UNIFIED COUNCIL
Foundation for Success

UNIFIED COUNCIL OLYMPIADS

UIMO
Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)

CLASS - 10
Question Paper Code : UM9264

KEY

1	2	3	4	5	6	7	8	9	10
C	B	Del	A	D	A	C	D	B	A
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	D	C	C	A	D	C	C	A	B
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
A	D	B	A	A	A	B	A	C	D
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
A,D	B,C	A,C,D	A,B,D	A,C	C	D	D	D	D
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
B	D	C	D	C	C	D	D	B	A

EXPLANATIONS

MATHEMATICS - 1

01. (C) Given $b^2 = 4ac$

$\therefore \quad = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-b \pm \sqrt{b^2 - b^2}}{2a} = \dfrac{-b}{2a}$

02. (B) Given $\angle A, \angle B, \angle C$ are in AP

$\therefore \quad \angle B = \dfrac{\angle A + \angle C}{2} \Rightarrow \angle A + \angle C = 2\angle B$

But $\angle A + \angle B + \angle C = 180°$

$\therefore \quad 2\angle B + \angle B = 180°$

$3\angle B = 180° \Rightarrow \angle B = \dfrac{180°}{3} = 60°$

03. (Delete)

04. (A) Given $\tan 6\theta = \cot 2\theta = \tan(90° - 2\theta)$

$\therefore \quad \tan 6\theta = 90° - 2\theta$

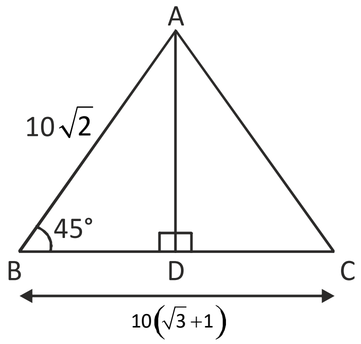
$\therefore \quad 6\theta = 90° - 2\theta$

$\therefore \quad 8\theta = 90°$

$\therefore \quad 4\theta = 45°$

$\therefore \quad \sec 4\theta = \sec 45° = \sqrt{2}$

05. (D)



Construction: $AD \perp BC$

In ΔABD, $\angle D = 90$

$\therefore \quad \sin 45^\circ = \dfrac{AD}{AB}$

$\dfrac{1}{\sqrt{2}} = \dfrac{AD}{10\sqrt{2}\text{ cm}}$

AD = 10 cm

In ΔABD, $\angle D = 90^\circ$ & $\angle B = 45^\circ \Rightarrow \angle BAD = 45^\circ$

$\therefore \quad$ BD = AD = 10 cm

$\therefore \quad$ DC = BC – BD = $(10\sqrt{3} + 10 - 10)$ cm = $10\sqrt{3}$ cm

In ΔADC, $\angle D = 90^\circ \Rightarrow = AC^2 = AD^2 + DC^2$

$= (10\text{ cm})^2 + (10\sqrt{3}\text{ cm})^2$

$= 100\text{ cm}^2 + 300\text{ cm}^2$

$AC = \sqrt{400\text{ cm}^2}$

= 20 cm

06. (A) Given A(–1, –1) B(2, 3) and C(8, 11)

$\therefore \quad$ Area of ABC =

$\dfrac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$

$= \dfrac{1}{2}\left|-1(3 - 11) + 2(11 + 1) + 8(-1 - 3)\right|$

$= \dfrac{1}{2}\left|8 + 24 - 32\right|$

$= \dfrac{1}{2}\left|32 - 32\right|$

$= \dfrac{1}{2} \times 0$

= 0

07. (C)

$(x - 2)\ \big|\ x^4 - 7x^3 + 13x^2 - 5x - 2\ \big|\ x^3 - 5x^2 + 3x + 1$

$x^4 - 2x^3$

(–) (+)

$-5x^3 + 13x^2 - 5x - 2$

$-5x^3 + 10x^2$

(+) (–)

$3x^2 - 5x - 2$

$3x^2 - 6x$

(–) (+)

$x - 2$

$x - 2$

(–) (+)

0

$x - 1\ \big|\ x^3 - 5x^2 + 3x + 1\ \big|\ x^2 - 4x - 1$

$x^3 - x^2$

(–) (+)

$-4x^2 + 3x + 1$

$-4x^2 + 4x$

(+) (–)

$-x + 1$

$-x + 1$

(+) (–)

0

$\therefore \quad x^2 - 4x - 1 = 0$

a = 1 $\quad$ b = –4 $\quad$ c = –1

$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-(-4) \pm \sqrt{16 - (4 \times 1 \times -1)}}{2 \times 1}$

$= \dfrac{4 \pm \sqrt{20}}{2}$

$= \dfrac{4 \pm 2\sqrt{5}}{2}$

$= \left(2 \pm \sqrt{5}\right)$

08. (D) Given $4x^2 + 0x - 1 = 0$

$\therefore \quad$ a = 4 $\quad$ b = 0 $\quad$ & $\quad$ c = –1

$\therefore \quad a + b = -\dfrac{b}{a} = -\dfrac{0}{4} = 0$

09. (B) Let $\frac{1}{\sqrt{x}} = a$ & $\frac{1}{\sqrt{y}} = b$

$\therefore$ $2a + 3b = \frac{13}{6}$

$\Rightarrow$ $12a + 18b = 13 \longrightarrow (1)$

$4a - 9b = \frac{-19}{6}$

$\Rightarrow$ $24a - 54b = -19 \longrightarrow (2)$

eq. (1) × 2 $\Rightarrow$ $24a + 36b = 26$
$24a - 54b = -19 \longrightarrow (2)$
(–) (+) (+)
$90b = 45$

$b = \frac{45}{90} = \frac{1}{2}$

$12a + 18\left(\frac{1}{2}\right) = 13 \longrightarrow (1)$

$12a + 9 = 13$

$12a = 13 - 9 = 4$

$a = \frac{4}{12} = \frac{1}{3}$

$\therefore$ $a = \frac{1}{3} = \frac{1}{\sqrt{x}} \Rightarrow \sqrt{x} = 3$

$\therefore$ $x = 9$

$\therefore$ $b = \frac{1}{2} = \frac{1}{\sqrt{y}} \Rightarrow \sqrt{y} = 2$

$\therefore$ $y = 4$

$2x - 5y = 2 \times 9 - 5 \times 4 = 18 - 20 = -2$

10. (A) Let the three consecutive integers be x, $(x + 1)$ & $(x + 2)$

Given $x^2 + (x + 1)(x + 2) = 277$

$\Rightarrow$ $x^2 + x^2 + 3x + 2 - 277 = 0$

$\Rightarrow$ $2x^2 + 3x - 275 = 0$

$\Rightarrow$ $2x^2 + 25x - 22x - 275 = 0$

$\Rightarrow$ $x(2x + 25) - 11(2x + 25) = 0$

$(2x + 25)(x - 11) = 0$

$x - 11 = 0$ (OR) $2x + 25 = 0$

$x = 11$ (OR) $2x = -25$

$x = \frac{-25}{2}$

$\therefore$ $x = 11$ [$x = \frac{-25}{2}$ is rejected because it is not a positive integer]

$\therefore$ $x + x + 1 + x + 2 = 11 + 12 + 13 = 36$

11. (C) Given 1001, 1005, ____ 9997 are the required numbers which are in Arithmetic progression.

$\therefore$ $a = 1001$, $d = 4$ & $a_n = 9997$

$\therefore$ $a_n = a + (n - 1)d = 9997$

$1001 + (n - 1)4 = 9997$

$(n - 1)\,4 = 9997 - 1001$

$n - 1 = \frac{8996}{4} = 2249$

$n = 2249 + 1 = 2250$

$s_n = \frac{n}{2}[a + a_n]$

$= \frac{2250}{2}[1001 + 9997]$

$= \frac{2250}{2} \times 10998^{5499}$

$= 1,23,72,750$

12. (D) Given $a_7 = 31$ & $a_1 = -5$

$\therefore$ $a + 6d = 31$

$-5 + 6d = 31$

$6d = 36$

$d = 6$

$x_1 = a + d = -5 + 6 = 1,$

$x_2 = x_1 + d$

$x_2 = 1 + 6 = 7$

$x_3 = x_2 + d$

$x_3 = 7 + 6 = 13$

∴ $x_4 = x_3 + d = 13 + 6 = 19$

$x_5 = x_4 + d = 19 + 6 = 25$

∴ $x_1 + x_2 + x_3 + x_4 + x_5 = 1 + 7 + 13 + 19 + 25 = 65$

13. (C) A remainder is always less than devisor but it can be zero also

∴ a = bq + r

where $0 \le r < b$

∴ Option (C) is correct

14. (C) Given (–1) is the zero of $p(x) = x^3 + ax^2 + bx + c$

$p(-1) = (-1)^3 + a(-1)^2 + b(-1) + c = 0$

$-1 + a - b + c = 0$

$c = (b - a + 1)$

But $\alpha\beta\gamma = \frac{-c}{a}$

⇒ $-1 \times \beta r = \frac{-(b - a + 1)}{1}$

∴ $\beta r = (b - a + 1)$

15. (A) Let the present ages of the father and the son be 'x' years and 'y' years respectively

Five years ago, their ages were $(x - 5)$ years and $(y - 5)$ years

According to the problem

$(x - 5) = 7(y - 5)$

⇒ $x - 5 + 35 - 7y = 0$

⇒ $x - 7y + 30 = 0$

⇒ $x = 7y - 30$ ...... (1)

Five years later, their ages will be $(x + 5)$ years and $(y + 5)$ years

According to the problem

$(x + 5) = 3(y + 5)$

⇒ $x + 5 = 3y + 15$

⇒ $x = 3y + 15 - 5$

⇒ $x = 3y + 10$ ...... (2)

From (1) and (2), we have

$7y - 30 = 3y + 10$

⇒ $4y = 40$

⇒ $y = 10$

⇒ $x = 7y - 30$

$= 7(10) - 30$

$= 40$

∴ The present ages of the father and the son are 40 years and 10 years respectively

16. (D) Given 63, 65, 67... are in AP.

a = 63, d = 65 – 63 = 2

$a_n = a + (n - 1)d = 63 + (n - 1)(2) = 63 + 2n - 2 = 61 + 2n$ ⟶ ①

Given 3, 10, 17... are in AP

b = 3 D = 10 – 3 = 7

$b_n = b + (n - 1)D = 3 + (n - 1)7 = 3 + 7n - 7$

$= 7n - 4$

Given $a_n = b_n$

⇒ $= 61 + 2n = 7n - 4$

$61 + 4 = 7n - 2n$

$5n = 65$

$n = 13$

17. (C) Given ABCD is a rectangle

∴ $BD = AC = \sqrt{(11 + 10)^2 + (15 + 5)^2}$

$= \sqrt{21^2 + 20^2}$

$= \sqrt{441 + 400}$

$= \sqrt{841}$

$= 29$

18. (C) In quadrilaterals ABCD and PQRS

$\frac{7}{20} = \frac{z}{16\frac{2}{3}} \Rightarrow \frac{7}{20} = \frac{3z}{50}$

⇒ $z = \frac{35}{6} = 5\frac{5}{6}$ units

19. (A) Given area of ΔABC = 70 square units

$$\therefore \frac{1}{2}\left|\lambda(2\lambda-6+2\lambda)-\lambda+1(6-2\lambda-2-2\lambda)-4-\lambda(2-2\lambda-2\lambda)\right|=70$$

$$\left|\lambda(4\lambda-6)-\lambda+1(4)-4-\lambda(2-4\lambda)\right|=2\times 70$$

$\Rightarrow 4\lambda^2 - 6\lambda - 4\lambda + 4 - 8 + 16\lambda - 2\lambda + 4\lambda^2 = \pm 140$

$8\lambda^2 + 4\lambda - 4 = \pm 140$

$4(2\lambda^2 + \lambda - 1) = \pm 140$

$2\lambda^2 + \lambda - 1 = \pm\frac{140}{4} = 35$

$2\lambda^2 + \lambda - 1 = 35$

$2\lambda^2 + \lambda - 36 = 0$

$2\lambda^2 + 9\lambda - 8\lambda - 36 = 0$

$\lambda(2\lambda + 9) - 4(2\lambda + 9) = 0$

$(2\lambda + 9)(\lambda - 4) = 0$

$\lambda = 4 \in z$ but $\lambda = \frac{-9}{2} \notin z$

(or)

$2\lambda^2 + \lambda - 1 = -35$

$2\lambda^2 + \lambda + 34 = 0$

$\Delta = -b^2 - 4ac$

$= (1)^2 - 4 \times 2 \times 34$

$= 1 - 272$

$\Delta = -271$

$\Delta < 0 \Rightarrow$ No real roots

$\therefore$ One integer satisfies λ value

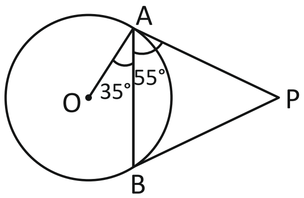
20. (B) $\angle BAP = \angle PAO - 35° = 90° - 35° = 55°$

[∵ A tangent is perpendicular to the radius]

In ΔAPB, AP = PB ⇒ $\angle ABP = \angle BAP = 55°$

In ΔAPB, $55° + 55° + \angle APB = 180°$

$\therefore$ $\angle APB = 180° - 110° = 70°$



21. (A) Given $\tan(A - B) = \frac{1}{\sqrt{3}} = \tan 30°$

$\therefore$ $A - B = 30° \longrightarrow$ (1)

Given $\cos(A + B) = 0 = \cos 90°$

$A + B = 90° \longrightarrow$ (2)

eq. (1) + (2) $\Rightarrow \angle A - \cancel{\angle B} + \angle A + \angle B$

$= 30° + 90°$

$2\angle A = 120°$

$\angle A = 60°$

$60° + \angle B = 90° \longrightarrow$ (2)

$\angle B = 90° - 60° = 30°$

$\therefore$ $\angle A + 2\angle B = 60° + 2 \times 30°$

$= 60° + 60° = 120°$

22. (D) Since, quadrilateral circumscribing a circle then opposite sides subtends supplementary angles at the centre of the circle.

$\therefore$ $\angle AOB + \angle COD = 180°$

$125° + \angle COD = 180°$

$\angle COD = 180° - 125° = 55°$

23. (B) Volume of cuboid = Volume of cylinder

$\Rightarrow$ $lbh = \pi r^2 h$

$\Rightarrow$ $r^2 = \frac{44 \times 30 \times 15 \times 7}{22 \times 28}$

r = 15 cm

Hence radius of the cylinder is equal to 15 cm

24. (A) Diameter of big semicircle

= (42 m + 7 m + 7 m) ÷ 56 m

Radius of big semicircle = $\frac{56\text{ m}}{2} = 28\text{ m}$

Length of rectangle

= 126 m – 28 m – 28 m = 70 m

Total area = $(\frac{22}{7} \times 28\text{ m} \times 28\text{ m}) + (70\text{ m} \times 56\text{ m})$

$= 2464\text{ m}^2 + 3920\text{ m}^2$

$= 6384\text{ m}^2$

Radius of small semicircle

= 42 m ÷ 2 = 21 m

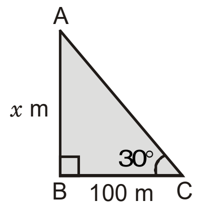
Unshaded area = $(\frac{22}{7} \times 21\text{ m} \times 21\text{ m}) +$ $(70\text{ m} \times 42\text{ m})$

= 1386 m^2 + 2940 m^2

= 4326 m^2

∴ Area of the running track = 6384 m^2 – 4326 m^2 = 2058 m^2

25. (A) Let AB be the height of the building (x m), BC be the distance of the observer from the foot of the building and the angle of elevation is 30°.



Then $\tan 30° = \frac{x}{100}$

$\Rightarrow$ $x = 100 \times \tan 30° = \frac{100}{\sqrt{3}}\text{m} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{100 \times 1.73}{3}$

= 57.66 m

26. (A) Given $\sec\theta + \tan\theta = 2$ .... (1)

but $\sec^2\theta - \tan^2\theta = 1$

$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$2(\sec\theta - \tan\theta) = 1$

$\sec\theta - \tan\theta = \frac{1}{2}$ .... (2)

$\sec\theta + \tan\theta = 2$ .... (1)

$\sec\theta - \tan\theta = \frac{1}{2}$ .... (2)

$2\tan\theta = 2 - \frac{1}{2} = \frac{4-1}{2} = \frac{3}{2}$

$\therefore \tan\theta = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$

27. (B) Given $\sqrt{7}, 3\sqrt{7}, 5\sqrt{7}$ _____ are in AP

∴ $a = \sqrt{7}$ $d = 3\sqrt{7} - \sqrt{7} = 2\sqrt{7}$

$s_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}\left[2\sqrt{7} + (n-1)2\sqrt{7}\right]$

$= \frac{n}{2}\left[\cancel{2\sqrt{7}} + 2\sqrt{7}\,n - \cancel{2\sqrt{7}}\right]$

$= \frac{n}{\cancel{2}} \times n \times \cancel{2}\sqrt{7}$

$s_n = n^2\sqrt{7}$

28. (A) $a = 5, b = -2\sqrt{6}$ $c = -2$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-2\sqrt{6} \pm \sqrt{(-2\sqrt{6})^2 - 4 \times 5 \times -2}}{2(5)}$

$= \frac{2\sqrt{6} \pm \sqrt{24 + 40}}{10}$

$= \frac{2\sqrt{6} \pm 8}{10} = \frac{2(\sqrt{6} \pm 4)}{10}$

$= \frac{4 + \sqrt{6}}{5}$ (OR) $\frac{-4 + \sqrt{6}}{5}$

29. (C)

```
144) 180 (1
     144
     ----
      36) 144 (4
          144
          ---
           0
```

∴ HCF of 144 and 180 = 36

Given 7m + 113 = 36

7m = 36 – 113

7m = –77

$m = \frac{-\cancel{77}^{11}}{\cancel{7}_1}$

m = –11

30. (D) Required polynomial = k[$x^2 - x(\alpha + \beta) + \alpha\beta$] where k is any real number other than zero.

$= k[x^2 - x(-3) - 10]$

$= k(x^2 + 3x - 10)$

$= x^2 + 3x - 10$ (OR) $2x^2 + 6x - 20$

$3x^2 + 9x - 30$ (OR) $\left(\frac{x^2}{2} + \frac{3x}{2} - 5\right)$

MATHEMATICS - 2

31. (A, D)

Given P divides the Join of AB in the ratio 1 : 2

A(–3, 2) B(9, 5) 1 : 2

$$P = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right) = \left(\frac{9-6}{1+2}, \frac{5+4}{3}\right)$$

= (1, 3)

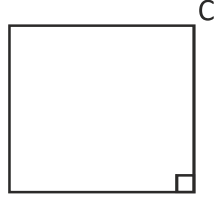
Given Q divides the join of AB in the ratio 2 : 1

$$\therefore Q = \left(\frac{9\times2-3\times1}{3}, \frac{2\times5+2\times1}{3}\right)$$

= (5, 4)

32. (B, C)

Given A(5, 4) & (–1, 6)



let B be (x, y)

AB = BC [Given ABCD is a square]

$$\sqrt{(x-5)^2 + (y-4)^2} = \sqrt{(x+1)^2 + (y-6)^2}$$

squaring on both sides.

$$\cancel{x^2} - 10x + 25 + \cancel{y^2} - 8y + 16 = \cancel{x^2} + 2x + 1 + \cancel{y^2} - 12y + 36$$

$-10x - 2x - 8y + 12y = 37 - 25 - 16$

$-12x + 4y = -4$

$3x - y = 1$

$3x - 1 = y$

But $AC^2 = AB^2 + BC^2$

$(5 + 1)^2 + (4 - 6)^2 = (x - 5)^2 + (y - 4)^2 + (x + 1)^2 + (y - 6)^2$

$36 + 4 = x^2 - 10x + 25 + y^2 - 8y + 16 + x^2 + 2x + 1 + y^2 - 12y + 36$

$2x^2 + 2y^2 - 8x - 20y + 78 = 40$

$2(x^2 + y^2 - 4x - 10y + 39) = 40$

$$x^2 + y^2 - 4x - 10y + 39 = \frac{\cancel{40}^{20}}{\cancel{2}}$$

$x^2 + (3x - 1)^2 - 4x - 10(3x - 1) + 39 = 20$

$\therefore x^2 + 9x^2 - 6x + 1 - 4x - 30x + 10 + 39 = 20$

$10x^2 - 40x + 30 = 0$

$10(x^2 - 4x + 3) = 0$

$x^2 - 4x + 3 = 0$

$x^2 - 3x - x + 3 = 0$

$x(x - 3) -1(x - 3) = 0$

$(x - 3)(x - 1) = 0$

$x = 3$ (or) $x = 1$

If $x = 1$ then $y = 3x - 1 = 2$

one vertex = (1, 2)

If $x = 3$ then $y = 3x - 1 = 8$

other vertex = (3, 8)

33. (A, C, D)

Given lines are parallel $\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\frac{\cancel{3}^1}{\cancel{6}_2} = \frac{+1}{+2} \neq \frac{+p}{+5}$$

$\therefore p \neq \frac{5}{2}$

$\therefore$ 'p' can be real number except $\frac{5}{2}$

$\therefore$ p = 5 or – 5 (or) 0

34. (A, B, D)

Option A sum of the roots = $\frac{-b}{a} = \frac{-3}{-1} = 3$

Option B sum of the roots = $\frac{-b}{a} = \frac{-(-6)}{2} = 3$

Option C sum of the roots = $\frac{-b}{a} = \frac{-15}{5} = -3$

Option D sum of the roots = $\frac{-b}{a} = \frac{-(-9)}{3} = \frac{9}{3} = 3$

35. (A, C)

Three units from B is C (5, 0)

let A be (x, y)

Given AB = AC

$AB^2 = AC^2$

$(x-2)^2 + (y-0)^2 = (x-5)^2 + (y-0)^2$

$\cancel{x^2} - 4x + 4 + \cancel{y^2} = \cancel{x^2} - 10x + 25 + \cancel{y^2}$

$10x - 4x = 25 - 4 = 21$

$6x = 21$

$x = \frac{\cancel{21}^{7}}{\cancel{6}_{2}} = \frac{7}{2}$

$\left(\frac{7}{2}, y\right)$ is 3 units from (2, 0)

$\therefore \sqrt{\left(2-\frac{7}{2}\right)^2 + (0-y)^2} = 3$

$\sqrt{\left(\frac{4-7}{2}\right)^2 + y^2} = 3$

Squaring on both sides $\left(\frac{-3}{2}\right)^2 + y^2 = 9$

$y^2 = 9 - \frac{9}{4} = \frac{36-9}{4}$

$y = \pm\sqrt{\frac{27}{4}} = \pm\frac{3\sqrt{3}}{2}$

REASONING

36. (C) The shapes are moving around the points of the polygon. The circle and arrow are both moving anti-clockwise 2 spaces, and the square is moving 3 spaces in a clockwise direction

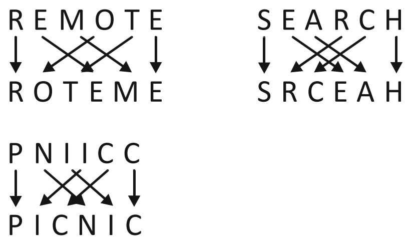
37. (D) Except option (D), remaining options are equally portioned.

38. (D) when P is selected that Z should also be selected and when R is selected than T should also be selected. Thus Z and T will be the other two members of the group. The only option that does not have Z and T is the option (D). So the correct answer is (D).

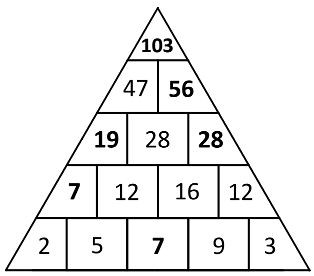
39. (D) More than 17

8 big square + 8 small square + 2 square in middle = 18

40. (D)

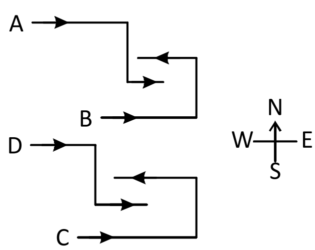


41. (B)



42. (D) 7155 × 7156

43. (C) A = East, B = West, D = East, C = West



44. (D) All urban boys play cricket.

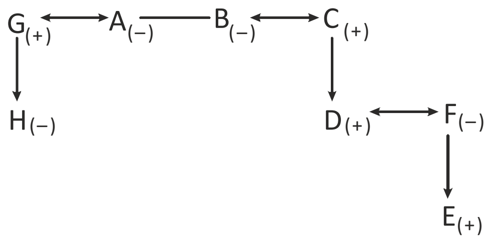
45. (C) (+) → male

(–) → female

⟷ → wife & husband

↓ → son/ daughter

—— → brother / sister

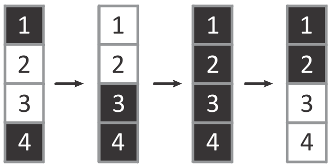


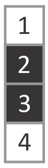
H is niece to B.

CRITICAL THINKING

46. (C) According to the statement, course of action I & II follow the given statement.

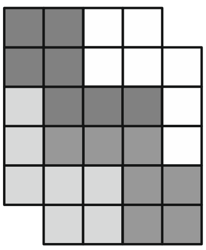
47. (D)



In the question the output is  1 and 3 blocks are in reverse position So, switch Q is fault option(D) is correct.

48. (D)

49. (B)



50. (A) A blended learning approach ensures that the learner is engaged and driving his or her individual learning experience.