01 **If α, β, γ are the roots of $x^3 - 2x^2 + 4x - 4 = 0$ then find $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$.**

$$\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 = (\alpha\beta + \beta\gamma + \alpha\gamma)^2 - 2$$
$$(\alpha\beta.\beta\gamma + \beta\gamma.\gamma\alpha + \gamma\alpha + \gamma\alpha.\alpha\beta)$$
$$= 9 - 2\alpha\beta\gamma\,(\alpha + \beta + \gamma)$$
$$= 9 - 2\,(4)(2) = 9 - 16 = -7$$

02 **Solve the equation $x^4 - 9x^3 + 27x^2 - 29x + 6 = 0$, one root being $2 - \sqrt{3}$.**

Let $f(x) = x^4 - 9x^3 + 27x^2 - 29x + 6$

$2 - \sqrt{3}$ is a root of $f(x) = 0 \Rightarrow 2 + \sqrt{3}$ is also a root of $f(x) = 0$

$2 - \sqrt{3}, 2 + \sqrt{3}$ are roots of $f(x) = 0$

$\Rightarrow x^2 - 4x + 1$ is a factor of $f(x)$

	1	−9	27	−29	6
4	−	4	−20	24	−
−1	−	−	−1	5	−6
	1	−5	6	0	0

$\therefore \quad x^4 - 9x^3 + 27x^2 - 29x + 6 = 0$

$\Rightarrow (x^2 - 4x + 1)(x^2 - 5x + 6) = 0$

$\Rightarrow x^2 - 4x + 1 = 0$ (or) $x^2 - 5x + 6 = 0$

$\Rightarrow x = 2 \pm \sqrt{3}$ (or) $x = 2, 3$

03 **Solve the equation $x^3 - 6x^2 + 7x + 2 = 0$, one root being $2 + \sqrt{5}$.**

$2 + \sqrt{5}$ is a root $\Rightarrow 2 - \sqrt{5}$ is also a root

The equation having $2 \pm \sqrt{5}$ as roots is $x^2 - 4x - 1 = 0$

$\therefore$ $x^2 - 4x - 1$ is a factor of $x^3 - 6x^2 + 7x + 2$

	1	−6	7	2
4	−	4	−8	−
1	−	−	1	−2
	1	−2	0	0

$\therefore$ $x^3 - 6x^2 + 7x + 2 = 0 \Rightarrow (x^2 - 4x - 1)(x - 2) = 0$

$\Rightarrow x = 2$ or $2 \pm \sqrt{5}$

$\therefore$ The roots are $2 \pm \sqrt{5}$, 2

04 **Solve the equation $x^3 - 11x^2 + 37x - 35 = 0$, one root being $3 + \sqrt{2}$.**

$3 + \sqrt{2}$ is a root $\Rightarrow 3 - \sqrt{2}$ is also a root

The equation having $3 + \sqrt{2}$ as roots is $x^2 - 6x + 7 = 0$

$\therefore x^2 - 6x + 7$ is a factor of $x^3 - 11x^2 + 37x - 35$

	1	−11	37	−35
6	−	6	−30	−
−7	−	−	−7	35
	1	−5	0	0

$x^3 - 11x^2 + 37x - 35 = 0 \Rightarrow (x^2 - 6x + 7)(x - 5) = 0$

$\Rightarrow x = 5$ or $x = 3 + \sqrt{2}$

$\therefore$ The roots are $3 + \sqrt{2}$, 5

05 **Find the roots of $x^4 - 16x^3 + 86x^2 - 176x + 105 = 0$.**

$3 + \sqrt{2}$ is a root $\Rightarrow 3 - \sqrt{2}$ is also a root.

The equation having $3 + \sqrt{2}$ as roots is $x^2 - 6x + 7 = 0$

$\therefore\ x^2 - 6x + 7$ is a factor of $x^3 - 11x^2 + 37x - 35$

	1	−11	37	−35
6		6	−30	
−7			−7	35
	1	−5	0	0

$x^3 - 11x^2 + 37x - 35 = 0 \Rightarrow (x^2 - 6x + 7)(x - 5) = 0$

$\Rightarrow x = 5$ or $x = 3 \pm \sqrt{2}$

$\therefore$ The roots are $3 \pm \sqrt{2}, 5$