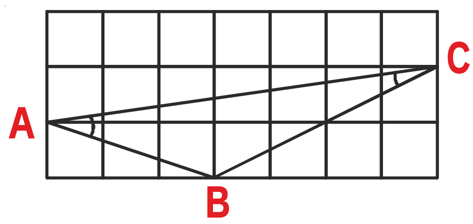
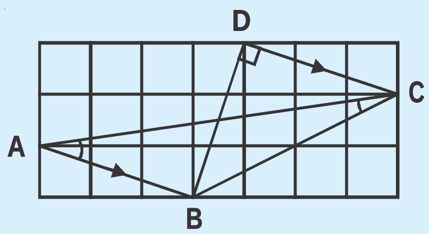
01

In the given figure (made up of unit squares), find the sum of ∠BAC and ∠BCA.



Add in 2 lines DB and DC



Since the grid is made up of unit squares,

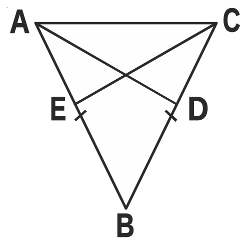
∠BDC = 90º, DC||AB

Thus ∠DCA = ∠BAC (alt. ∠s)

and DB = DC ⇒ ∠DCB = 45°

∠BAC +   = 45°

02 **AD and CE are the medians of the isosceles triangle ABC shown in figure, where AB = CB. Show that the medians are equal in measure.**



Given AB = BC

$\Rightarrow \frac{AB}{2} = \frac{BC}{2}$

$\Rightarrow$ AE = CD $\rightarrow$ (1)

In ΔABC, AB=BC

$\Rightarrow \angle ACB = \angle BAC$ $\rightarrow$ (2)

In ΔACE & ΔACD

$\overline{AE} = \overline{CD}$ [$\because$ side & from eq (1)]

$\angle BAC = \angle ACB$ [$\because$ angle & from eq (2)]

$\overline{AC} = \overline{AC}$ [$\because$ side & common]

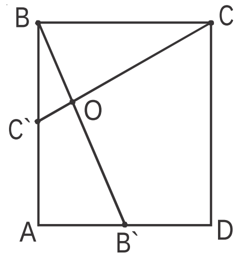
$\therefore \Delta ACE \cong \Delta CAD$ [$\because$ SAS congruency]

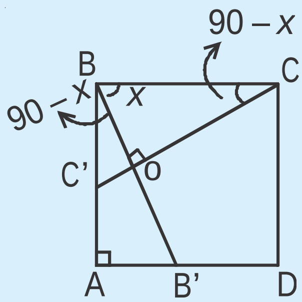
$\therefore \overline{CE} = \overline{AD}$

[$\because$ corresponding parts of congruent triangles]

03

ABCD is a square C' is a point on BA and B' is a point on AD such that BB' and CC' are perpendicular. Show that AB'B and BC'C are congruent.





Given $\angle BOC = 90°$

In ΔBOC, let $\angle OBC = x$

In ΔBOC, $x + 90° + \angle BCO = 180°$

$\angle BCO = 90° - x$ ..... (1)

$\angle ABC = 90°$

$\angle ABB' + x = 90°$

$\angle ABB' = 90° - x$ ..... (2)

In $\Delta ABB'$ & $\Delta BCC'$

$\angle A = \angle ABC = 90°$ [$\because$ angle]

$\overline{AB} = \overline{BC}$ [$\because$ side of square]

$\angle ABB' = \angle BCC'$

[$\because$ angle & from eq (1) & (2)]

$\therefore$ $\Delta ABB' \cong \Delta BCC'$ [$\because$ ASA congruency]

04 Show that the medians of a triangle pass through the same point which divides each of the medians in the ratio 2 : 1.

PQR is a triangle.

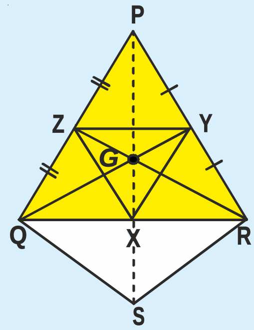
QY and ZR are the medians intersecting at G.

PG when extended meets QR at X

It is required to prove that

i) PX is the third median meeting at G

ii) $\frac{PG}{GX} = \frac{QG}{GY} = \frac{RG}{GZ} = \frac{2}{1}$



Extend PX to S, such that GX = XS

Join QS and RS

Now in ΔPQS, Z is the midpoint of PQ and G is the midpoint of PS,

$\therefore$ ZG || QS $\Rightarrow$ GR || QS ... (1)

Similarly, in ΔPRS, Y is the midpoint of PR and G is the midpoint of PS,

$\therefore$ YG || RS $\Rightarrow$ GQ || RS ... (2)

From (1) and (2), QGRS is a parallelogram

QR and GS are the diagonals. QX = XR (since, diagonals of a parallelogram bisect each other)

Hence X is the midpoint of QR

$\therefore$ PX is the median of ΔPQR

Hence the medians of a triangle pass through the same point i.e., they are concurrent

Also GX = XS $\Rightarrow$ GS = 2GX

and PG = GS $\Rightarrow$ PG = 2GX

$$\Rightarrow \frac{PG}{GX} = \frac{2}{1}$$

Similarly, $\frac{QG}{GY} = \frac{2}{1}$ and $\frac{RG}{GZ} = \frac{2}{1}$

Hence, the medians of a triangle pass through the same point which divides each of the median in the ratio 2 : 1