

01

If $x > \frac{1}{2}$, find the simplest radical form expression for $\frac{1 + \sqrt{2x-1}}{\sqrt{x + \sqrt{2x-1}}}$.

$$\text{Let } x = \frac{1 + \sqrt{2x-1}}{\sqrt{x + \sqrt{2x-1}}}$$

Squaring on both sides of this equation

$$\text{you'll get } y^2 = \frac{1 + 2x - 1 + 2\sqrt{2x-1}}{x + \sqrt{2x-1}} = 2$$

$$\text{Since } y > 0, y = \sqrt{2}$$

02

Find the value of $20202021 \times 20212020 - 20202020 \times 20212021$.

$$\text{Let } a = 20202020; b = 20212020$$

$$20202021 \times 20212020 - 20202020 \times 20212021$$

$$= (a + 1)b - a(b + 1)$$

$$= ab + b - ab - a$$

$$= b - a$$

$$= 20212020 - 20202020$$

$$= 1000$$

03

For what ordered triple of integers (a, b, c) is $x = \frac{3}{\sqrt[3]{7}-2}$ a solution of $x^3 + ax^2 + bx + c = 10$?

$$\text{Since } x = \frac{3}{\sqrt[3]{7}-2} \Rightarrow x(\sqrt[3]{7}-2) = 3 \Rightarrow x\sqrt[3]{7} = 3+2x$$

cubing on both sides, we get

$$7x^3 = 8x^3 + 36x^2 + 54x + 27$$

$$\text{So, } x^3 + 36x^2 + 54x + 27 = 0$$

$$\text{and } (a, b, c) = (36, 54, 27)$$

04

What are all values of x that satisfy $x\sqrt{x} - 2\sqrt{x} = x$?

$$\text{Since } \Rightarrow x\sqrt{x} - 2\sqrt{x} - \sqrt{x}\sqrt{x} = 0$$

$$\Rightarrow \sqrt{x}(x - 2 - \sqrt{x}) = 0$$

$$\therefore \sqrt{x} = 0 \text{ (from which } x = 0) \text{ or } x - 2 = \sqrt{x}$$

squaring on both sides,

$$(x - 2)^2 = (\sqrt{x})^2$$

$$x^2 + 4 - 2x = x$$

$$x^2 - 3x + 4 = 0$$

$$(x - 1)(x - 4) = 0$$

$$\Rightarrow x = 1, 4 \text{ (reject 1)}$$

05

If x is a real number, atmost how many integers can lie between the number

$$x - \frac{2020}{2021} \text{ and } x + \frac{2020}{2021} ?$$

The length of the interval from

$$x - \frac{2020}{2021} \text{ to } x + \frac{2020}{2021} \text{ is } \frac{4040}{2021}$$

Since $1 < \frac{4040}{2021} < 2$, the number of integers

this interval can span is atmost 2

06

If $a^2 \neq b^2$, what are both solutions of $(a^2 - b^2)x^2 - (a^2 + b^2)x + ab = 0$?

Looking at the first and last terms,

$$[(a + b)x - a] [(a - b)x - b] = 0$$

Since the middle term is also correct,

$$x = \frac{a}{a+b}, \frac{b}{a-b}$$