01 **The polynomial f(x) has roots of equations 3, –3, –k. Given that the coefficient of x^3 is 2, and that f(x) has a remainder of 8 when divided by $x + 1$, find the value of k. Hence, or otherwise, find the remainder when f(x) is divided by $x - 10$.**

$f(x) = (x - 3)(x + 3)(2x + k)$

Since the coefficient of x^3 is 2

$f(-1) = 8$ if $f(x) = (x^2 - 9)(2x + k)$

$2x^3 + kx^2 - 18x - 9k = 8$

$-2 + k + 18 - 9k = 8$

$k = 1$ (or) $f(-1) = 4$ if $f(x) = (x^2 - 9)(x + k)$

$f(-1) = x^3 + kx^2 - 9x - 9k = 4$

$\Rightarrow -8k = -4 \Rightarrow k = \frac{1}{2}$

$f(x) = (x^2 - 9)(2x + 1)$

$f(x) = 2x^3 + x^2 - 18x - 9$

3	2	1	–18	–9
	0	6	21	9
–3	2	7	3	0
	0	–6	–3	
	2	1	0	

Factors are $(x - 3)(x + 3)(2x + 1)$

Other factor $= \frac{1}{2}$

$f(10) = 2(10^3) + 10^2 - 18(10) - 9 = 1911$

∴ Remainder when divided by

$x - 10$ is 1911

02 **The cubic polynomial f(x) is such that (x + 1), (x – 2), (x + k) are factors of f(x) and the coefficient of x^3 is –2. Given that f(x) has a remainder of 20 when divided by x – 4, find k for {k : k ≠ 0, k < 1}. Hence or otherwise, solve f(x) = 0.**

Given coefficent of x^3 is –2

$\therefore$ f(x) = (x + 1) (x – 2) (–2x + p)

= ($x^2 - x - 2$) (–2x + p)

= $-2x^3 + px^2 + 2x^2 - px + 4x - 2p$

$\therefore$ f(4) = 20

$\Rightarrow -2(4)^3 + (p + 2)(4)^2 - (p - 4)(4) - 2p = 50$

$\Rightarrow$ 10p – 80 = 20

$\Rightarrow$ p = 10

The equation is (x + 1) (x – 2) (x + k)

where k = $\frac{-p}{2} = \frac{-10}{2} = -5$

Hence, the required equation is (x + 1) (x – 2) (x – 5)

$\therefore$ The roots are –1, 2, 5

03 **Find the x-coordinates of the points of intersection of the curve $y = 2x^2 + 1$ and $y = 5x - \frac{2}{x}$. Hence, solve the equation $\frac{t^2+2}{t^3} = \frac{5-2t^2}{t^2}$.**

Given that $y = 2x^2 + 1$ and $y = 5x - \frac{2}{x}$ based on the given information:

$5x^2 - 2 = 2x^3 + x$

$\Rightarrow 2x^3 - 5x^2 + x + 2 = 0$

1	2	−5	1	2
	0	2	−3	−2
2	2	−3	−2	0
	0	4	2	
	2	1	0	

Factors are $(x - 1)(x - 2)(2x + 1)$

$\therefore x = 1, 2, \frac{-1}{2}$

$\therefore \frac{t^2+2}{t^3} = \frac{5-2t^2}{t^2}$

$\Rightarrow 2t^3 + t^2 - 5t + 2 = 0$

Factors are $(t - 1)(t + 2)(2t - 1)$

$\therefore t = 1, -2, \frac{1}{2}$

04 **If $x^2 = y + z$, $y^2 = z + x$ and $z^2 = x + y$, find the value of $\left(\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}\right)$.**

$$x^2 = y + z \Rightarrow x + x^2 = x + y + z$$

$$\Rightarrow x(1 + x) = x + y + z$$

$$y^2 = z + x \Rightarrow y + y^2 = x + y + z$$

$$\Rightarrow y(1 + y) = x + y + z$$

$$z^2 = x + y \Rightarrow z + z^2 = x + y + z$$

$$\Rightarrow z(1 + z) = x + y + z$$

$$\therefore x(1 + x) = y(1 + y) = z(1 + z)$$

$$= x + y + z = k$$

$$\therefore x = \frac{k}{1+x},\ y = \frac{k}{1+y},\ z = \frac{k}{1+z}$$

$$\text{or, } \frac{x}{k} = \frac{1}{1+x},\ \frac{y}{k} = \frac{1}{1+y},\ \frac{z}{k} = \frac{1}{1+z}$$

$$\therefore \left(\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}\right) = \frac{x}{k} + \frac{y}{k} + \frac{z}{k}$$

$$= \frac{1}{k}(x + y + z) = \left(\frac{x+y+z}{x+y+z}\right) = 1$$