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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION (UPDATED)

CLASS - 11 (PCM)

Question Paper Code : UN487

KEY

1. B	2. B	3. B	4. A	5. B	6. B	7. A	8. A	9. A	10. D
11. C	12. A	13. A	14. C	15. D	16. A	17. C	18. A	19. B	20. B
21. A	22. C	23. A	24. C	25. B	26. D	27. D	28. D	29. B	30. B
31. B	32. D	33. C	34. A	35. A	36. B	37. B	38. D	39. B	40. A
41. A	42. C	43. C	44. A	45. D	46. D	47. C	48. D	49. A	50. B
51. C	52. B	53. C	54. C	55. B	56. C	57. A	58. B	59. B	60. C

SOLUTIONS

MATHEMATICS

01. (B) Clarity X = {0, 9, 54, 243....) and Y = {0, 9, 18, 27,.....)

$X \cup Y = Y$.

02. (B) Let $z \in C$

gof : $A \to C$ is onto ; $z \in C$

$\exists x \in A \ni (gof)(x) = z$.

Let $f(x) = y$. Then $z = (gof)(x) = g[f(x)]$

$= g(y)$.

$\therefore z \in C$

$\exists y \in B \ni g(y) = z$

g : $B \to C$ is onto

Let $A = \{1, 2, 3\}$, $B = \{a, b, c, d\}$, $C = \{x, y, z\}$ and

f: $1 \to a$, $2 \to b$, $3 \to c$, g: $a \to x$, $b \to y$, $c \to z$, $d \to z$.

Then gof : $1 \to x$, $2 \to y$, $3 \to z$.

Now gof : $A \to C$ is onto, But f : $A \to B$ is not onto.

03. (B) $-3 + ix^2y$, $x^2 + y + 4i$ are conjugate

$x^2 + y = -3$,

$x^2y = -4$

$x^2 = 1$,

$y = -4$

$x = \pm 1$

04. (A) Let α be the common root. Then

$2bx^2 + 3c\alpha - \alpha = 0,\ 2a\alpha^2 + 3b\alpha + 4c = 0$

$$\frac{\alpha^2}{12c^2+3bd} = \frac{\alpha}{-2ad-8bc} = \frac{1}{6b^2-6ac}$$

$$\alpha^2 = \frac{12c^2+3bd}{6b^2-6ac},\quad \alpha = \frac{-2ad-8bc}{6b^2-6ac}$$

$$\therefore\quad \alpha^2 = \left(\frac{-2ad-8bc}{6b^2-6ac}\right)^2 = \frac{12c^2+3bd}{6b^2-6ac}$$

$$= \frac{4bc+ad}{3\left(b^2-ac\right)} = \frac{3\left(bd+4c^2\right)}{2\left(4bc+ad\right)}$$

$$\frac{2}{9}\,\frac{4bc+ad}{b^2-ac} = \frac{bd+4c^2}{4bc+ad}$$

$$k = \frac{9}{2}$$

05. (B) The number of one digit number formed = 4

The number of two digit numbers formed = 4 × 5 = 20

The number of three digit numbers formed = 4 × 5^2 = 100

The number of four digit numbers less than 7000 formed = 2 × 5^3 = 250

∴ The number of natural numbers less than 7000 is 4 + 20 + 100 + 250 = 374

06. (B) The numbers in the committee may be of the following.

(1) 1 Indian, 1 American, 4 Australians

(2) 1 Indian, 2 American, 3 Australians

(3) 1 Indian, 3 American, 2 Australians

(4) 1 Indian, 4 American, 1 Australians

(5) 2 Indian, 1 American, 3 Australians

(6) 2 Indian, 2 American, 2 Australians

(7) 2 Indian, 3 American, 1 Australians

(8) 3 Indian, 1 American, 2 Australians

(9) 3 Indian, 2 American, 1 Australians

(10) 4 Indian, 1 American, 1 Australians

∴ The number of ways of forming the committee

$= 3 \times {}^5C_1 \times {}^5C_1 \times {}^5C_4 + 6 \times {}^5C_1 \times {}^5C_2 \times {}^5C_3 + {}^5C_2 \times {}^5C_2 \times {}^5C_2 = 375 + 3000 + 1000 = 4375$

07. (A) $S_k = \dfrac{1+2+3+.....+k}{k} = \dfrac{k(k+1)}{2k}$

$$= \frac{(k+1)}{2}$$

$$= S_1^2 + S_2^2 + ...+ S_{10}^2 = \frac{5}{12}A$$

$$1^2 + \left(\frac{3}{2}\right)^2 + 2^2 + \left(\frac{5}{2}\right)^2 + ...+ \left(\frac{11}{2}\right)^2$$

$$= \left(\frac{5}{12}\right)A$$

$$\therefore\quad \frac{1}{4}[2^2 + 3^2 + ... + 11^2] = \frac{5}{12}A$$

$$= \left[\frac{11\times12\times23}{6} - 1\right] = \frac{5}{3}A\times4$$

$$A = \frac{3}{5}[506 - 1] = 303$$

08. (A) $(1 - x - x^2 + x^3)^6 = (1 - x)^6(1 - x^2)^6$

$(1 - {}^6C_1x + {}^6C_2x^2 - {}^6C_3x^4 + {}^6C_4x^4 - {}^6C_5x^5 + {}^6C_6x^6) \times (1 - {}^6C_1x^2 + {}^6C_2x^4 - {}^6C_3x^6 + {}^6C_4x^8 - {}^6C_5x^{10} + {}^6C_6x^{12})$

Coefficient of $x^7 = (-{}^6C_1 \times -{}^6C_3) + (-{}^6C_3 \times -{}^6C_2) + (-{}^6C_5 \times -{}^6C_1) = 120 - 300 + 36$

$= -144$

09. (A) Let d_1, d_2 be the common differences of the two A.P.s respectively.

$x_3 = 8$

$x_1 + 2d_1 = 8$

$x_8 = 20$

$x_1 + 7d_1 = 20$

$x_1 + 2d_1 = 8,\quad x_1 + 7d_1 = 20$

$x_1 = 3.2,\quad d_1 = 2.4$

$x_5 = x + 4d_1 = 3.2 + 9.6$

$x_5 = 12.8$

$h_2 = 8$

$\frac{1}{h_2} = \frac{1}{8}$,

$\frac{1}{h_1} + d_2 = \frac{1}{8}$, $h_7 = 20$

$\frac{1}{h_7} = \frac{1}{20}$

$\frac{1}{h_1} + 6d_2 = \frac{1}{20}$

$\frac{1}{h_1} + d_2 = \frac{1}{8}$, $\frac{1}{h_1} + 6d_2 = \frac{1}{20}$

$5d_2 = \frac{1}{20} - \frac{1}{8} = \frac{2-5}{40} = \frac{-3}{40}$

$d_2 = \frac{-3}{200}$

$\frac{1}{h_1} + d_2 = \frac{1}{8}$

$\frac{1}{h_1} = \frac{1}{8} + \frac{3}{200} = \frac{28}{200} = \frac{7}{50}$

$\frac{1}{h_{10}} = \frac{1}{h_1} + 9d_2 = \frac{7}{50} - \frac{27}{200} = \frac{1}{200}$

$h_{10} = 200$

$\therefore x_5 h_{10} = (12.8)(200) = 2560$

10. (D) $\underset{x\to 0}{\text{Lt}} \frac{x\cot(4x)}{\sin^2 x \cot^2(2x)}$

$= \underset{x\to 0}{\text{Lt}} \frac{x\tan^2 2x}{\sin^2 x \tan 4x}$

$= \underset{x\to 0}{\text{Lt}} \frac{\frac{\tan^2 2x}{x^2}}{\left(\frac{\sin^2 x}{x^2}\right)\left(\frac{\tan 4x}{x}\right)} = \frac{2^2}{4} = 1$

11. (C) $f(x)$ is continuous at $x = 1$

$\underset{x\to 1-}{\text{Lt}} f(x) = \underset{x\to 1+}{\text{Lt}} f(x)$

$\underset{x\to 1-}{\text{Lt}} (ax + b) = \underset{x\to 1+}{\text{Lt}} (ax^2 + c)$

$a + b = a + c$

$b = c$ ________ (1)

$f(x)$ is continuous at $x = 2$

$\underset{x\to 2-}{\text{Lt}} f(x) = \underset{x\to 2-}{\text{Lt}} f(x)$

$\underset{x\to 2-}{\text{Lt}} (ax^2 + c) = \underset{x\to 2-}{\text{Lt}} \frac{dx^2+1}{x}$

$4a + c = \frac{4d+1}{2}$

$8a + 2c = 4d + 1$ ________ (2)

$f(x)$ is differentiable at $x = 1$

$f'(1-) = f'(1+)$

$\left[\frac{d}{dx}(ax+b)\right]_{x=1} = \left[\frac{d}{dx}(ax^2+c)\right]_{x=1}$

$a = 2a$

$a = 0$

$f(x)$ is differentiable at $x = 2$

$f'(2-) = f'(2+)$

$\left[\frac{d}{dx}(ax^2+c)\right]_{x=2} = \left[\frac{d}{dx}\left(dx+\frac{1}{x}\right)\right]_{x=2}$

$4a = d - \frac{1}{4}$

$d = \frac{1}{4}$

$(2) \Rightarrow 0 + 2c = 4\left(\frac{1}{4}\right) + 1$

$\Rightarrow 2c = 2$

$c = 1$

and hence h = 1,

$\therefore ad - bc = 0\left(\frac{1}{4}\right) - (1)(1) = -1$

12. (A) Median M = 24. Mean deviation about median = $\frac{\Sigma f_i[x_i - M]}{N}$ = $\frac{510}{68}$ = 7.5

13. (A) Probability of obtaining at least one head and a least one tail

= 1 – [Probability of obtaining all heads + Probabilistic of obtaining all tails] = 1

$-\left[\frac{1}{2^8}+\frac{1}{2^8}\right]$ = 1 – $\frac{1}{2^7}$ = $\frac{127}{128}$

14. (C) G. E = $\frac{\sin(-660°)\tan(1050°)\sec(-420°)}{\cos(225°)\text{cosec}(315°)\cos(510°)}$

$$= \frac{-\sin(720°-60°)\tan(3.360-30°)\sec(360°+60°)}{\cos(180°+45°)\text{cosec}(360°-45°)\cos(360°+90°+60°)}$$

$$= \frac{\sin(60°)(-\tan 30°)\sec 60°}{-\cos 45°(-\text{cosec}45°)(-\sin 60°)}$$

$$= \frac{1/\sqrt{3}}{(1/\sqrt{2})\sqrt{2}} = \frac{2}{\sqrt{3}}$$

15. (D) 0 < α, β < π/4, cos(α + β) = 3/5, sin(α – β) = 5/13

0 < α + β < π/2, 0 < α – β < π/2 and tan (α + β)

= 4/3, tan(α – β) = 5/12

= tan2α = tan[(α + β) + (α – β)]

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$= \frac{\left(\frac{4}{3}\right)+\left(\frac{5}{12}\right)}{1-\left(\frac{4}{3}\right)+\left(\frac{5}{12}\right)} = \frac{48+15}{36-20}$$

$$= \frac{63}{16}$$

16. (A) cos A = $-\frac{60}{61}$, A ∉ Q_2 ⇒ A ∉ Q_3

Tan A = $\frac{11}{60}$, cos A = $-\frac{60}{61}$

Tan B = $-\frac{7}{24}$, B ∉ Q_2

B ∉ Q_4 ⇒ $\frac{B}{2}$ ∉ Q_2

$$\frac{2\text{Tan}\frac{B}{2}}{1-\text{Tan}^2\frac{B}{2}} = -\frac{7}{24}$$

= 48 Tan $\frac{B}{2}$ = –7 + 7 Tan² $\frac{B}{2}$

= 7 Tan² $\frac{B}{2}$ – 48 Tan $\frac{B}{2}$ – 7 = 0

$$= \left(\text{Tan}\frac{B}{2}-7\right)\left(7\text{Tan}\frac{B}{2}+1\right) = 0$$

= $-\frac{1}{7}$, cos $\frac{B}{2}$ = $-\frac{7}{\sqrt{50}}$, sin $\frac{B}{2}$ = $\frac{1}{\sqrt{50}}$

$$\text{Tan}\left(A+\frac{B}{2}\right) = \frac{\text{Tan } A+\text{Tan}\frac{B}{2}}{1-\text{Tan } A\,\text{Tan}\frac{B}{2}}$$

$$= \frac{\frac{11}{60}-\frac{1}{7}}{1+\frac{11}{60}\frac{1}{7}} = \frac{77-60}{420+11} > 0$$

$$\cos\left(A+\frac{B}{2}\right) = \text{Cos } A \text{ Cos}\frac{B}{2} - \sin A \sin\frac{B}{2}$$

$$= \left(-\frac{60}{61}\right)\left(-\frac{7}{\sqrt{50}}\right)-\left(-\frac{11}{61}\right)\left(\frac{1}{\sqrt{50}}\right) > 0$$

= A + $\frac{B}{2}$ ∈ Q_1

17. (C) $\frac{\sin(2A+B)}{\sin B} = \frac{5}{1}$

By components and dividends we shall get

$\frac{\tan(A+B)}{\tan A} = \frac{3}{2}$

18. (A) Given, $a\cos^2\frac{c}{2} + c\cos^2\frac{A}{2} = \frac{3b}{2}$

$a\frac{s(s-c)}{ab} + \frac{c.s(s-a)}{bc} = \frac{3b}{2}$

$s = \frac{3b}{2}$

2s = 3b

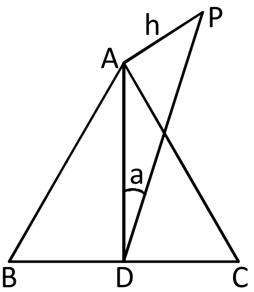
2b = a + c

a, b, c are in AP.

19. (B) Let ABC he the triangular park, AP the pole at A , and D the midpoint of BC. Let each side of the equilateral triangle ABC be a

The $AD^2 - AB^2 - BD^2 = a^2 - \frac{a^2}{a} = \frac{3a^2}{4}$

$Ad = \frac{\sqrt{3}a}{2}$



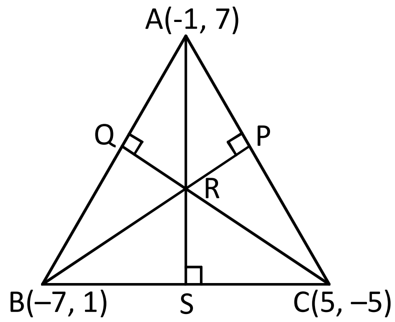
and since AP = h and $\angle ADP = \alpha$

We have AD = h cot α

Therefore, $\frac{\sqrt{3}a}{2}$ = h cot α

$a = \frac{2}{\sqrt{3}}(h\cot\alpha)$

20. (B)



$m_{BC} = \frac{6}{-12} = -\frac{1}{2}$

∴ Equation of AS is $y - 7 = 2(x + 1)$

$y = 2x + 9$ __________ (1)

$m_{AC} = \frac{12}{-6} = -2$

∴ Equation of BP is $y - 1 = \frac{1}{2}(x + 7)$

$y = \frac{x}{2} + \frac{9}{2}$ __________ (2)

From equation (i) and (ii),

$2x + 9 = \frac{x+9}{2}$

$4x + 18 = x + 9$

$3x = 9$

$x = -3$

∴ $y = 3$

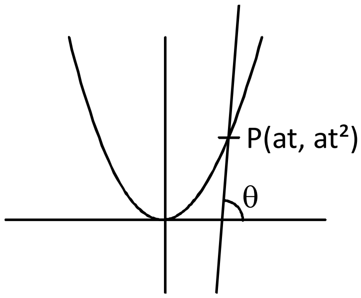
21. (A) Truth table

p	q	~p	$p \vee q$	$(\sim p)\wedge(p\vee q)$	$(\sim p)\wedge(p\vee q)\rightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

∴ $(\sim p)\wedge(p\vee q)\rightarrow q$ be a tautology

Other options are not tautology

22. (C)



$x^2 = 8y$

Then, equation of tangent at P

$tx = y + at^2$

$y = tx - at^2$

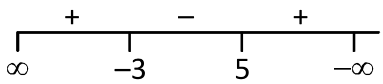
Then, slope t = tanθ

Now, $y = \tan\theta x - 2\tan^2\theta$

$\cot\theta y = x - 2\tan\theta$

$x = y\cot\theta + 2\tan\theta$

23. (A) $A = (m \in R : x^2 - (m+1)x + m + 4 = 0$ has real roots)



$D \geq 0$

$(m+1)^2 - 4(m+4) \geq 0$

$m^2 - 2m - 15 \geq 0$

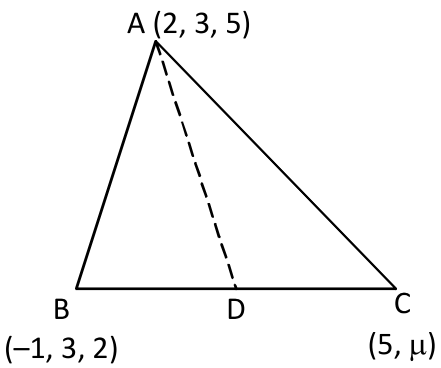
$A = ((-\infty, -3) \cup (5, \infty))$

$B = (-3, 5)$

$A - B = (-\infty, -3) \cup (5, \infty)$

24. (C) If D be the mid–point of BC, then

$$D = \left(\frac{\lambda-1}{2}, 4, \frac{\mu+2}{2}\right)$$



Direction ratios of AD are $\left(\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}\right)$

Since median AD is equally inclined with coordinate axed, therefore direction ratios of AD will be equal. i.e.,

$$\left(\frac{\left(\frac{\lambda-5}{2}\right)^2}{\left(\frac{\lambda-5}{2}\right)^2 + 1 + \left(\frac{\mu-8}{2}\right)^2}\right)$$

$$= \left(\frac{1}{\left(\frac{\lambda-5}{2}\right)^2 + 1 + \left(\frac{\mu-8}{2}\right)^2}\right)$$

$$= \left(\frac{\left(\frac{\mu-8}{2}\right)^2}{\left(\frac{\lambda-5}{2}\right)^2 + 1 + \left(\frac{\mu-8}{2}\right)^2}\right)$$

$$\left(\frac{\lambda-5}{2}\right)^2 = 1 = \left(\frac{\mu-8}{2}\right)^2$$

$\lambda = 7, 3$ and $\mu = 10, 6$

If $\lambda = 7$ and $\mu = 10$

Then $\frac{\lambda}{\mu} = \frac{7}{10}$

$10\lambda - 7\mu = 0$

25. (B) $\log_{10}(\tan 40° \times \tan 41° \times \ldots\ldots \tan 50°)$

$= \tan_{10}{}^{1} = 0$

PHYSICS

26. (D) The negative value of area of the a – t graph shows that the change in velocity in the time interval is negative. The final velocity depends on the initial velocity of the particle.

i.e., = $v_f = v_t$ [area of a – t graph$]_i^f$

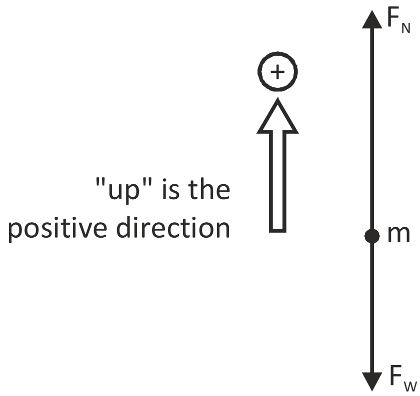
Thus, it may happen that the final velocity of the particle becomes zero.

The negative value of area of the v – t graph shows that the change in displacement in the time interval is negative. The final displacement is given by

$x_f = x_i$ + [area of v – t graph]

The particle may return to its original position if area of v – t graph is 0. If this area is negative the particle will always cross its original position.

27. (D) First draw a free-body diagram.



The person exerts a downward force on the scale, and the scale pushes up on the person with an equal (but opposite) force, F_N. Thus, the scale reading is F_N, the magnitude of the normal force. As $F_N - F_w = ma$, we have

$F_N = F_w + ma$ = (800 N) + [800 N/(10 $\frac{m}{s^2}$)](5 m/s^2) = 1200 N.

28. (D) Let m_1 be the mass of the bullet and m_2 that of the block.

Total mass,

$M = m_1 + m_2 = 0.01 + 4.0 = 4.01$ kg

$\mu = 0.25$ and $g = 9.8$ m s^{-2}

Frictional force,

$F = \mu R = 0.25 \times 4.01 \times 9.8$

Acceleration, $a = \frac{F}{M} = \frac{0.25 \times 4.01 \times 9.8}{4.01}$

= 2.45 m s^{-2}

Let u_1 be the initial velocity of the bullet.

K.E. of the bullet = $\frac{1}{2} m_1 u_1^2$

The kinetic energy of the bullet is transferred to the wooden block.

Let the block and bullet move with the velocity V.

$$\frac{1}{2} m_1 u_1^2 = \frac{1}{2} MV^2$$

$$V^2 = \frac{m_1 u_1^2}{M} = \frac{0.01 \times u_1^2}{4.01} \quad ......(i)$$

The block comes to rest after covering 20 m.

$0 = V^2 - 2 \times 2.45 \times 20$

$V^2 = 2 \times 2.45 \times 20$ ......(ii)

From equation (i) and (ii)

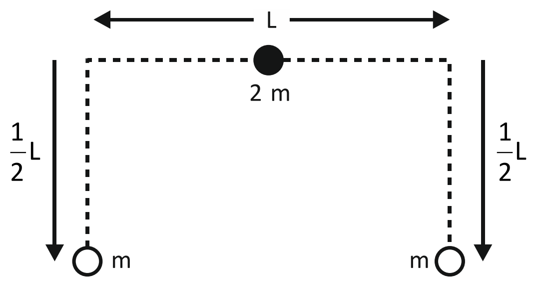
$$\frac{0.01 u_1^2}{4.01} = 2 \times 2.45 \times 20$$

$$u_1^2 = \frac{2 \times 2.45 \times 20 \times 4.01}{0.01} = 39298$$

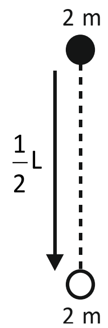
$u = \sqrt{39298}$ = 198.24 m/s

29. (B) In case of a viscous liquid, the strain produced increases with decrease in stress, hence portion QR of the graph agrees with viscous liquid.

30. (B) First replace each rod by concentrating its mass at its center of mass position.



The center of mass of the two m's is at their midpoint, at a distance of $\frac{1}{2}L$ below the center of mass of the rod of mass 2 m.



Now, applying the equation for locating the center of mass (letting y = 0 denote the position of the center of mass of the top horizontal rod), we find

$$y_{cm} = \frac{(2m)(0) + (2m)\left(\frac{1}{2}L\right)}{2m + 2m} = \frac{1}{4}L$$

31. (B) Radius of hydrogen atom,

r = 0.5 Å = 0.5 × 10^{-10} m

Volume of hydrogen atom = $\left(\frac{4}{3}\right)\pi r^3$

$$= \left(\frac{4}{3}\right) \times \left(\frac{22}{7}\right) \times \left(0.5 \times 10^{-10}\right)^3$$

= 0.524 × 10^{30} m^3

1 mole of hydrogen contains 6.023 × 10^{23} hydrogen atoms

∴ Volume of 1 mole of hydrogen atoms = 6.023 × 10^{23} × 0.524 × 10^{-30}

= 3.16 × 10^{-7} m^3

32. (D) The range of both the balls will be equal because the angles of projection 60° and 30° are complementary. Therefore, the ball Y also travels 40 m in the horizontal direction.

For complementary angles, the time of flight T_1 and T_2 are related as

$$T_1 T_2 = \frac{2R}{g}$$

$$\therefore\ T_2 = \frac{2(40\,m)}{(10\,m\,s^{-2})(4\,s)} = 2\,s$$

33. (C) In addition to acceleration due to gravity, gravitational intensity also provides the necessary centripetal force.

$$\therefore\ g = g_0 - \omega^2 R\cos^2\theta$$

34. (A) As the hemispherical bowl just floats without sinking in a liquid, therefore, it is completely immersed in the liquid.

Let d and D be the inner and outer diameter of the bowl, then

$$\frac{2}{3}\pi\frac{(D^3 - d^3)}{8}\rho = \frac{2}{3}\pi\left(\frac{D^3}{8}\right)\rho_l$$

or $$\frac{D^3 - d^3}{D^3} = \frac{\rho_l}{\rho}$$

or $$\frac{d^3}{D^3} = 1 - \frac{\rho_l}{\rho}$$

or $$d = D\left[1 - \frac{\rho_l}{\rho}\right]^{1/3}$$

Here D = 1 m; $\frac{\rho_l}{\rho} = \frac{1.2 \times 10^3}{6 \times 10^3} = 0.2$

∴ d = (1) $(1 - 0.2)^{1/3}$ = 0.93 m

35. (A) From v = u + at = 100 – 10 × 5 = 50 m/s

This is the velocity at the time of explosion. According to the principle of conservation of linear momentum,

$$1 \times 50 = \frac{400}{1000} \times (-25) + \frac{600}{1000} \times v_2$$

$50 + 10 = 0.6\, v_2$

$$v_2 = \frac{60}{0.6} = 100 \text{ m/s}$$

∴ The second fragment will go upwards with a speed of 100 m/s.

36. (B) As the gas is confined, n remains constant, and the volume is fixed, V remains constant as well. As R is a universal constant, the ideal gas law, PV = nRT, represents that P and T are proportional. Therefore, if T increases by a factor of 2, then so does P.

37. (B) A + B = 16,

$$8\sqrt{3} = (A^2 + B^2 + 2\,AB\cos\theta)^{1/2}$$

$$\text{And } \tan 90^\circ = \frac{B\sin\theta}{A + B\cos\theta}$$

or $$\infty = \frac{B\sin\theta}{A + B\cos\theta}$$

or $A + B\cos\theta = 0$

or $B\cos\theta = -A$

∴ $8\sqrt{3} = [A^2 + B^2 + 2A(-A)]^{1/2}$

or $192 = B^2 - A^2 = (B - A)(B + A) = (B - A) \times 16$

or $B - A = 192/16 = 12$

On solving, A = 2 and B = 14.

38. (D) Option (A) Process BC : $V \alpha T$, therefore,

$$T_C = \left(\frac{V_C}{V_B}\right) T_B = 6T_0$$

Option (B) Process DA : $V \alpha T$, therefore,

$$T_D = \left(\frac{V_D}{V_A}\right) T_A = 3T_0$$

Option (C) Process AB : $P \alpha T$, therefore,

$$T_B = \left(\frac{P_B}{P_A}\right) T_A = 2T_0$$

39. (B) Here, $m_1 = m_2 = 100$ kg; r = 100 m

Acceleration of first astronaut,

$$a_1 = \frac{Gm_1m_2}{r^2} = \frac{1}{m_1} = \frac{Gm_2}{r^2}$$

Acceleration of second astronaut,

$$a_2 = \frac{Gm_1m_2}{r^2} = \frac{1}{m_2} = \frac{Gm_1}{r^2}$$

Net acceleration of approach

$$a = a_1 + a_2 = \frac{Gm_2}{r^2} + \frac{Gm_1}{r^2} = \frac{2Gm_1}{r^2}$$

$$= \frac{2 \times (6.67 \times 10^{-11}) \times 100}{(100)^2}$$

$= 2 \times 6.67 \times 10^{-13}$ m/s² As $s = \frac{1}{2}at^2$

$$\therefore\ t = \left(\frac{2s}{a}\right)^{1/2} = \left[\frac{2 \times (1/100)}{2 \times 6.67 \times 10^{-13}}\right]^{1/2} \text{ second}$$

On solving we get t = 1.41 days

40. (A) 1. Stefan's constant $\sigma = \frac{E}{T^4}$

$$= \frac{ML^2T^{-2}}{T.L^2K^2} = [ML^0T^{-3}K^{-4}]$$

2. Coefficient of volume expansion

$$\gamma = \frac{\Delta v}{V \times T}$$

$$= \frac{L^3}{L^3K} = [M^0L^0T^0K^{-1}]$$

3. Work done = $[M^1 L^2 T^{-2}]$

4. Velocity gradient =

$$\frac{\text{Velocity}}{\text{Distance}} = \frac{LT^{-1}}{L} = T^{-1}$$

The correct order is 2, 4, 3, 1.

CHEMISTRY

41. (A) $\Delta = E_{C-H} - \sqrt{E_{H-H} \times E_{C-C}}$

$= 98.8 - ((104.2) \times 83.1)^{1/2} = 5.75$ k cal

$x_C - x_H = 0.18\sqrt{\Delta} = 0.18\,(5.75)^{1/2} = 0.43$

$\therefore$ $x_C = 0.43 + x_H = 0.43 + 2.1 = 2.53$

42. (C) Decrease of K with rise of temperature means that the forward reaction is exothermic or the backward reaction (formation of HI) is endothermic. As the given reaction is exothermic, energy of HI is greater or stability is less than H_2 and I_2.

43. (C) H_3O^+ = Pyramidal, $H_2C = NH$ = Planar, ClO_2^- = Angular, NH_4^+ = Tetrahedral, PCl_5 = Trigonal bipyramidal

44. (A) $HO—CH_2—CH = CH_2$ is Vinylcarbinol

45. (D) Mass of the given sample (impure) of sodium chloride = 6.5 g

Mass of silver chloride formed = 14.35 g

Percentage purity of sodium chloride = ?

The chemical equation for the reaction is,

	NaCl	+ AgNO3 ⟶	AgCl(s)	+ $NaNO_3$
Stoichiometric mass	(23 + 35.5) g 58.5 g		(108 + 35.5) g 143.5 g	

Thus, 143.5 g of silver chloride is obtained from = 58.5 g of pure NaCl

1 g of silver chloride is obtained from

$= \dfrac{58.5}{143.5}$

14.35 g of silver chloride is obtained from $= \dfrac{14.35 \times 58.5}{143.5} g = 5.85\,g$

Mass of pure NaCl in 6.5 g of impure sample = 5.85 g

Percentage purity of sodium chloride

$= \dfrac{5.85}{6.5} \times 100 = 90$

46. (D) $CuSO_4\,(aq) + 2KI(aq) \longrightarrow CuI_2 + K_2SO_4$

$2CuI_2 \longrightarrow Cu_2I_2 + I_2$

$2Na_2S_2O_3 + I_2 \longrightarrow 2Na_2S_4O_6 + 2NaI$

(evolved iodine: I_2; The evolved iodine is reduced)

47. (C) The spectral series in hydrogen spectra which appears in the visible region is the Balmer series. For Balmer series, in the equation.

$\bar{v} = R_H \left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]$ where $n_2 > n_1$

$n_1 = 2$ and $n_2 = 3, 4, 5$ .... 1.

So, the lowest energy transition in Balmer series will be that from 3rd shell ($n_2 = 3$) to the second shell ($n_1 = 2$). Then,

$\bar{v} = 1.1 \times 10^7 m^{-1}\left[\dfrac{1}{2^2} - \dfrac{1}{3^2}\right]$

So, $\bar{v} = 15.28 \times 10^5\ m^{-1}$

Now, the energy for 1.0 g atom (1 mol of H atoms) is given by,

$E = N_A h v = N_A h c \bar{v} = 6.02 \times 10^{23} \times 6.62 \times 10^{-34} \times 3 \times 10^8 \times 15.28 = 10^5$ J

$E = 18.26 \times 10^4$ J/g atom = 182.6 kJ/g atom

48. (D) In the stomach, the medium is acidic while in the small intestine, the medium is basic. Hence, acetyl salicylic acid is almost unionised in the stomach but ionized in the small intestine.

49. (A) Given $C = \dfrac{12}{13} \times 100\% = 92.3\%$

$H = \dfrac{1}{13} \times 100\% = 7.69\%$

$C = \dfrac{92.3}{12} = 7.69 = 1$

$H = \dfrac{7.69}{1} = 7.69 = 1$

So, empirical formula is CH

As P decolourises $Br_2 - H_2O$, but Q does not. Therefore, $P = C_2H_2$ (acetylene) and $Q = C_6H_6$ (benzene).

50. (B) Ethyl alcohol undergoes combustion according to the reaction,

$C_2H_5OH + 3O_2 \longrightarrow 2CO_2 + 3H_2O$

$\Delta H = -1367$ kJ mol^{-1}

Then $\Delta_c H = \Sigma aH_{products} - \Sigma bH_{reactants}$

As the enthalpy of a compound is taken as equal to its heat of formation, and the enthalpy of an element is taken as zero, we can write,

$-1367 = [2\Delta_f H(CO_2)\ 3\Delta_f H(H_2O)] - [\Delta_f H(C_2H_5OH) + 0]$

Therefore, $\Delta_f H(C_2H_5OH) = 2(-393.4) + 3(-285.9) + 1367 = -277.5$ kJ mol^{-1}

51. (C)

Element	%	% / At. wt.	Ratio
N	30.5	30.5 / 14 = 2.18	1
O	69.5	69.5 / 16 = 4.34	2

Empirical formula = NO_2. E.F. wt. = 46

$\therefore$ $n = \frac{92}{46} = 2.$ Hence, mol. formula = N_2O_4

52. (B) By Hannay and Smith equation, % ionic character

$= 16(4-1.2) + 3.5(4-1.2)^2$

$= 44.8 + 27.44 = 72.24.$

53. (C) Isotones are the atoms of different elements which have equal number of neutrons but different number of protons inside their nuclei.

${}^{39}_{19}K$ and ${}^{40}_{20}Ca$ contain 20 neutrons in their nuclei and 19 and 20 protons respectively.

54. (C) The reaction is,

Initial amount :

$H_2O(g) + CO(g) \rightleftharpoons H_2(g) + CO_2(g)$

1 mol 1mol 0 0

Amounts reacted up to equilibrium :

$\frac{40}{100} \times 1 \text{ mol} = \frac{40}{100} \times 1 \text{ mol}$

= 0.4 mol = 0.4 mol

Amounts at equilibrium:

(1 – 0.4) mol (1 – 0.4) mol 0.4 mol 0.4 mol

= 0.6 mol = 0.6 mol 0.4 mol 0.4 mol

Volume of the reaction vessel = 10 L.

Equilibrium concentration :

$\frac{0.6 \text{ mol}}{10 \text{ L}}$ $\frac{0.6 \text{ mol}}{10 \text{ L}}$ $\frac{0.4 \text{ mol}}{10 \text{ L}}$ $\frac{0.4 \text{ mol}}{10 \text{ L}}$

0.06 mol L^{-1} 0.06 mol L^{-1} 0.04 mol L^{-1} 0.04 mol L^{-1}

The equilibrium constant of this reaction is

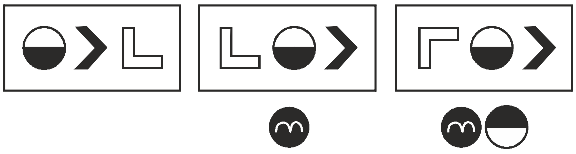
then given by, $K = \frac{[H_2(g)][CO_2(g)]}{[H_2O(g)][CO(g)]} =$

$\frac{(0.04 \text{ mol L}^{-1})(0.04 \text{ mol L}^{-1})}{(0.06 \text{ mol L}^{-1})(0.06 \text{ mol L}^{-1})} = \frac{16}{36} = 0.44$

55. (B) In alkali metals, the reactivity increases down the group due to decrease in IE_1. But in case of halogens, the reactivity decreases down the group due to decrease in their electrode potentials.

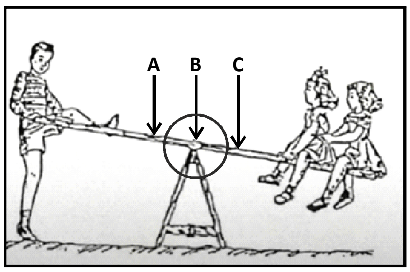
CRITICAL THINKING

56. (C)



57. (A) In case your visualized figure didn't match any of the available options, identify a unique characteristic (e.g., shape) of one of the provided pieces and try to locate that same characteristic in one of the options. In this example, it is the Trapezoid.

58. (B) Most of the tension is creating at point B and secondly due to the edge at point B it is more likely to break so the answer is B



59. (B) The delivery boy needs to walk from the grocery store to the firehouse. First, locate the grocery store and the firehouse. The grocery store is on Second Avenue between B1 Street and C1 Street. The firehouse is on D1 Street between Second and Third Avenues. Since the delivery boy is walking, you can ignore the one-way streets. Tracea route. Beginning at the grocery store, the delivery boy should walk east on Second Avenue to D1 Street, turn right, and go half a block to the firehouse.

Now read the answer choices. Choice B is the route you would have directed the delivery boy to use to get from the grocery store to the firehouse. Choices A and C have him walking west on Second Avenue, which is not the correct direction from the grocery store to the firehouse. Choice D has the delivery boy walking on First Avenue, which is not where the entrance to the grocery store is located, and left on D1 Street, which will not take him to the firehouse.

60. (C) From I, we conclude that weight of each pole = (4 × 5) kg = 20 kg.

So, total weight of 10 poles

= (20 × 10) kg = 200 kg.

From II, we conclude that:

Weight of each pole = (weight of 3 poles) – (weight of 2 poles) = 20 kg.

So, total weight of 10 poles = (20 × 10) kg = 200 kg.

The End