**01** Two candidates attempt to solve a quadratic equation of the form $x^2 + px + q = 0$. One starts with a wrong value of p and finds the roots to be 2 and 6. The other starts with a wrong value of q and finds the roots to be 2 and –9. Find the correct roots and the equation.

When p is wrong i.e., $\frac{-b}{a}$ $(= \alpha + \beta)$ is wrong but $\frac{c}{a}$ $(= \alpha . \beta)$ is correct

Hence $\alpha\beta = \frac{c}{a} = 2 \times 6 = 12$

Again when q is wrong i.e., $\frac{c}{a} = (\alpha\beta)$ is wrong but $\frac{-b}{a} = \alpha + \beta$ is correct

$\therefore \frac{-b}{a} = \alpha + \beta = 2 + (-9) = -7$

$\therefore$ The required correct quadratic equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - (-7)x + 12 = 0$

$\Rightarrow x^2 + 7x + 12 = 0$

The correct roots of this equation are $-3, -4$

**02** The ratio of the roots of the equation $ax^2 + bx + c = 0$ is same as the ratio of the roots of the equation $px^2 + qx + r = 0$. If $D_1$ and $D_2$ are the discriminants of $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ respectively, find the ratio of $D_1 : D_2$.

Let $\alpha_1, \beta_1$ be the roots of $ax^2 + bx + x = 0$ and $\alpha_2, \beta_2$ be the roots of $px^2 + qx + r = 0$, then,

$$\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} \Rightarrow \frac{\alpha_1 + \beta_1}{\alpha_1 - \beta_1} = \frac{\alpha_2 + \beta_2}{\alpha_2 - \beta_2}$$

$$\Rightarrow \frac{(\alpha_1 + \beta_1)^2}{(\alpha_1 - \beta_1)^2} = \frac{(\alpha_2 + \beta_2)^2}{(\alpha_2 - \beta_2)^2}$$

$$\Rightarrow \frac{(\alpha_1 + \beta_1)^2}{(\alpha_1 + \beta_1)^2 - 4\alpha_1\beta_1} = \frac{(\alpha_2 + \beta_2)^2}{(\alpha_2 + \beta_2)^2 - 4\alpha_2\beta_2}$$

$$\Rightarrow \frac{b^2/a^2}{\frac{b^2 - 4ac}{a^2}} = \frac{q^2/q^2}{\frac{q^2 - 4rp}{p}} \Rightarrow \frac{b^2}{D_1} = \frac{q^2}{D_2}$$

$$\Rightarrow \frac{D_1}{D_2} = \frac{b^2}{q^2}$$

Hence, the ratio of $D_1 : D_2 = b^2 : q^2$

**03** If α, β are the roots of $ax^2 + bx + c = 0$ and α + k, β + k are the roots of $px^2 + qx + r = 0$, what is the value of k ?

We have $\alpha + \beta = \frac{b}{a}$, $\alpha\beta = \frac{c}{a}$

and $\alpha + k + \beta + k = -\frac{q}{p}$

and $(\alpha + k)(\beta + k) = \frac{r}{p}$

$$\Rightarrow \alpha + \beta + 2k = -\frac{q}{p}$$

$$\Rightarrow -\frac{b}{a} + 2k = -\frac{q}{p}\left(\because \alpha + \beta = -\frac{b}{a}\right)$$

$$\therefore \quad k = \frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)$$

04 Solve $(x - 2)(x - 4)(x + 3)(x + 5) = 120$.

Solve $(x - 2)(x - 4)(x + 3)(x + 5) = 120$

$(x - 2)(x + 3)(x - 4)(x + 5) = 120$

$(x^2 + x - 6)(x^2 + x - 20) = 120$

$(a - 6)(a - 20) = 120$ where $a = x^2 + x$

$a^2 - 26a + 120 = 120$

$a^2 - 26a = 0$

$a(a - 26) = 0$

$a = 0$ (OR) $a - 26 = 0$

$x^2 + x = 0$

$x(x + 1) = 0$

$x = 0$ (or) $x = -1$

$x^2 + x - 26 = 0$

$$x = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= -\frac{1 \pm \sqrt{1 - 104}}{2}$$

$$= -\frac{1 \pm \sqrt{105}}{2}$$

## 05

**Solve $3x^4 - 20x^3 - 94x^2 - 20x + 3 = 0$.**

Solve $3x^4 - 20x^3 - 94x^2 - 20x + 3 = 0$

Divide both sides by $x^2$

$$3x^2 - 20x - 94 - \frac{20}{x} + \frac{3}{x^2} = 0$$

$$3x^2 + \frac{3}{x^2} - 20x - \frac{20}{x} - 94 = 0$$

$$3\left(x^2 + \frac{1}{x^2}\right) - 20\left(x + \frac{1}{x}\right) - 94 = 0$$

Let $x + \frac{1}{x} = a$, $\quad x^2 + \frac{1}{x^2} = a^2 - 2$

$3(a^2 - 2) - 20a - 94 = 0$

$3a^2 - 6 - 20a - 94 = 0$

$3a^2 - 20a - 100 = 0$

$\Rightarrow 3a^2 - 30a + 10a - 100 = 0$

$3a(a - 10) + 10(a - 10) = 0$

$(a - 10)(3a + 10) = 0$

$a = 10$ (or) $a = -\frac{10}{3}$