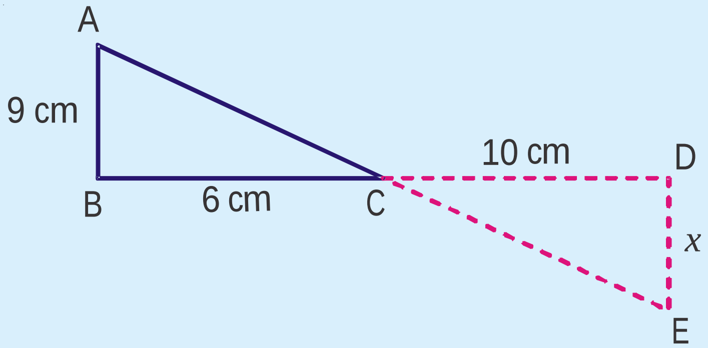
01 **A toy of height 9 cm is placed at a distance of 6 cm from the optic centre of a convex lens. A real image of the toy is formed at a dis tance of 10 cm from the optic centre on the other side of the lens. What is the height of the image ?**



Let height of image be 'x'

Given $\Delta ABC \sim \Delta EDC$

$$\Rightarrow \quad \frac{9\text{ cm}}{ED} = \frac{BC}{DC}$$

$$\Rightarrow \quad \frac{9\text{ cm}}{x\text{ cm}} = \frac{6\text{ cm}}{10\text{ cm}}$$

$$\Rightarrow \quad x = 15\text{ cm}$$

02

If ABC is a right triangle with right angle at B; AC = 2 units, BC = 1 unit and BD is perpendicular to AC. What is the area of the rectangle with BD as one of its diagonal?

$$AB^2 = AC^2 - BC^2 = 4 - 1 = 3$$

$$AB = \sqrt{3}$$

$$ar\left(\Delta ABC\right) = \frac{1}{2} \times AB \times BC$$

$$= \frac{1}{2} \times \sqrt{3} \times 1 = \frac{\sqrt{3}}{2}$$

$$\text{also, } ar(\Delta ABC) = \frac{1}{2} \times BD \times AC$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{1}{2} \times BD \times 2$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = BD$$

Consider DBDF & ΔACB

$\angle BFD = \angle ABC = 90°$

$\angle DBC + \angle DCB = 90° = \angle BAC + \angle BCA$

$\Rightarrow \angle DBF + \angle DCB = \angle BAC + \angle BCD$

$\Rightarrow \angle DBF = \angle BAC$

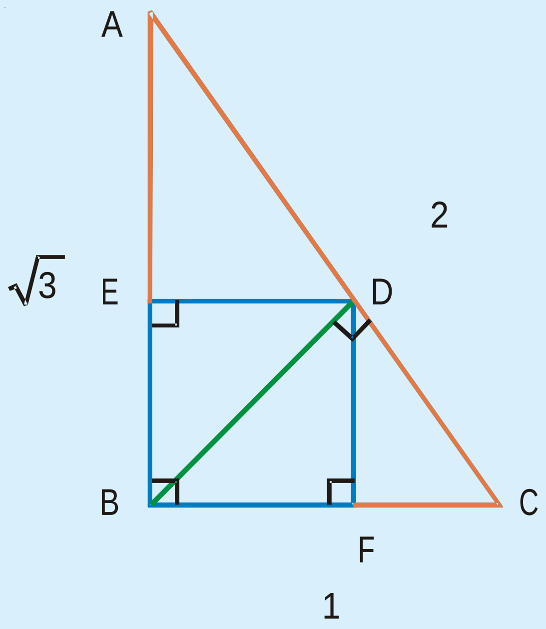
$\Rightarrow \Delta BDF \sim \Delta ACB$

(A.A. axiom of similarity).

$$\Rightarrow \frac{DF}{CB} = \frac{BD}{AC} = \frac{BF}{AB}$$

$$\Rightarrow \frac{DF}{1} = \frac{\sqrt{3}}{4} = \frac{BF}{\sqrt{3}}$$

$$\Rightarrow DF = \frac{\sqrt{3}}{4} \;\&\; BF = \frac{3}{4}$$



$$ar\,(BEDF) = BF \times DF = \frac{3}{4} \times \frac{\sqrt{3}}{4}$$

$$= \frac{3\sqrt{3}}{16}$$

03 **How many triplets (x, y, z) of positive real numbers can be found such that $x^y = z$, $y^z = x$ and $z^x = y$?**

Given $x^y = z \Rightarrow x^{xy} = z^x$

$x^{xy} = y \Rightarrow x^{xyz} = x^z$

$x^{xyz} = x^1 \Rightarrow xyz = 1$

Suppose one of x, y or z, say x is larger than 1.

Since $y^z = x$, either y or z must be larger than 1.

If y is larger than 1, then $z = x^y$ is also larger than 1.

If z is larger than 1, then $y = zx$ is also larger than 1.

Hence, if one of the numbers is larger than 1, they are all, but as the product of the numbers equals 1, they cannot all be larger than 1.

Hence they are all less than or equal to 1. If one of them is less than one, it no follows that the product is less than 1.

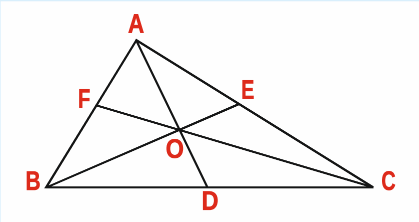
$\therefore$ it must be $x = y = z = 1$

i.e., the number of triplets possible is only one.

04 ABC is a triangle and 'O' is the point of intersection of the medians. Prove that $AB^2 + BC^2 + CA^2 = 3(OA^2 + OB^2 + OC^2)$.

By appolonius theorem $AB^2 + AC^2 = 2(AD^2 + BD^2)$

$$\Rightarrow AB^2 + AC^2 = 2\left(\left(\frac{3}{2}OA\right)^2 + \left(\frac{BC}{2}\right)^2\right)$$



$$\Rightarrow AB^2 + AC^2$$

$$= \frac{9}{2}OA^2 + \frac{1}{2}BC^2 \quad \ldots \quad (1)$$

Similarly

$$BC^2 + BA^2 = \frac{9}{2}OB^2 + \frac{1}{2}AC^2 \quad \ldots \quad (2)$$

and

$$CA^2 + CB^2 = \frac{9}{2}OC^2 + \frac{1}{2}AB^2 \quad \ldots \quad (3)$$

Adding (1), (2) and (3)

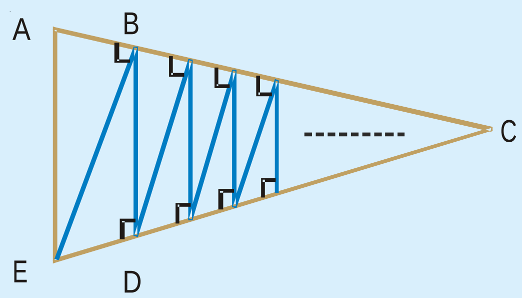
$$2(AB^2 + BC^2 + CA^2) = \frac{9}{2}(OA^2 + OB^2 + OC^2) + \frac{1}{2}(AB^2 + BC^2 + CA^2)$$

$$\Rightarrow \frac{3}{2}(AB^2 + BC^2 + Ca^2) = \frac{9}{2}(OA^2 + OB^2 + OC^2)$$

$$\Rightarrow AB^2 + BC^2 + CA^2 = 3\,(OA^2 + OB^2 + OC^2)$$

05 **In the given figure, an infinite series of similar right triangles converges to point C. If AE = 16, and ED = 8. What is the sum of all the vertical segment (AE + BD + ...) ?**

Since all right triangles are similar



$\Rightarrow \Delta ABE \sim \Delta EDB$

$\Rightarrow \frac{EB}{AE} = \frac{BE}{DB} \Rightarrow \frac{AB}{AE} = \frac{BD}{EB}$

Let $\frac{EB}{AE} = x$, then $EB = AEx = 16x$ and $BD = EBx = 16x^2$

$\Rightarrow AE = 16, BD = 16x^2$, the next vertical segment is $16x^2 \times x^2 = 16x^4$ and soon

Sum of all vertical segments is a infinite geometric series

Also $EB^2 = ED^2 + BD^2$

$\Rightarrow (16x)^2 = 8^2 + (16x^2)^2$

$\Rightarrow 4x^2 = 1 + 4x^4$

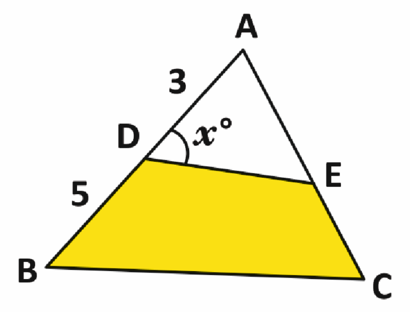
$\Rightarrow (2x^2)^2 - 2(2x^2) + 1$

$\Rightarrow (2x^2 - 1)^2 = 0 \quad \Rightarrow x = \frac{1}{\sqrt{2}}$

$\therefore AE + BD + ... = 16 + 16x^2 + 16x^4 + ... = \frac{16}{1 - x^2}$

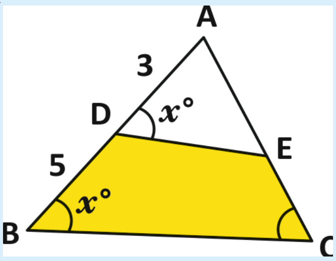
$\Rightarrow \frac{16}{1 - \frac{1}{2}} = 32$

06 **In the given figure, find the value of AE if AC = 5.6 cm.**



Since, $\angle ADE = \angle ABC$

So, DE || BC and



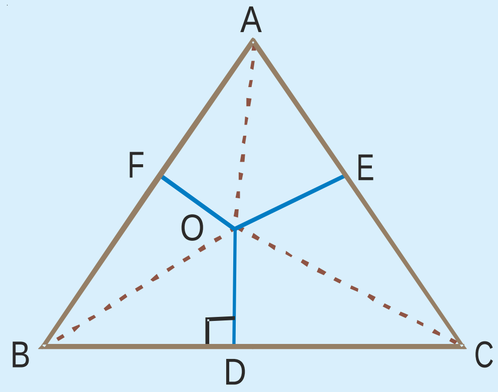
$\Rightarrow \Delta ADE \sim \Delta ABC$

$$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC} \Rightarrow \frac{3}{3+5} = \frac{AE}{5.6}$$

$\Rightarrow AE = 2.1$ cm

Hence, the value of AE is 21 cm

07 **In the given figure, from a point O, OD, OE, OF are drawn perpendicular to the sides BC, CA and AB respectively of a DABC, prove that $BD^2 - DC^2 + CE^2 - EA^2 + AF^2 - FB^2 = 0$.**



$BD^2 = OB^2 - OD^2$

and $DC^2 = OC^2 - OD^2$

$\Rightarrow BD^2 - DC^2 = OB^2 - OC^2$ ... (1)

Similarly,

$CE^2 - EA^2 = OC^2 - OA^2$ ... (2)

$AF^2 - FB^2 = OA^2 - OB^2$ ... (3)

On adding (1), (2) and (3), we get

$BD^2 - DC^2 + CE^2 - EA^2 + AF^2 - FB^2 = 0$