overrightarrow{DC} = K\overrightarrow{PQ}$

$\Rightarrow$ k = – 6

24. **(A)** ∴ The vectors are ⊥ $r(\bar{i}-2x\bar{j}-3y\bar{k})$

$.(\bar{i}+3x\bar{j}+2y\bar{k}) = 0$

$\Rightarrow 1 - 6x^2 - 6y^2 = 0$

$\Rightarrow x^2 + y^2 = 1/6$ which represents a circle

25. **(C)** $S.D = \frac{\left(\bar{a}-\bar{c}\ \bar{b}\ \bar{d}\right)}{\left|\bar{b}\times\bar{d}\right|}$

$\bar{b}\times\bar{d} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$

$= \bar{i}(-3) - \bar{j}(-3) + \bar{k}(-3)$

$= -3\bar{i} + 3\bar{j} - 3\bar{k}$

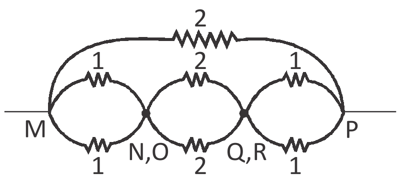
$\left[\bar{a}-\bar{c}\ \bar{b}\ \bar{d}\right] = \left(-3\bar{i}-3\bar{j}-3\bar{k}\right).\left(-3\bar{i}+3\bar{j}-3\bar{k}\right)$

= 9 – 9 + 9 = 9

∴ S.D. = $\frac{9}{3\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$

PHYSICS

26. **(C)** N and O are equipotential points and so are Q and R. Here the circuit can be redrawn as shown below 1 Ω and 1 Ω in parallel sum, upto 1/2 W; 2 W and 2 W in parallel sum up to 1 W; 1/2 W, 1 W, 1/2 W in series sum up to 1/2 + 1 + 1/2 = 2 Ω; 2Ω and 2Ω in parallel sum upto = 1 Ω.



27. **(A)** Here, $M = 0.24 JT^{-1}$; r = 10 cm = 0.1 m

The magnetic field due to a short bar magnet on its axis,

$$B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$$

$$= 10^{-7} \times \frac{2 \times 0.24}{(0.1)^3} = 4.8 \times 10^{-5} T$$

28. **(D)** The sun produces the whole range of EM waves. Visible light for us to see, infrared radiations to give us heat and thus keeps us warm and ultraviolet rays to make our body produce vitamin D plus giving our skin a tanned complexion.

29. **Deleted**

30. **(D)** The work done in moving the loops away from each other is converted into the electric energy. Thus, the current in each loop increases.

31. **(D)** Proton is deflected towards positive x-axis even if only electric field is switched on. Therefore $\vec{E}$ is along positive x-axis or option (B) is correct. Proton is deflected along positive x-axis when only magnetic field is switched on i.e., magnetic force $\vec{F}_m$ is along positive x-axis. This is possible in the following two cases

(i) $\vec{v}$ is along positive y-axis and $\vec{B}$ along positive z-axis

(ii) $\vec{v}$ is along negative y-axis and $\vec{B}$ along negative z-axis because $\vec{F}_m = q(\vec{v} \times \vec{B})$. Therefore, options (A) and (C) are also correct.

32. **(C)** Dipole moment = p = charge × separation

$= q \times 2l = 1.602 \times 10^{-19} \times 4 \times 10^{-10} = 6.408 \times 10^{-29}$ Cm

Torque = pE sin θ

$= 6.408 \times 10^{-29} \times 3 \times 10^5 \times \sin 30^\circ = 9.612 \times 10^{-24}$ Nm.

33. **(B)** As there is no current through AB, all the points of AB are at the same potential, say, V_1. For the same reason all the points of CD are at the same potential, say, V_2. But $V_1 > V_2$ because current is flowing from A to C or B to D. Therefore, $V_E > V_F$. This is why on connecting E and F by a wire, current will flow from E to F.

34. **(C)** Two sources should have the same wavelength, nearly the same amplitude and have a constant phase difference. If the phase difference between two interfering waves does not remain constant, interference pattern will not be sustained.

35. **(A)** In fact there are two capacitors in parallel-one between the inner sphere and the inner surface of the outer sphere and the other between the outer surface of the outer sphere and distant objects which may be treated as a spherical body of large radius.

C_1(capacitance of the first)

$$= \frac{4\pi\varepsilon_0 \times 2 \times 4 \times 10^{-4}}{(4-2) \times 10^{-2}} = 4\pi\varepsilon_0 \times 0.04 \text{ F}$$

C_2 (capacitance of the second) =

$4\pi\varepsilon_0 b = 4\pi\varepsilon_0 \times 0.04$ F

Total capacitance = $4\pi\varepsilon_0 (0.04 + 0.04)$

$= 4\pi\varepsilon_0 \times 0.08$ F

∴ Potential of the outer sphere =

$$\frac{q}{C} = \frac{2\times10^{-6}}{4\pi\varepsilon_0 \times 0.08}$$

$$= 9\times10^9 \frac{2\times10^{-6}}{0.08} = 2.25\times10^5 \text{ V}$$

Charge on the outer surface of the outer sphere

$= 2.25 \times 10^5 \times (4\pi\,\varepsilon_0 \times 0.04) = 10^{-6}$ C $= 1\ \mu$C

36. **(C)** From $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we have

$$\frac{1}{b} + \frac{1}{a} = \frac{1}{f} \quad \text{or} \quad f = \frac{ab}{a+b} \quad .... (1)$$

Further $AC^2 + BC^2 + AB^2$

or $(a^2 + c^2) + (b^2 + c^2) = (a + b)^2$

or $a^2 + b^2 + 2c^2 = a^2 + b^2 + 2ab$

$\therefore$ $ab = c^2$

Substituting this in Eq. (1), we get

$$f = \frac{c^2}{a+b}$$

37. **(A)** The change in angular momentum of the electron,

$\Delta L = n_2\,(h/2\pi) - n_1\,(h/2\pi) = (n_2 - n_1)\ h\,/\,2\pi$

Here, $n_1 = 4$; $n_2 = 3$ and

$h = 6.62 \times 10^{-34}$ J s

$$\therefore \Delta L = \frac{(3-4)\times6.62\times10^{-34}}{2\pi}$$

$= -\ 1.054 \times 10^{-34}$ J s

38. **(C)** Average power consumed in an a.c. circuit = (VI/2) cos α where α is the phase difference between current and emf. Here power factor (i.e., cos α) = 0. Therefore, power consumed is zero.

39. **(A)** The neutral temperature is $\theta_n = -\frac{a}{b}$

$$= \frac{16.3\times10^{-6}}{0.042\times10^{-6}}\ ^\circ C = 388^\circ C.$$

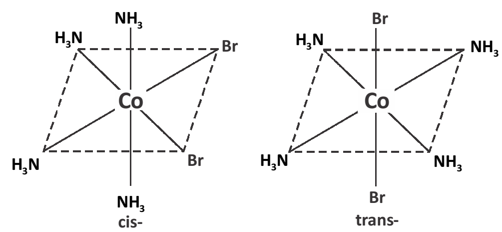
The inversion temperature is double the neutral temperature i.e., 776°C.

40. **(D)** Being a conductor, each plate has the same potential at each point. As $E \propto \sigma$, so surface charge density σ is higher at the closer end because electric field being the conductor gradient of potential is highest where the plates are closest to each other.

CHEMISTRY

41. **(D)** A calculated amount of spiegeleisen (an alloy of Fe, Mn and C) is added during the production of steel.

42. **(A)** $[Co(NH_3)_4Br_2]Cl$ and $[Co(NH)_3)_4BrCl]Br$ are ionization isomers. The geometrical isomers are



43. **(C)** Gold has higher reduction potential than iron. Hence, Fe is easily oxidized to Fe^{2+}.

44. **(B)** $CH_3CHOHCH_2CH_3 \xrightarrow{[O]} CH_3COCH_2CH_3$

45. **(B)** The number of atoms in a unit cell are: simple cubic lattice = 1, body centred = 2 and face centred = 4.

46. **(D)** SF_4 is trigonal bipyramidal (hybridisation sp^3d) with one lone-pair of electrons. CF_4 is tetrahedral (due to sp^3 hybridisation of orbitals on carbon) with no lone-pair.

XeF_4 is a planar molecule with 2 lone-pairs, (sp^3d^2 hybridisation of Xe).

47. **(C)** Propan-l-ol gives propanal on passing over heated copper. Aldehydes give positive Fehling solution test.

Propan-2-ol is a secondary alcohol. On oxidation by heating with copper, it gives propanone. Propanone (a ketone) does not give Fehling's test.

48. **(A)** Amount of K_2CO_3 = 20.7 g

Molar mass of K_2CO_3 = 138

Moles of K_2CO_3 = $\frac{20.7}{138} = 0.15$

Mass of solution

= (500 ml) × (1 g ml^{-1}) = 500 g

Amount of water = 500 – 20.7 = 479.3g

Molality = $\frac{\text{Moles of solute}}{\text{Mass of solvent in gram}} \times 1000$

= $\frac{0.15}{479.3} \times 1000 = 0.313$ m.

49. **(B)** Formaldehyde is an emulsifier.

50. **(C)** $2K\overset{+7}{Mn}O_4 + 3H_2SO_4 \rightarrow$

$K_2SO_4 + 2Mn^{2+}SO_4 + 3H_2O + 5|O|$

$5H_2C_2O_4 + 5|O| \rightarrow 10\ CO_2 + 5\ H_2O$

Here Mn^{+7} in $KMnO_4$ has been reduced to Mn^{2+} in $MnSO_4$.

51. **(A)** $C_6H_5CH_2NH_2$ is an aralkylamine and hence is more basic than arylamines which, in turn, are more basic than amides.

52. **(B)** Bulky groups on the carbon atom attached to the halogen atom will hinder attack by S_N2 mechanism due to steric hindrance and hence would favour S_N1 mechanism.

53. **(D)** Rate = Rate constant × Concentration of the reactant

2.40×10^{-5} mol L^{-1} s^{-1}

= 3.0×10^{-5} s^{-1} [N_2O_5]

So, [N_2O_5] = $\frac{2.40 \times 10^{-5}\ \text{mol L}^{-1}\text{s}^{-1}}{3.0 \times 10^{-5}\text{s}^{-1}}$

= 0.8 mol L^{-1}

54. **(A)** H(O) $P(OH)_2$ contains one P—H group and two P—OH units, thus it is reducing and dibasic.

55. **(A)** Formaldehyde which does not contain α-hydrogen does not undergo aldol condensation.

CRITICAL THINKING

56. **(D)** 57. **(A)**

58. **(C)** 59. **(B)** 60. **(A)**

THE END