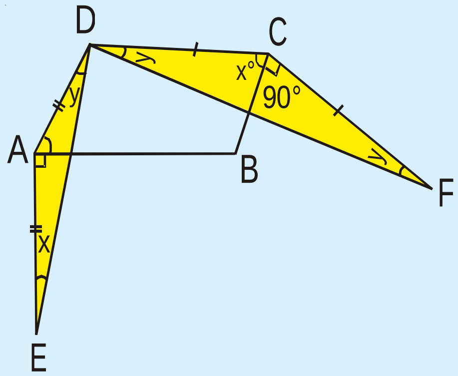
01 **ABCD is a parallelogram. From A, line $\overline{AE}$ is drawn perpendicular to $\overline{AB}$ and equal to $\overline{AD}$, E being on the opposite side of $\overline{AB}$ as D. From C, a line $\overline{CF}$ is drawn perpendicular to BC and equal to CD, F being on the opposite side of $\overline{BC}$ from D. Prove that the angles ADE, CDF are equal and that ∠EDF is a right angle.**



In ΔDAE, AD = AE given

$\therefore \angle ADE = \angle AED$ ... (1)

In ΔDCF, CD = CF given

$\therefore \angle CDF = \angle CFD$ ... (2)

ABCD is a prallelogram

$\angle BAD = \angle BCD$

In Δ^s DAE, DCF,

$\angle ADE + \angle AED = \angle CDF + \angle CFD$

But $\angle ADE = \angle AED$; $\angle CDF = \angle CFD$

$\therefore \angle ADE = \angle AED = \angle CDF = \angle CFD$

$\Rightarrow \angle ADE = \angle CDF$

ABCD is a parallelogram

$\therefore \angle ADC + \angle DCB = 180°$

$\Rightarrow \angle EDF + 2y + x = 180°$ ... (1)

In ΔDCF, $2y + x + 90 = 180°$ ... (2)

From (1) and (2)

$\angle EDF + 2y + x = 2y + x + 90$

$\Rightarrow \angle EDF = 90°$

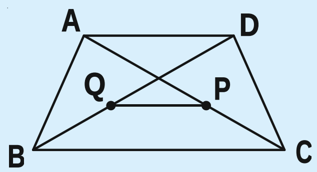
02 **ABCD is a trapezium and P, Q are the mid-points of the diagonals AC and BD. Which of the following is equal to PQ ?**

(A) $\frac{1}{2}(AB)$ **(B)** $\frac{1}{2}(CD)$ **(C)** $\frac{1}{2}(AB - CD)$ **(D)** $\frac{1}{2}(AB + CD)$

Key is (C)

Since AB || DC and transversal AC cuts them at A and C respectively

$\therefore \angle 1 = \angle 2$ ... (i)



[$\therefore$ Alternate angles are equal]

Now, in ΔAPR and ΔDPC, $\angle 1 = \angle 2$

AP = CP [Since, P is the mid-point of AC] and $\angle 3 = \angle 4$

[Vertically opposite angles]

So, $\Delta APR \neq \Delta DPC$ [ASA]

$\Rightarrow$ AR = DC and PR = DP ... (ii)

Again, P and Q are the mid-points of sides DR and DB respectively in ΔDRB

$\therefore PQ = \frac{1}{2}RB = \frac{1}{2}(AB - AR)$

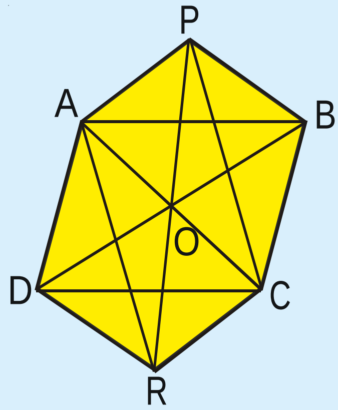
[Since, AR = DC]

$\therefore PQ = \frac{1}{2}(AB - DC)$

03 **ABCD and APCR are the two parallelograms with AC as the common diagonal. Prove that PBRD is a parallelogram.**

Construction

Let the diagonal AC is drawn and "O", the mid-point of AC is taken. PR and BD are also joined.



Since ABCD is a parallelogram the diagonals AC and BD bisect each other at "O".

∴ "O" is the mid-point of AC and also of BD.

Since APCR is a parallelogram, the diagonals AC and PR bisect each other at "O"

∴ "O" is the mid-point of PR [as "O" is the mid-point of AC]

Now, in quadrilateral PBRD, "O" is the mid-point of BD and also of PR. That is BD and PR bisect each other (at "O")

But they are the diagonals of quad PBRD

Hence, PBRD is a parallelogram

Hence it is proved

04 **ABCD is a trapezium in which AB || CD. Which of the following is equal to $AC^2 + BD^2$?**

(A) $AD^2 + BC^2 - 2AB \cdot CD$ **(B)** $AD^2 + BC^2 + 2AB \cdot CD$

(C) $AD^2 - BC^2 + 2AB \cdot CD$ **(D)** $AD^2 - BC^2 - 2AB \cdot CD$

Key is (B)

From C, draw CL $\perp$ AB and from D, drawn DM $\perp$ AB

Then CL = DM

In ΔACB, since $\angle B$ is an acute angle,

$\therefore AC^2 = AB^2 + BC^2 - 2AB.BL$ ... (1)

Similarly, In ΔABD, Since $\angle A$ is an acute angle,

$\therefore BD^2 = AD^2 + AB^2 - 2AB \cdot AM$ ... (2)

Adding (1) and (2), we get

$AC^2 + BD^2 = AD^2 + BC^2 + 2AB^2 - 2AB \cdot BL - 2AB.AM$

$= AD^2 + BC^2 + 2AB(AB - BL - AM)$

$= AD^2 + BC^2 + 2AB(AL - AM)$

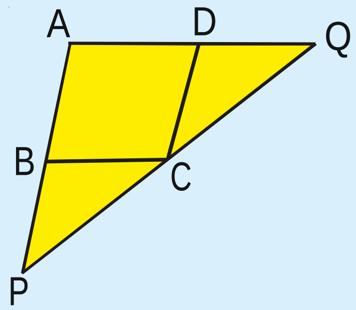
$= AD^2 + BC^2 + 2AB \cdot ML$

$= AD^2 + BC^2 + 2AB \cdot CD$

05 **ABCD is a parallelogram. AB and AD are produced to P and Q respectively such that BP = AB and DQ = AD. Prove that P, C, Q lie on a straight line.**

Construction

CP and CQ are joined



In triangles PBC and CDQ

BC = DQ [since, BC = AD opposite sides of a parallelogram and AD = DQ by hypothesis]

BP = DC

$\angle PBC = \angle CDQ$

[since, $\angle ABC = \angle ADC$, opp $\angle^s$ of a $||^{gm}$ and hence, their supplementary angles are equal]

Δ^s are congruent (SAS congruency)

$\angle BPC = \angle DCQ$ and $\angle BCP = \angle DQC$

Again, $\angle BCD$ = alt $\angle PBC$ [since, AP || DC]

Now, $\angle BCP + \angle BCD + \angle DCQ$

$= \angle BCP + \angle PBC + \angle BPC = 2$ right angles

i.e., $\angle PCQ$ is a straight angle

i.e., P, C, Q lie on a straight line