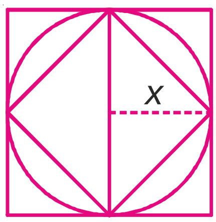
01 **The difference of the area of the circumscribed and the inscribed squares of a circle is 35 sq.cm. Find the area of the circle.**



Let the radius of the circle be x cm

$\therefore$ Diameter = $2x$ cm

Now, diagonal of the inscribed square = $2x$ cm

$\therefore$ Area of the inscribed square

$$= \frac{\text{diagonal}^2}{2} = \frac{(2x)^2}{2} = 2x^2 \text{ sq.cm}$$

Again side of the circumscribed square = $2x$ cm

$\therefore$ Area of the circumscribed square

$= (2x)^2$ sq. cm $= 4x^2$ sq. cm

So, by the problem,

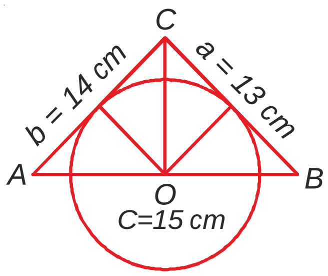
$4x^2 - 2x^2 = 35$ or $2x^2 = 35$

$$x^2 = \frac{35}{2}$$

$\therefore$ Area of circle = px^2

$$= \frac{22}{7} \times \frac{35}{2} = 55 \text{ sq. cm}$$

02 **The sides of a triangle are a = 13 cm, b = 1 4 cm, c = 15 cm, the sides a and b are the tangents to a circle , whose centre lies on the third side. Find the circumference of the circle.**



The centre O of the circle lies an AB and let r cm be the radius of the cirlce.

Since, radius is perpendicular to the tangent at the point of contact.

The area of $\Delta BOC = \frac{1}{2} \times 13 \times r$ sq. cm and $\Delta AOC = \frac{1}{2} \times 14 \times r$ sq. cm

Hence the total area of the $\Delta ABC = \Delta BOC + \Delta AOC$

$= \left(\frac{1}{2}.13.r + \frac{1}{2}.14.r\right)$ sq. cm $= \frac{1}{2} \times r \times 27$ sq. cm

Again if s = semi-perimeter,

$s = \frac{13+14+15}{2}$ cm = 21 cm

The area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(8)(7)(6)} = 84$ sq. cm

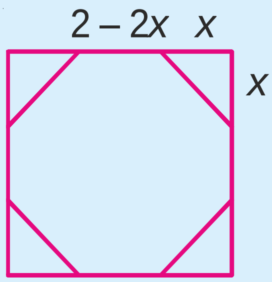
Comparing these two, $\frac{1}{2} \times r \times 27 = 84 \Rightarrow r = \frac{56}{9}$ cm

$\therefore$ Circumference of the circle $= 2\pi r = 2 \times \frac{22}{7} \times \frac{56}{9} = 39\frac{1}{9}$ cm

03

A square, whose side is 2 metres, has its corners cut away so as to form an octagon with all sides equal. Find the length of each side of the octagon, in metres.

Let the position of the edge cut at each corner be x m.
Since the resulting figure is a regular octagon.

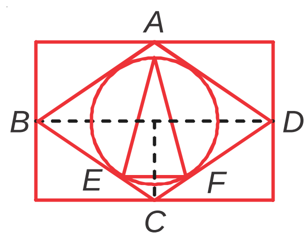


$$\therefore \sqrt{x^2+x^2} = 2-2x \Rightarrow x\sqrt{2} = 2-2x$$

$$\Rightarrow \sqrt{2}\times\left(1+\sqrt{2}\right) = 2 \Rightarrow x = \frac{\sqrt{2}}{\sqrt{2}+1}$$

04

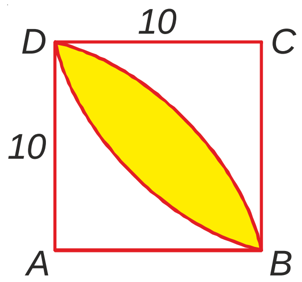
What is the area of the inner equilateral triangle if the side of the outermost square is '*a*'? (ABCD is a square)



BD = a, EF = $\frac{a}{2}$ (Since, EF || BD and EF = $\frac{1}{2}$BD)

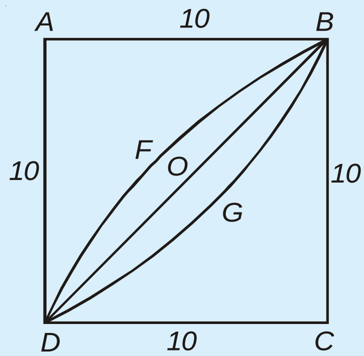
$\therefore$ Area of the equilateral triangle EFG $= \frac{4\sqrt{3}a^2}{64} = \frac{\sqrt{3}a^2}{16}$

05 **In the figure, ABCD is a square with side 10 cm. BFD is an arc of a circle with centre C. BGD is an arc of a circle with centre A. What is the area of the shaded region ?**



Area of the portion

$\Delta FBC = \frac{1}{4} \times \pi \times (10)^2 = 25\pi$



$\therefore$ Area of the $\Delta BCD = \frac{1}{2} \times 10 \times 10 = 50$

$\therefore$ Area of the portion DFBOD

= Are of the portion DFBC – Area of $\Delta BCD = 25\pi - 50$

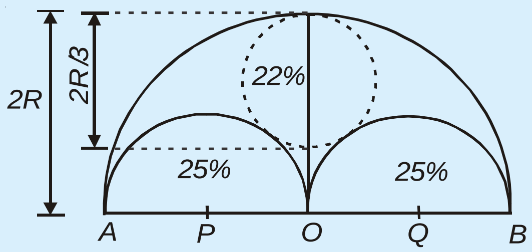
$\therefore$ Area of the portion DFBGD

= 2 × Area of the portion DFBOD

$= 2(25\pi - 50) = 50\pi - 100$

06 **Three horses are grazing within a semi-circular field. In the diagram given, AB is the diameter of the semi-circular field with centre at O. Horses are tied up at P, R and S such that PO and RO are the radii of semi-circles with centres at P and R respectively, and S is the centre of the circle touching the two semi-circles with diameters AO and OB. The horses tied at P and R can graze within the respective semi-circle and the horse tied at S can graze within the circle centred at S. What is the percentage of the area of the semi-circles with diameter AB that cannot be grazed by the horses is nearest to ?**

Roughly taking the radius of smallest circle having radius r as 1/3rd of radius of biggest circle (i.e., radius of semicircular field).



Ratio of radius = 2R: R : R : $\frac{2R}{3}$

Ratio of area = 4 : 1 : 1 : 2$\left(\frac{4}{9}\right)$

= 4 : 1 : 1 : $\frac{8}{9}$

= 100 : 25 : 25 : 22 % urgrazed area

= 100% – (25% + 25% + 22%) = 28%