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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION (UPDATED)

CLASS - 12 (PCM)
Question Paper Code : UN487

KEY

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SOLUTIONS

MATHEMATICS

01. (C) Domain and codomain = {1, 2, 3....20}

There are five multiple of 4 as 4, 8, 12, 16 and 20

and there are 6 multiple of 3 as 3, 6, 9, 12, 15, 18.

Since, whenever k is multiple of 4 then, f(k) is multiple of 3 then total number of arrangement

$= {}^6c_5 \times 5! = 6!$

Remaining !5 elements can be arranged in 15! ways

Since, for every input, there is an output function f(k) in onto

$\therefore$ Total number of arrangement = 15!. 6!

02. (A) $S = \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right)$ +...... up to 10 terms

$= \tan^{-1}\left(\frac{2-1}{1+2.1}\right) + \tan^{-1}\left(\frac{3-2}{1+3.2}\right)$

$= (\tan^{-1} 2 - \tan^{-1} 1) + (\tan^{-1} 3 - \tan^{-1} 2) + (\tan^{-1} 4 - \tan^{-1} 3) + ... + (\tan^{-1} 11 - \tan^{-1} 10)$

$$(\tan^{-1} 11 - \tan^{-1} 1 = \tan^{-1}\left(\frac{11-1}{1+11.1}\right)$$

$$= \tan^{-1}\left(\frac{5}{6}\right)$$

$$\therefore \tan(S) = \left(\frac{5}{6}\right)$$

03. (A) $$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 6 & 1 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 6 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 6 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 90 & 12 & 1 \end{bmatrix}$$

$$P^5 = \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 90 & 12 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 15 & 1 & 0 \\ 135 & 15 & 1 \end{bmatrix}$$

$\because$ $Q - P^5 = I$

$Q = I + P^5$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 15 & 1 & 0 \\ 135 & 15 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{bmatrix}$$

$$= \frac{q_{21} + q_{31}}{q_{32}} = \frac{15+135}{15} = 10$$

04. (D) $$\Delta_1 = \begin{bmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{bmatrix}$$

$x(-x^2 - 1) - \sin\theta(-x \sin\theta - \cos\theta) + \cos\theta (-\sin\theta + x \cos\theta)$

$= -x^3 - x + x \sin^2\theta + \sin\theta \cos\theta - \cos\theta \sin\theta + x \cos^2\theta$

$- x^3 - x + x = -x^3$

Similarly, $\Delta_2 = -x^3$

Then, $\Delta_1 + \Delta_2 = -2x^3$

05. (B) Let common ratio of G.P be r

$a_2 = a_1 r,\ a_3 = a_1 r^2, ...a^{10} = a_1 r^9$

$C_1 \to C_1 - C_2,\ C_2 \to C_2 - C_1$

$$\Delta = \begin{vmatrix} \log_e\left(\frac{a_1^r a_2^k}{a_2^r a_3^k}\right) & \log_e\left(\frac{a_1^r a_2^k}{a_2^r a_3^k}\right) & \log_e a_3^r a_4^k \\ \log_e\left(\frac{a_4^r a_5^k}{a_5^r a_6^k}\right) & \log_e\left(\frac{a_5^r a_6^k}{a_6^r a_7^k}\right) & \log_e a_6^r a_7^k \\ \log_e\left(\frac{a_7^r a_8^k}{a_8^r a_9^k}\right) & \log_e\left(\frac{a_8^r a_9^k}{a_9^r a_{10}^k}\right) & \log_e a_9^r a_{10}^k \end{vmatrix}$$

$$\Delta = \begin{vmatrix} \log\frac{1}{r^{r+k}} & \log\frac{1}{r^{r+k}} & \log a_3^r a_4^k \\ \log\frac{1}{r^{r+k}} & \log\frac{1}{r^{r+k}} & \log a_6^r a_7^k \\ \log\frac{1}{r^{r+k}} & \log\frac{1}{r^{r+k}} & \log a_9^r a_{10}^k \end{vmatrix} = 0$$

$\forall r, K \in N$

Hence, number of elements in S is infinitely many.

06. (A) $$\lim_{x\to 0} x\left[\frac{4}{x}\right] = A$$

$$\lim_{x\to 0} x\left[\frac{4}{x} - \left\{\frac{4}{x}\right\}\right] = A$$

$$\lim_{x\to 0} 4 - x\left\{\frac{4}{x}\right\} = A$$

$4 - 0 = A$

As, $f(x) = [x^2]\sin(\pi x)$ will be discontinuous at non-integers

And, when $x = \sqrt{A+1}$

$x = \sqrt{5}$

which is not an integer.

Hence, $f(x)$ is discontinuous when x is equal to $\sqrt{A+1}$

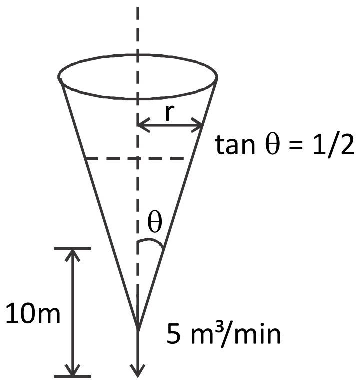
07. (B) Since, f and g both are continuous function on [0, 1] and differentiable on (0, 1) then $3c \in (0, 1)$ such that

$$f'(c) = \frac{f(1)-f(0)}{1} = \frac{6-2}{1} = 4$$

and $g'(c) = \frac{g(1)-g(0)}{1} = \frac{2-0}{1} = 2$

Thus, we get $f'(c) = 2g'(c)$

08. (D) Given that water is poured into the tank at a constant rate of 5 m^3/minute



$\therefore \frac{dv}{dt} = 5\ m^3/min$

Volume of the tank is,

$$V = \frac{1}{3}\pi r^2 h \qquad \_\_\_\_\_\_\_(1)$$

where r is radius and h is height at any time

By the diagram.

$$\tan\theta = \frac{r}{h} = \frac{1}{2}$$

$h = 2r$

$$\frac{dh}{dt} = \frac{2dr}{dt}$$

Differentiate eqn (1) w.r.t 't', we get

$$\frac{dV}{dt} = \frac{1}{3}\left(\pi^2 r\frac{dr}{dt}h + \pi r^2\frac{dh}{dt}\right)$$

Putting h = 10, r = 5 and $\frac{dV}{dt} = 5$ in the above equation,

$$5 = \frac{75\pi}{3}\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{5\pi}\ m/min$$

09. (C) $I = \int \frac{d\theta}{\cos^2\theta(\tan 2\theta + \sec 2\theta)}$

$$= \int \frac{\sec^2\theta}{\frac{1+\tan^2\theta}{1-\tan^2\theta} + \frac{2\tan\theta}{1-\tan^2\theta}}d\theta$$

$$= \int \frac{\sec^2\theta(1-\tan^2\theta)}{(1+\tan\theta)^2}d\theta$$

$$= \int \frac{\sec^2\theta(1-\tan\theta)}{1+\tan\theta}d\theta$$

Let $\tan\theta = t$,

$\sec^2\theta\, d\theta = dt$, then

$$I = \int\left(\frac{1-t}{1+t}\right)dt = \int\left(-1+\frac{2}{1+t}\right)dt$$

$= -t + 2\log(1 + t) + C$

$= -\tan\theta + 2\log(1 + \tan\theta) + C$

Hence, by comparison $\lambda = -1$ and $f(x) = 1 + \tan\theta$

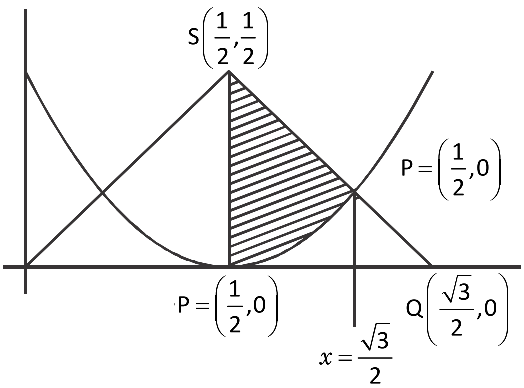
10. (C) $$\int_{\pi/6}^{\pi/3}\left[\frac{1}{2}\frac{d(\tan^4 x)}{dx}\sin^4 3x + \frac{1}{2}\tan^4 x\frac{d(\sin^4 3x)}{dx}\right]dx$$

$$= \frac{1}{2}\int_{\pi/6}^{\pi/3} d(\tan^4 x \sin^4 3x)dx$$

$$= \left[\frac{\tan^4 x \sin^4 3x}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{9.0}{2} - \frac{\frac{1}{9}1}{2} = \frac{-1}{18}$$

11. (B) Coordinates of $P\left(\frac{1}{2}, 0\right)$, $Q\left(\frac{\sqrt{3}}{2}, 0\right)$,

$R\left(\frac{\sqrt{3}}{2}, 1 - \frac{\sqrt{3}}{2}\right)$ and $S\left(\frac{1}{2}, \frac{1}{2}\right)$



Required area = Area of trapezium PQRS

$$\int_{1/2}^{\sqrt{3}/2} \left(x - \frac{1}{2}\right)^2 dx$$

$$= \frac{1}{2}\left(\frac{\sqrt{3}-1}{2}\right)\left(\frac{1}{2} + 1 - \frac{\sqrt{3}}{2}\right) - \frac{1}{3}\left(\left(x - \frac{1}{2}\right)^3\right)_{1/2}^{\sqrt{3}/2}$$

$$= \frac{\sqrt{3}}{4} - \frac{1}{3}$$

12. (A) $\sqrt{1+x^2}.\sqrt{1+y^2} = -xy\frac{dy}{dx}$

$$\int \frac{\sqrt{1+x^2}}{x} dx = -\int \frac{y}{\sqrt{1+y^2}} dy$$

Let $x = \tan\theta$

$dx = \sec^2\theta \, d\theta$

$$\int \frac{\sec^2\theta d\theta}{\tan\theta} = -\int \frac{2y}{2\sqrt{1+y^2}} dy$$

$$\int \frac{\sec^2\theta + \cos^2\theta}{\sin\theta.\cos^2\theta} d\theta = -\sqrt{1+y^2}$$

$$\int (\tan\theta.\sec\theta + \text{cosec}\theta) d\theta = -\sqrt{1+y^2}$$

$$\sec\theta + \log_e|\text{cosec}\theta - \cot\theta| = -\sqrt{1+y^2} + C$$

$$\therefore \quad \sqrt{1+x^2} + \log_e\left(\frac{\sqrt{1+x^2}-1}{x}\right)$$

$$= -\sqrt{1+y^2} + C$$

$$\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$$

13. (B) Let vector be $\lambda\left[\left(\bar{a}+\bar{b}\right)\times\left(\bar{a}-\bar{b}\right)\right]$

Given $\bar{a} = \left(3\hat{i} + 2\hat{j} + 2\hat{k}\right)$ and

$\bar{b} = \left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$

$\therefore \; \bar{a} + \bar{b} = 4\hat{i} + 5\hat{j}$ and $\bar{a} - \bar{b} = 2\hat{i} + 4\hat{k}$

$\therefore$ vector $= \lambda\left[\left(4\hat{i} + 4\hat{j}\right)\times\left(2\hat{i} + 4\hat{k}\right)\right]$

$= \lambda\left[16\hat{i} - 16\hat{j} - 8\hat{k}\right] = 8\lambda\left[2\hat{i} - 2\hat{j} - \hat{k}\right]$

Given that magnitude of the vector is 12.

$\therefore 12 = 8|\lambda|\sqrt{4+4+1} \Rightarrow |\lambda| = \frac{1}{2}$

$\therefore$ required vector is $= \pm 4\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$

14. (B) Let $\vec{v_1}$ and $\vec{v_2}$ be the vectors perpendicular to the plane OPQ and PQR respectively.

$$\vec{v_1} = \overline{PQ}\times\overline{OQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix}$$

$= 5\hat{i} - \hat{j} - 3\hat{k}$

$$\vec{v_2} = \overline{PQ} \times \overline{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ -2 & -1 & 1 \end{vmatrix}$$

$$= \hat{i} - 5\hat{j} - 3\hat{k}$$

$$\because \cos\theta = \frac{\vec{v_1}.\vec{v_2}}{|\vec{v_1}|.|\vec{v_2}|} = \frac{5+5+9}{25+1+9} = \frac{19}{35}$$

$$\therefore \theta = \cos^{-1}\left(\frac{19}{35}\right)$$

15. (C) P(out came is head) = $\frac{1}{2}$

P(outcome is tail) = $\frac{1}{2}$

P(7 or 8 is the sum of two dice)

$$= \frac{6}{36} + \frac{5}{36} = \frac{11}{36}$$

P(7 or 8 is the number of card)

$$= \frac{1}{9} + \frac{1}{9} = \frac{2}{9}$$

Required probability = $\frac{1}{2} \times \frac{11}{36} + \frac{1}{2} \times \frac{2}{9}$

$$= \frac{1}{2}\left(\frac{11+8}{36}\right) = \frac{19}{72}$$

16. (A) A = (3, 6, 9, 12) and (3, 3), (6, 6), (9, 9) (12, 12) $\in$ R

R is reflexive

(6, 12) $\in$ R but (12, 6) $\notin$ R

R is not symmetric

(3, 6) $\in$ R, (6, 12) $\in$ R

(3, 12) $\in$ R .

$\therefore$ R is transitive

17. (B) $f(x + y) = f(x)f(y)$

$f(x) = ax$ for some constant a,

f(1) = 2

a = 2

$f(x) = 2x$

$$\sum_{k=1}^{10} f(a+k) = 16\left(2^{10} - 1\right)$$

$$2^{a+1} + 2^{a+2} + ...+ 2^{a+10} = 16(2^{10} - 1)$$

$$= \frac{2^{a+1}\left(2^{10} - 1\right)}{2-1} = 16(2^{10} - 1)$$

$$2^{a+1} = 16$$

a = 3

18. (B) $\sin^{-1}\frac{\sqrt{3}}{2} = A$; $\sin^{-1}\frac{\sqrt{2}}{3} = B$

$\sin A = \frac{\sqrt{3}}{2}$; $\quad \sin B = \frac{\sqrt{2}}{3}$

$x = \frac{\sqrt{3}}{2}$, $\quad y = \frac{\sqrt{2}}{3}$

$$x^2 + y^2 = \frac{3}{4} + \frac{2}{3} = \frac{17}{12} > 1$$

$\cos A = \frac{1}{2}$; $\cos B = \frac{1}{\sqrt{3}}$, $\frac{\pi}{2} < A + B < \pi$

$$\sin(A + B) = \frac{\sqrt{3}}{2}\frac{1}{\sqrt{3}} + \frac{1}{2}\frac{\sqrt{2}}{3} = \frac{\sqrt{3}+\sqrt{2}}{2\sqrt{3}}$$

$$A + B = \sin^{-1}\left(\frac{\sqrt{3}+\sqrt{2}}{2\sqrt{3}}\right)$$

$$\sin^{-1}\frac{\sqrt{3}}{2} + \sin^{-1}\frac{\sqrt{2}}{3} = \pi - \sin^{-1}\left(\frac{\sqrt{3}+\sqrt{2}}{2\sqrt{3}}\right)$$

19. (B) A_1, A_2, A_3 be the events of selecting bags A,B and C respectively, E be the event of selecting black ball from the selected bag.

Clearly $P(A_1) = \frac{1}{3}$, $P(A_2) = \frac{1}{3}$, $P(A_3) = \frac{1}{3}$

$P(E|A_1) = \frac{3}{5}$, $P(E|A_2) = \frac{2}{6} = \frac{1}{3}$, $P(E|A_3) = \frac{2}{5}$

Required probability $P(E) = P(A_1)P(E|A_1) + P(A_2)P(E|A_2) + P(A_3)P(E|A_3)$

$$\frac{1}{3}\frac{3}{5}+\frac{1}{3}\frac{1}{3}+\frac{1}{3}\frac{2}{5}$$

$$= \frac{1}{3}\left(\frac{3}{5}+\frac{1}{3}+\frac{2}{5}\right).\left(\frac{9+5+6}{15}\right)=\frac{1}{3}.\frac{20}{15}=\frac{4}{9}$$

20. (B) Given vectors lie in a plane

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

$a(0 - c) - a(b - c) + c(c - 0) = 0$

$-ac - ab + ac + c^2 = 0$

$c^2 = ab$

c is the geometric mean of a, b

21. (A) $l + 3m + 5n = 0$

$l = -3m - 5n$

$5lm - 2mn + 6ln = 0$

$5m(-3m - 5n) - 2mn + 6n(-3m - 5n) = 0$

$-15m^2 - 25mn - 2mn - 18mn - 30n^2 = 0$

$-15m^2 - 45mn - 30n^2 = 0$

$m^2 + 3mn + 2n^2 = 0$

$(m + n)(m + 2n) = 0$

$m + n = 0$ or $m + 2n = 0$

If $m + n = 0$ then $m = -n$ and $l = 3n - 5n$

$= -2n$

$l : m : n = -2n : -n : n = 2 : 1 : -1$

If $m + 2n = 0$ then $m = -2n$ and

$l = 6n - 5n = n$

$l : m : n = n : -2n : n = 1 : -2 : 1$

If θ is the angle between the lines then

$$\cos\theta = \left|\frac{2(1)+(1)(-2)+(-1)(1)}{\sqrt{4+1+1}\sqrt{1+4+1}}\right| = \frac{1}{6}$$

$$\theta = \cos^{-1}\left(\frac{1}{6}\right)$$

22. (A) $$\frac{dy}{dx} = \frac{y(\sin x - y)}{\cos x}$$

$$\frac{dy}{dx} = y\tan x - y^2\sec x$$

$$\frac{dy}{dx} - y\tan x = -y^2\sec x$$

$$\frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y}\tan x = -\sec x$$

Put $\frac{1}{y} = \nu$, Then $-\frac{1}{\nu^2}\frac{dy}{dx} = \frac{dy}{dx}$

$$\therefore -\frac{dy}{dx} - \nu\tan = -\sec x$$

$$\frac{dy}{dx} + (\tan x)\nu = -\sec x,$$

Which is linear in ν

I.F = $e^{\int \tan x\,dx} = e^{\ln \sec x} = \sec x$

The solution is $\nu \times \sec x$

$$= \int +\sec^2 x\,dx + k$$

$\nu \times \sec x = +\tan x + C$

$$= \frac{\sec x}{y} = \tan x + C$$

$\sec x = (\tan x + c)y$

23. (B) $$I = \int_0^1 \cos\left(2\cot^{-1}\frac{\sqrt{1-x}}{1+x}\right)dx$$

Put $x = \cos\theta$

$dx = -\sin\theta\, d\theta$; $x = 0, 1$

$\theta = \pi/2, 0$

$$l = -\int_{\pi/2}^{0} \cos\left(2\cot^{-1}\frac{\sqrt{2\sin^2\frac{\theta}{2}}}{2\cos^2\frac{\theta}{2}}\right)\sin\theta\, d\theta$$

$$= \int_{0}^{\pi/2} \cos\left[2\cot^{-1}\left(\tan\frac{\theta}{2}\right)\right]\sin\theta\, d\theta$$

$$= \int_{0}^{\pi/2} \cos\left[2\cot^{-1}\left(\frac{\pi}{2}-\frac{\theta}{2}\right)\right]\sin\theta\, d\theta$$

$$= \int_{0}^{\pi/2} \cos\left[2\left(\frac{\pi}{2}-\frac{\theta}{2}\right)\right]\sin\theta\, d\theta$$

$$= \int_{0}^{\pi/2} \cos(\pi-\theta)\sin\theta\, d\theta$$

$$= -\frac{1}{2}\int_{0}^{\pi/2} 2\sin\theta\cos\theta\, d\theta$$

$$= -\frac{1}{2}\int_{0}^{\pi/2} \sin 2\theta\, d\theta$$

$$= \frac{1}{4}(\cos 2\theta)_0^{\pi/2}$$

$$= \frac{1}{4}(-1-1) = \frac{-1}{2}$$

24. (A) $g(3-) = \left[\frac{k}{2\sqrt{x+1}}\right]_{x=3} = \frac{k}{4}$

$g'(3+) = m$

$\therefore g'(3-) = g'(3+)$

$$\frac{k}{4} = m$$

$$g'(3+) = \lim_{h\to 0}\frac{g(3+h)-g(3)}{h}$$

$$m = \lim_{h\to 0}\frac{m(3+h)+2-2k}{h}$$

$$= \lim_{h\to 0}\frac{(3m-2k+2)+mh}{h}$$

$3m - 2k + 2 = 0$

$3m - 8m + 2 = 0$

$$m = \frac{2}{5},\ k = \frac{8}{5}$$

$$\therefore k + m = \frac{8}{5} + \frac{2}{5} = 2$$

25. (C) $(p\wedge \sim q)\vee q\vee(\sim p\wedge q)$

$\equiv [(p\wedge \sim q)\vee q]\vee(\sim p\wedge q)$

$\equiv [(p\vee q)\wedge(\sim q\vee q)]\vee(\sim p\wedge q)$

$\equiv [(p\vee q)\wedge t]\vee(\sim p\wedge q)$

$\equiv (p\vee q)\vee(\sim p\wedge q)$

$\equiv (p\vee q\vee \sim p)\wedge(p\vee q\vee q)$

$\equiv (q\vee t)\wedge(p\vee q)$

$\equiv t\wedge(p\vee q)$

$\equiv p\vee q$

PHYSICS

26. (C) $R_1 = R,\ R_2 = 2R,\ t_1 = 20°\ C,\ t_2 = ?$

$$\alpha = 3.8 \times 10^{-3}\ °C^{-1},\quad \alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$$

$$3.8 \times 10^{-3} = \frac{2R - R}{Rt_2 - 2R\times 20};$$

$$Rt^2 - 40R = \frac{R}{3.8\times 10^{-3}}$$

$$t_2 = \frac{1000}{3.8} + 40 = 303°\ C$$

27. (A) $E = 100 \cos 100\, t$ volt

$E_0 = 100$ V, $\omega = 100$ rad/s

$R = 10\ \Omega$, $L = 100$ mH $= 10^{-1}$ H

$$\tan\phi = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{100\times 10^{-1}}{10} = 1$$

$$\phi = \frac{\pi}{4}$$

28. (C) Inside a magnet, its magnetic lines of force move from south pole of a magnet towards its north pole.

29. (C) Here, $\vec{E} = 2\times10^3\,\hat{k}\,V/m$

$\vec{ds} = (10\times20)\times10^{-4}\,\hat{k}m^2$

$d\phi = \vec{E}\cdot\vec{ds} = 2\times10^3\,\hat{k}\cdot(10\times20\times10^{-4})\hat{k}$

= 40 V-m

30. (C) In hydrogen atom, $E_n = -\dfrac{Rhc}{n^2}$

Also, $E_n \propto m$, where m is the mass of the electron.

Here, the electron has been replaced by a particle, whose mass is double the mass of an electron. Therefore, for this hypothetical atom, energy in nth orbit will be given by,

$$E_n = -\frac{2Rhc}{n^2}$$

The longest wavelength (or minimum energy) photon will correspond to the transition of particle from n = 3 to n = 2

$$\Rightarrow \frac{hc}{\lambda_{max}} = E_3 - E_2 = 2Rhc\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$= 2\,R\,hc \times \frac{5}{36}$$

$$\therefore \lambda_{max} = \frac{hc}{\frac{5}{18}Rhc} = \frac{18}{5R}$$

31. (B) $n = \dfrac{150}{30\times10^{-2}} = 500\text{ turns/m}$

Total length of the copper wire = L = 2 $\pi r n l$ where l is the length of the solenoid and r its radius

l = 0.30 m, r = 0.03 m

L = 2 × 3.14 × 0.03 × 500 × 0.30 = 28.26 m

Resistance of the wire

= R = 0.01 × 28.26 = 0.2826 Ω

Current = I = $\dfrac{V}{R} = \dfrac{12}{0.2826}$ = 42.46 A

B = $\mu_0 n l = 4\pi \times 10^{-7} \times 500 \times 42.46$

= 0.027 T

32. (A) The critical angle for total internal reflection is computed as follows:

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{1.45}{2.90} = \frac{1}{2} \Rightarrow \theta_c = 30^\circ$$

Total internal reflection can be happen only if the incident beam originates in the medium with the higher index of refraction and strikes teh interface of the other medium at an angle of incidence greater than the critical angle.

33. (C) No. of half lives, $n_1 = \dfrac{80}{20} = 4$

$$n_2 = \frac{80}{40} = 2$$

$$\frac{N_1}{N_0} = \left(\frac{1}{2}\right)^{n_1} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

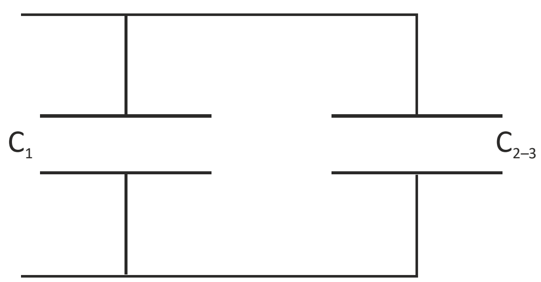
$$\frac{N_2}{N_0} = \left(\frac{1}{2}\right)^{n_2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\frac{N_1}{N_2} = \frac{1}{16}\times\frac{4}{1} = \frac{1}{4}$$

34. (D) From the "crossed" position, Nicol-prism is rotated through 60°. Therefore, angle between two Nicols-prisms will be 30°.

$$\therefore\ I = I_0\cos^2 30^\circ = \frac{3}{4}I_0 = 75\%\text{ of }I_0$$

35. (D) Observe that C_2 and C_3 are in series, and they are in parallel with C_1. That is the capacitor equivalent to the series combination of C_2 and C_3 (which we'll call C_{2-3}) is in parallel with C_1. We can represent this as follows:



So, the first step is to find C_{2-3}:

$$\frac{1}{C_{2-3}}=\frac{1}{C_2}+\frac{1}{C_3}\Rightarrow C_{2-3}=\frac{C_2C_3}{C_2+C_3}$$

Now this is in parallel with C_1, so the overall equivalent capacitance (C_{1-2-3}) is

$$C_{1-2-3}=C_1+C_{2-3}=C_1+\frac{C_2C_3}{C_2+C_3}$$

Substituting in the given numerical values, we get,

$$C_{1-2-3}=(2\,\mu F)+\frac{(4\,\mu F)(6\,\mu F)}{(4\,\mu F)+(6\,\mu F)}=4.4\,\mu F$$

36. (C) Frequency of electromagnetic wave does not change win change in medium for a ray of light but wavelength of wave with change in medium.

$$\nu_{vacuum}=\frac{1}{\sqrt{\mu_0\in_0}}=\nu\lambda_0 \text{ and in medium}$$

$$\nu_{med}=\nu\lambda_m=\frac{1}{\sqrt{\mu_0\mu\in_0\in_r}}$$

or $\nu\lambda_m=\frac{\nu}{\sqrt{\in_r}}=\frac{\nu}{\sqrt{4}}=\frac{\nu}{2}$

$\therefore$ $\frac{\lambda_m}{\lambda_0}=\frac{1}{2}$ or $\lambda_m=\frac{\lambda_0}{2}$

37. (A) Max. K.E. = $h\nu-\phi_0$

$= 6.63\times10^{-34}\times8\times10^{14}-3.2\times10^{-19}$

$= 2.1\times10^{-19}$ J.

38. (B) The coil of a moving coil galvanometer is wound over a metal frame in order to provide electromagnetic damping by which the galvanometer becomes dead beat.

39. (A) For the curved surface, θ = 90°

$\therefore$ ϕ = E ds cos 90° = 0

40. (A) Here, A = 10 cm² = 10×10^{-4} m² = 10^{-3} m²

l = 20 cm = 0.2 m, n_1 = 300, n_2 = 400, M = ?

$$M=\frac{\mu_0n_1n_2A}{l}$$

$$=\frac{(4\pi\times10^{-7})\times300\times400\times10^{-3}}{0.2}$$

$= 2.4\pi\times10^{-4}$ H

CHEMISTRY

41. (B) $Zn + Fe^{2+} \longrightarrow Zn^{2+} + Fe$ (n = 2)

$$E=E^\circ-\frac{0.0591}{n}\log K_c$$

$$0.2905=E^\circ-\frac{0.0591}{2}\log\frac{0.01}{0.001}$$

or E° = 0.2905 + 0.0295 = 0.32 volt

$$E^\circ=\frac{0.0591}{2}\log K_{eq}$$

$$0.32=\frac{0.0591}{2}\log K_{eq}$$

$$K_{eq}=10^{\frac{0.32}{0.0295}}$$

42. (D) $K_2Cr_2O_7 + 4\,KCl + 6\,H_2SO_4 \longrightarrow 2\,CrO_2Cl_2 + 6\,KHSO_4 + 3\,H_2O$

43. (C) CH_3CHO (Acetaldehyde (A)) $\xrightarrow[\text{(ii) } H^+/H_2O]{\text{(i) } CH_3MgI}$ $CH_3-CHOH-CH_3$ (Isopropyl alcohol (2°) (B)) $\xrightarrow{\text{Victor Meyer's test}}$ Blue colour

44. (A) Half of the reaction is completed in 100 seconds

$\therefore$ $t_{1/2}$ = 100 sec.

$\therefore$ $K=\frac{0.693}{100}\sec^{-1}$

For a first order reaction

$$K=\frac{2.303}{t}\log\frac{a}{a-x}$$

a = 100, x = 99, $t_{99\%}$ = ?

$$t_{99\%} = \frac{2.303}{K}\log\frac{100}{100-99}$$

$$=\frac{2.303\times 100}{0.693}\log\ 100\ \text{sec.}$$

$$=\frac{2.303\times 100\times \log 100\ \text{sec}}{0.693} = 664.64\ \text{sec.}$$

45. (D) The structural formula of the complex X is $[Cr(H_2O)_4Br_2]\ Cl.\ H_2O$, one mole of which gives 2 moles of particles $[Cr(H_2O)_5Cl]\ Br_2$, one mole of which gives 3 moles of particles $[Cr(H_2O)_5Cl]^{2+}$ + 2 Br^-

46. (C) $\Delta T_f = \frac{1000\times K_f\times w_2}{w_1\times M_2}$

ΔT_f (canesugar) = 273.15 – 271 = 2.15°

∴ $2.15 = \frac{1000\times K_f\times 5}{95\times 342}$ ..... (i)

$\Delta T_f(\text{Glucose}) = \frac{1000\times K_f\times 5}{95\times 180}$ ..... (ii)

Dividing (ii) by (i), we get,

$\frac{\Delta T_f(\text{Glucose})}{2.15} = \frac{342}{180}$ or ΔT_f (Glucose) = 4.08°

∴ Freezing point of glucose solution

= 273.15 – 4.08 = 269.07 K.

47. (A) As $C_3H_6O_2$ (B) on soda-lime distillation give C_2H_6 therefore, (B) must be a monocarboxylic acid. As (B) is obtained from (A) by loss of a molecule of $CO_2(C_4H_6O_4 - C_3H_6O_2)$, therefore, A must be a 1, 3-dicarboxylic acid in accordance with Blanc's rule. Thus, (A) most be CH_3–$CH(COOH)_2$ and (B) must be CH_3CH_2COOH.

48. (B) From 1 and 4, keeping [B] constant, [A] is made 4 times, rate also becomes 4 times. Hence rate ∝ [A].

From 2 and 3, keeping [A] constant, [B] is doubled, rate becomes 4 times. Hence, rate ∝ $[B]^2$. Overall rate law will be : rate = k [A] $[B]^2$.

49. (B) S_N1 reaction does not involve inversion of configuration of the optically active substrate.

50. (D) In aq. sol, HCl dissociates but in benzene it does not.

51. (A) $\overset{+7}{KMnO_4} \longrightarrow \overset{+4}{MnO_2}$; $\overset{+7}{KMnO_4} \longrightarrow Mn^{2+}$

$\overset{+7}{KMnO_4} \longrightarrow \overset{+3}{Mn(OH)_3}$; $\overset{+7}{KMnO_4} \longrightarrow \overset{6+}{MnO_4^{2-}}$

52. (B)

$$CH_3-CH_2-CH_2-Br \xrightarrow[\text{(Dehydrobromination) (–HBr)}]{\text{Conc. alc. NaOH, 80°C (X)}}$$

$$CH_3CH=CH_2 \xrightarrow[\text{(Mark. addition)}]{\text{HBr, acetic acid, 20°C (Y)}} CH_3\text{-}CHBr\text{-}CH_3$$

53. (B) For emf to be +ve, oxidation should occur at iron electrode.

E_{cell} = 1.23 + 0.44 V = 1.67 V

Δ G° = – nF E°cell = – 2 × 96500 × 1.67 J

= – 322 kJ

54. (A) Molar mass of the complex, $H_{12}O_6Cl_3Cr$ = 266.5 g mol^{-1}

$$P \xrightarrow{H_2SO_4\ (conc.)} \text{No reaction}$$

$$Q \xrightarrow{H_2SO_4\ (conc.)} 6.75\%\ \text{loss in weight}$$

$$R \xrightarrow{H_2SO_4\ (conc.)} 13.5\%\ \text{loss in weight}$$

Assuming the whole of H and O to be present as water (H_2O),

Mass of water in the compound = (12 + 96) g = 108 g

So, No. of molecules of water per molecule of the compound = $\frac{108\ g}{18\ g} = 6$

Then, Loss due to the loss of 1 H_2O =

$\frac{18\ g}{266.5\ g}\times 100$

= 6.75%

Loss due to the loss of 2 H_2O molecules

$$= \frac{2 \times 18}{266.5} \times 100$$

= 13.50%

Thus, complexes Q and R lose one water molecule and two water molecules respectively when treated with concentrated H_2SO_4.

Thus, the formulae of P, Q and R are :

P : $[Cr(H_2O)_6]Cl_3$ - all molecules of water are present in the coordination sphere.

Q : $[Cr(H_2O)_5Cl]Cl_2.H_2O$ - five water molecules and one Cl^- are present in the coordination sphere; one H_2O molecule is loosely held by the compound.

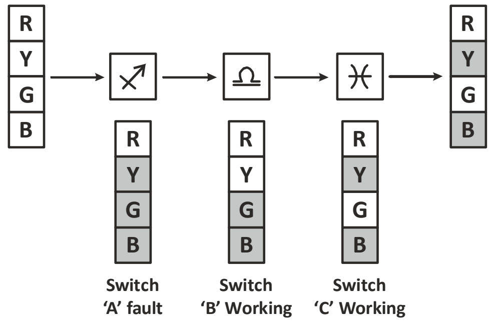
R : $[Cr(H_2O)_4Cl_2]Cl.2H_2O$ - four water molecules and two Cl^- ions are present in the coordination sphere; two H_2O molecules are loosely held by the compound.

55. (D) To produce R–CO–Ar, the acid chloride should be RCOCl and the hydrocarbon should be Ar–H, i.e.,

$$RCOCl + H - Ar \xrightarrow{AlCl_3} RCOAr + HCl$$

CRITICAL THINKING

56. (A)



57. (A) "Humans can be violent towards animals"

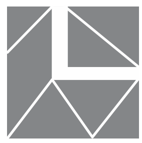
The paragraph presents the following logic: A tiger has been spotted in the empty village.

It is no longer endangered by conflicts with humans.

The missing assumption here is that humans can be violent towards animals.

∴ Option (A) is correct

58. (A)



59. (B) From II, we know that Kiran's mother is married to Janu's husband, which means that Janu is Kiran's mother.

60. (D) All the above statement are true regarding the context of the passage.

The End