01 **If you save Rs. 1 today, Rs. 2 the next day, Rs. 3 the succeeding day and so on, what will be your total savings in 365 days ?**

1 + 2 + 3 + ... 365

$\therefore$ Sum of 365 natural number is

$$S_{365} = \frac{365 \times 366}{2} \left(S_n = \frac{n(n+1)}{2} \right)$$

$$S_{365} = 66795$$

$\therefore$ Total savings = Rs. 66795

02 **Find the sum to n terms of the series 1 + (1 + 2) + (1 + 2+ 3) + (1 + 2 + 3 + 4) + ...**

The n^{th} term of series is 1 + 2 + 3 + 4 + ... + n

$$= \Sigma_n = \frac{n(n+1)}{2} = \frac{n^2 + n}{2}$$

$$\therefore \quad \text{Sum of n terms} = \Sigma \frac{(n^2 + n)}{2} = \frac{\Sigma n^2 + \Sigma n}{2}$$

$$= \frac{n(n+1) + [2n+1+3]}{12} = \frac{n(n+1)(2n+4)}{12}$$

$$= \frac{n(n+1)(n+2)}{6}$$

03

A bacteria gives birth to two new bacterias in each second and the life span of each bacteria is 5 seconds. The process of the reproduction is continuous until the death of the bacteria. Initially there is one newly born bacteria at time t = 0, find the total number of live bacterias just after 10 seconds.

Total number of bacterias after 10 seconds

$= 3^{10} - 3^{5} = 3^{5}(3^{5} - 1)$

since just after 10 seconds all the bacterias (i.e., 3^5) are dead after living for 5 seconds

04 **Find the sum of n terms of $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + (1^2 + 2^2 + 3^2 + 4^2) + \ldots$ from that find the sum of the first 10 terms.**

n^{th} term of the series $= 1^2 + 2^2 + 3^3 + \ldots\ldots + n^2 = (\Sigma n^2)$

$$= \frac{n(n+1)(2n+1)}{6} = \frac{2n^3 + 3n^2 + n}{6}$$

Sum of n terms of the series

$$= \frac{2\Sigma n^3 + 3\Sigma n^2 + \Sigma n}{6}$$

$$= \frac{2.\frac{n^2(n+1)^2}{4} + 3.\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}}{6}$$

$$= \frac{1}{2}[n(n+1)(n^2 + 3n + 2)]$$

Sum to 10 terms of the series

$$= \frac{1}{2}[10(11)(10^2 + 30 + 2)]$$

$$= \frac{1}{2} \times 10 \times 11 \times 132 = 7260$$

05 **The sum of an infinite geometric series is 162 and the sum of its first n terms is 160. The inverse of its common ratio is an integer, then how many values of the common ratio(r) are possible ?**

$$\frac{a}{1-r} = 162, \quad \frac{a(1-r^n)}{1-r} = 160$$

$$\therefore\ 1 - r^n = \frac{80}{81} \Rightarrow r^n = \frac{1}{81}$$

Now, since $\frac{1}{r} \in Z$

$$\therefore\ \frac{1}{r^n} = 81 \Rightarrow \frac{1}{r} = 3^{4/n}$$

$\therefore\ n = 1, 2, 4$

Here 1, 2, 4 are the factors of 4

n	r	a
1	$\frac{1}{81}$	160
2	$\frac{1}{9}, -\frac{1}{9}$	144 & 180
4	$\frac{1}{3}, -\frac{1}{3}$	108 & 216

Hence, 5 common ratio(r) values are possible

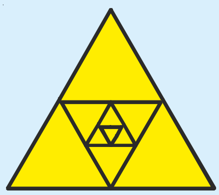
06

One side of an equilateral triangle is 24 cm. The mid-points of its sides are joined to form another triangle whose mid points are in turn joined to form still another triangle. This process continues indefinitely. Find the sum of the perimeters of all the triangles.

Perimeter of the outer most triangle is 72 cm and perimeter of the second outer most triangle is 36 cm.

Similarly perimeter of the third outermost triangle is 18 cm

$\therefore$ Sum of perimeter of all such triangles



$$= 72 + 36 + 18 + 9 + 4.5 + \ldots \infty \quad \left(\text{since, } S_\infty = \frac{a}{1-r}\right)$$

$$= \frac{72}{1-\frac{1}{2}} = \frac{72}{\frac{1}{2}} = 144 \text{ cm}$$