01 My house number is a three-digit number. The sum of this number and its three individual digits is 429. What is the product of the three digits which make up the house number ?

The sum of three digits is atmost 27 (as the largest digit is 9)

So, the first digit of the house number is 4.

Hence the second digit is less than 3

If this second digit were even, then the house number and the sum of its digits would have the same parity (i.e., both even or both odd). So, their sum would be even.

However, there sum is odd so the second digit is 1.

Now if the last digit is d then 410 + d + (4 + 1 + d) = 429

So d = 7 and the product of the digits is 28.

02 Integers from 1 to 20 are listed below in such a way that the sum of each adjacent pair is a prime number. Missing numbers are marked as stars.

20, ★ 16, 15, 4, ★, 12, ★, 10, 7, 6, ★, 2, 17, 14, 9, 8, 5, 18, ★.

Which number goes in the place of each star ?

The missing numbers are 1, 3, 11, 13 and 19.

Replacing the stars by letters gives

20, a, 16, 15, 4, b, 12, c, 10, 7, 6, d, 2, 17, 14, 9, 8, 5, 18, e

a cannot be 1 or 11 or 13 or 19, so it must be 3. Leaving 1, 11, 13, 19

b cannot be 1 or 11 or 13, so it must be 19

c cannot be 11 or 13, so it must be 1. Leaving 11, 14

$\Rightarrow$ d = 13 and e = 11

03 The sum of 2021 and a number n is divisible by 23 and 19. Find the smallest of number n.

If (2021 + n) is divisible by 23 and 19, it will then be divisible by the product of 23 and 19 i.e., 437.

We therefore, only need to find a multiple of 437 that is greater than 2021.

$437 \times 4 = 1748$ [Smaller than 2021]

$437 \times 5 = 2185$ [Greater than 2021]

$2185 - 2021 = 164$

$\Rightarrow n = 164$

The smallest value of n is 164

04 If n is the product of two primes whose sum is 999, compute the value of n.

Let the two primes be 'a' and 'b'

Given that $a + b = 999$, which is odd

Since the sum is odd, one of a, b is even and the other must be odd

But we know that there is only one even prime number

$\therefore a = 997$ and $b = 2$

or $a = 2$ or $b = 997$ ($\because a + b = 999$)

$\therefore n = ab = 997 \times 2$

$= 1994$

05 If the number N = 30a0b03 is divisible by 13, then 'a' cannot take which values among the digits 0 to 9.

N = 30a0b03 is divisible by 13

= 3000003 + 10000a + 100b

= $13Q_1 + 6 + 13Q_2 + 3a + 13Q_3 + 9b$

= $13 (Q_1 + Q_2 + Q_3) + (6 + 3a + 9b)$

= $13 (Q_1 + Q_2 + Q_3) + 3(2 + a + 3b)$= 2 + a + 3b should be divisible by 13

as a, b are the digits

2 + a + 3b = 13, 26

we have to try out values of 0 to 9 for a and b

a	8	2	6	0	5	9	3
b	1	3	6	8	2	5	7

a cannot be 1 or 4

06 Find the tens digit in the following product.
4 × 5 × 6 × 7 × 8 × 9 × 10 × 11 × 12 × 13 × 14.

4 × 5 = 20, 20 × 10 = 200

⇒ There are two zeroes at the end of this product as

4 × 5 × 6 × 7 × 8 × 9 × 10 × 11 × 12 × 13 × 14

⇒ 10 × 2 × 6 × 7 × 8 × 9 × 10 × 11 × 12 × 13 × 14

⇒ 2 × 6 × 7 × 8 × 9 × 11 × 12 × 13 × 14 × 100

⇒ tens digit of the product is 0