

01

The sum of two positive integers is 52 and their LCM is 168. Find the numbers.

Your solution here:

02

Let  $S = 1^4 + 2^4 + \dots + 2021^4$ . Find the remainder when  $S$  is divided by 10.

Your solution here:

03

What are the remainders when a square of any number is divided by 8.

Your solution here:

04

What are both primes  $p > 0$  for which  $\frac{1}{p}$  has a purely periodic decimal expansion with a period 5 digits long? [NOTE:  $\frac{1}{37} = 0.\overline{027}$  starts to repeat immediately, so it's purely periodic. Its period is 3 digits long.]

Your solution here:

05

A theorem due to Fermat (but first proven by Euler) says that

“Every prime which is 1 more than a multiple of 4 can be written as the sum of two squares in one and only one way”.

For the prime 818101, what are the two positive integers (which in this case, differ by 19) whose squares satisfy this theorem of Fermat ?

Your solution here: