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UIMO

Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)

CLASS - 8

Question Paper Code : UM9264

KEY

1	2	3	4	5	6	7	8	9	10
B	B	B	B	A	**Delete**	B	B	C	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
D	A	D	D	C	A	D	B	D	B
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
C	C	A	A	C	C	B	D	A	C
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
A,B,D	B,C,D	A,D	**A,B,C,D**	B,C,D	D	C	A	A	B
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
A	C	B	D	D	A	**C**	C	D	B

EXPLANATIONS

MATHEMATICS - 1

01. (B)

```
     5 | ‾32 ‾26 ‾69 ‾44 ‾16 | 56804
       |  25
       |  ------
   106 |   726
       |   636
       |  ------
  1128 |    9069
       |    9024
       |  ------
 11360 |     4544
       |        0
       |  ------
113604 |   454416
       |   454416
       |  ------
       |      (0)
```

02. (B) Given $x + y = 4$

Cubing on both sides

$x^3 + y^3 + 3xy(x + y) = 64$

$x^3 + y^3 + 3(-2)(4) = 64$

$x^3 + y^3 = 64 + 24 = 88$

and $x + y = 4$

squaring on both sides

$x^2 + y^2 + 2xy = 16$

$x^2 + y^2 + 2(-2) = 16$

$x^2 + y^2 = 16 + 4 = 20$

$$x+\frac{x^3}{y^2}+\frac{y^3}{x^3}+y=\frac{x^3}{x^2}+\frac{x^3}{y^2}+\frac{y^3}{x^2}+\frac{y^3}{y^2}$$

$$=\left(\frac{x^3}{x^2}+\frac{y^3}{x^2}\right)+\left(\frac{x^3}{y^2}+\frac{y^3}{y^2}\right)$$

$$=\left(\frac{x^3+y^3}{x^2}\right)+\left(\frac{x^3+y^3}{y^2}\right)$$

$$=\left(x^3+y^3\right)\left(\frac{1}{x^2}+\frac{1}{y^2}\right)$$

$$=88\times\frac{20}{4}$$

= 440

03. (B) Thosand place can be any single digit whole number except zero.

∴ Thousands place can be filled with 9 numbers Hundreds place can be any single digit whole number except the digit filled in thousands place.

∴ Hundreds place can be filled with 9 numbers

Similarly Tens place can be filled with 8 numbers

Units place can be filled with 7 number

∴ Total number of 4 digit numbers with different digits = 9 × 9 × 8 × 7 = 4536

04. (B) Given $x-\frac{1}{x}=\sqrt{21}$

squaring on both side

$$\left(x-\frac{1}{x}\right)^2=\left(\sqrt{21}\right)^2$$

$$x^2-2x\times\frac{1}{x}+\frac{1}{x^2}=21$$

$$x^2+\frac{1}{x^2}=21+2=23$$

$$x^2+2+\frac{1}{x^2}=23+2$$

$$\Rightarrow\left(x+\frac{1}{x}\right)^2=(5)^2\quad\Rightarrow x+\frac{1}{x}=5$$

$$\therefore\left(x^2+\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)=23\times5=115$$

05. (A) Area of shaded region

$$=\frac{\angle A}{360°}\times\pi r^2+\frac{\angle B}{360°}\times\pi r^2+\frac{\angle C}{360°}\times\pi r^2$$

$$=\frac{\pi r^2}{360°}(\angle A+\angle B+\angle C)$$

$$=\frac{22}{7}\times7\times7cm^2\times\frac{1}{360°}\times180°$$

= 77 cm^2

06. (Delete)

07. (B) Area of four walls = 2h(l + b)

= 2 × 10 m (50 + 40)m

= 1800 m^2

Total cost for colouring

= 1800 m^2 × $\frac{₹15}{m^2}$ = ₹ 27,000

08. (B) If an isosceles triangle with one angle is 60°, then it is equilateral triangle

∴ Area of ΔABC

$$=\frac{\sqrt{3}}{4}a^2=\frac{\sqrt{3}}{4}\times20\times20\ cm^2$$

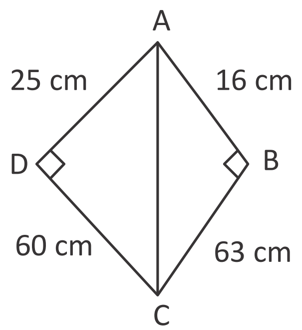
$$=100\sqrt{3}\ cm^2$$

09. (C) Let 20222023 = a, 20222024 = b & 20222025 = c

∴ (a + b)(a – b) + (b – c)(b + c) + (c – a)(c + a)

= $a^2 - b^2 + b^2 - c^2 + c^2 - a^2 = 0$

10. (C) Const :- Join AC



Area of ΔABC $=\frac{1}{2}\times AB\times BC$

$$=\frac{1}{2}\times16\times63\ cm^2$$

= 504 cm^2

Area of ΔACD

$= \frac{1}{2} \times AD \times DC = \frac{1}{2} \times 25\,cm \times 60\,cm = 750\ cm^2$

∴ Area of the quadrilateral ABCD = Area of ΔABC + Area of ΔACD

$= 504\ cm^2 + 750\ cm^2 = 1254\ cm^2$

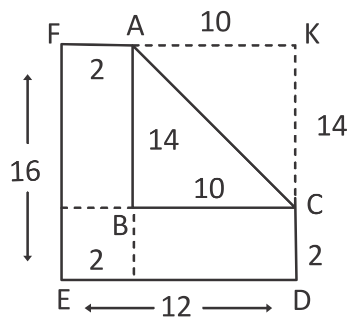
11. (D) $\frac{2^{x+1} + 2^x}{2^{x-1} - 2^x} = \frac{2^x \times 2 + 2^x}{\frac{2^x}{2} - 2^x}$

$= \frac{2^x(2+1)}{2^x\left(\frac{1}{2} - 1\right)}$

$= \frac{3}{\left(\frac{1-2}{2}\right)}$

$= 3 \times \frac{2}{-1} = -6$

12. (A) Cognitive Level: UE



Let $\overline{FA}$ and $\overline{DC}$ extended meet at K.

Area of rectangle FEDK = 12 × 16 = 192 cm^2

Area of ΔAKC $= \frac{1}{2} \times 14 \times 10 = 70\ cm^2$

∴ Required area = 192 – 70 = 122 cm^2

Hence, the correct answer is 122

13. (D) Given $3\angle A = 4\angle B = 6\angle C = k$

∴ $3\angle A = k \Rightarrow \angle A = \frac{k}{3}$

Similarly $\angle B = \frac{k}{4}$ & $\angle C = \frac{k}{6}$

But $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \frac{k}{3} + \frac{k}{4} + \frac{k}{6} = 180°$

$\Rightarrow \frac{4k + 3k + 2k}{12} = 180°$

$9k = 180° \times 12$

$k = \frac{180° \times 12}{9} = 240°$

$\angle A = \frac{k}{3} = \frac{240°}{3} = 80°$

14. (D) x is the number of families with less than 3 children, i.e., the number of families with no children, (= 0 children), 1 child and 2 children.

$\therefore\ x = 2 + 5 + 11 = 18$

15. (C) The second piece is longer than the first piece and the third piece is longer than the second piece. Hence, the shortest piece is the first piece.

Length of first piece = $2x$ cm

Length of second piece = $2(2x) = 4x$ cm

Length of third piece = $(4x + 20)$ cm

Length of rod = $(2x) + (4x) + (4x + 20)$ cm

$= 10x + 20$ cm

$= \frac{10(x+2)}{100}\ m = \frac{x+2}{10}\ m$

16. (A) Clearly, area of unshaded region

= area of circle – area of square

$= \pi r^2 - \frac{1}{2} \times (2r) \times (2r)$

[∵ Area of square = $\frac{1}{2} d^2$]

$= \pi r^2 - 2r^2 = r^2(\pi - 2)$ sq units

17. (D) Given $\pi r^2 h = x\ cm^3$ & $r = y$ cm

$\Rightarrow$ curved surface area = $2\pi rh$

$= 2\pi rh \times \frac{r}{r}$

$= \frac{2\pi r^2 h}{r} = \frac{2x}{y}\ cm^2$

18. (B) Given $x = y^z = (z^x)^z$ $[\because y = z^x]$

$= z^{xz}$

$= (x^y)^{xz}$

$x = x^{xyz}$

$\therefore\ xyz = 1$

19. (D) Sum of the digits = 5 + 6 + 7 + 8 + 9 + 1 +0 + 1 + 1 + 1 + 2 + 1 + 3 + 1 + 4 = 50

If sum of digits is 45 then the given number is divisible by 9

$\therefore$ Remainder = 50 – 45 = 5

20. (B) $\left(x+\frac{1}{x}\right)^2 = \left(x-\frac{1}{x}\right)^2 + 4x\times\frac{1}{x}$

$= \left(\frac{48}{7}\right)^2 + 4$

$= \frac{2304}{49} + 4$

$= \frac{2304+196}{49}$

$= \frac{2500}{49}$

$x+\frac{1}{x} = \sqrt{\frac{2500}{49}} = \frac{50}{7}$

$\therefore x^2 - \frac{1}{x^2} = \left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right) = \frac{48}{7}\times\frac{50}{7} = \frac{2400}{49}$

21. (C) Given AB || DC

$\Rightarrow \angle DAB + \angle ADC = 180°$ $\rightarrow(1)$

But AD || BC

$\Rightarrow \angle DAB + \angle ABC = 180°$ $\rightarrow(2)$

from (1) & (2) $\angle DAB + \angle ADC$

$= \angle DAB + \angle ABC$

$\angle ADC = \angle ABC$

$\therefore \frac{4x}{3} - \frac{x}{2} + \frac{67°}{2} = \frac{x}{2} + \frac{5x}{3} - \frac{53°}{2}$

$\frac{67°}{2} + \frac{53°}{2} = \frac{x}{2} + \frac{5x}{3} - \frac{4x}{3} + \frac{x}{2}$

$= \frac{3x+10x-8x+3x}{6}$

$\frac{120°}{2} = \frac{8x}{6}$

$x = 60°\times\frac{3}{4} = 45°$

$\therefore \angle ABC = \frac{x}{2} + \frac{5x}{3} - \frac{53°}{2} = \frac{45°}{2} + \frac{5\times45°}{3} - \frac{53°}{2}$

$= 75° + \frac{45°-53°}{2}$

$= 75° - \frac{8°}{2} = 75° - 4° = 71°$

22. (C) $\sqrt{10,41,01,209} = 10203$

1	$\overline{10},\overline{41},\overline{01},\overline{2}\overline{09}$ → 10,41,01,209	10203
	1	
20	04	
	0	
202	410	
	404	
2040	612	
	0	
20403	61209	
	61209	
	(0)	

23. (A)

4	2,08,393	456
	16	
85	483	
	425	
906	5893	
	5436	
	457	

24. (A) Let the principal be ₹ p

Given $p\left(1+\frac{r}{100}\right)^n - p = CI$

$\Rightarrow p\left(1+\frac{5}{100}\right) - p =$ ₹ 1324.05

$p\left(\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}\right) - p =$ ₹ 1324.05

$\frac{9261p}{8000} - p =$ ₹ 1324.05

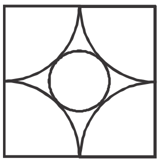
$$\frac{9261p-8000p}{8000}=₹\,1324.05$$

1261p = 8000 × ₹ 1324.05

$$p=\frac{₹\,10592400}{1261}=₹\,8400$$

$$SI=\frac{PTR}{100}=₹\,1260$$

25. (C) Total area = 14 × 14 = 196 m^2



Grazed area = $\left(\frac{\pi\times r^2}{4}\right)\times 4=\pi r^2$

= 22 × 7 m^2 = 154 m^2

Ungrazed area = (196 – 154) = 42 m^2

Ungrazed area excluding pond area

= 42 m^2 – 20m^2 = 22m^2

26. (C) $\frac{3}{4}(a+y)\left[y+a-\frac{1}{3}\left(y+a-\frac{1}{4}a-\frac{1}{4}y\right)\right]$

$$=\frac{3}{4}(a+y)\left[y+a-\frac{y}{3}-\frac{a}{3}+\frac{a}{12}+\frac{y}{12}\right]$$

$$=\frac{3}{4}(a+y)\left[\frac{2y}{3}+\frac{2a}{3}+\frac{a}{12}+\frac{y}{12}\right]$$

$$=\frac{3}{4}(a+y)\left[\frac{9y+9a}{12}\right]=\frac{3}{4}(a+y)\left[\frac{9}{12}(y+a)\right]$$

$$=\frac{9}{16}(a+y)^2$$

27. (B) $\frac{a^2-(b-c)^2}{(a+c)^2-b^2}=\frac{[a+(b-c)][a-(b-c)]}{(a+c-b)(a+b+c)}$

$$=\frac{(a+b-c)(a-b+c)}{(a+b+c)(a-b+c)}$$

$$=\frac{a+b-c}{a+b+c}$$

$$\therefore\ \frac{b^2-(a-c)^2}{(a+b)^2-c^2}=\frac{(b-a+c)(b+a-c)}{(a+b+c)(a+b-c)}$$

$$\frac{c^2-(a-b)^2}{(b+c)^2-a^2}=\frac{(c-a+b)(c+a-b)}{(a+b+c)(b+c-a)}=\frac{(c+a-b)}{(a+b+c)}$$

∴ Required result

$$=\frac{(a+b-c)}{a+b+c}+\frac{(b-a+c)}{a+b+c}+\frac{(c+a-b)}{(a+b+c)}$$

$$=\frac{a+b-c+b-a+c+c+a-b}{(a+b+c)}$$

$$=\frac{(a+b+c)}{(a+b+c)}=1$$

28. (D) Let the actual distance be x km. Then, more distance on the map, more is the actual distance.

$$\frac{x_1}{y_1}=\frac{x_2}{y_2}$$

$$\frac{0.6cm}{6.6km}=\frac{80.5cm}{xkm}\Rightarrow 0.6x=80.5\times 6.6$$

$$\Rightarrow x=\frac{80.5\times 6.6}{0.6}\Rightarrow x=885.5\text{ km}$$

29. (A) $\frac{-1}{2\times2\times2}+\frac{2\times2\times2-1+3\times3}{3\times3-2\times2}+\frac{7\times7\times7}{11\times11\times11}\times\frac{121}{98}$

$$=\frac{-1}{8}+\frac{8-1+9}{9-4}+\frac{7}{22}$$

$$=\frac{-1}{8}+\frac{16}{5}+\frac{7}{22}$$

$$=\frac{-55+1408+140}{440}$$

$$=\frac{1493}{440}=3\frac{173}{440}$$

30. (C) $SP=\frac{MP(100-d)}{100}$

$$₹x=\frac{MP(100-y)}{100}$$

$$\therefore MP=₹\frac{100x}{(100-y)}$$

MATHEMATICS - 2

31. (A, B, D)

In a square & rhombus the diagonals bisect each other perpendicularly

In a kite the diagonals intersect at right angle

32. (B, C, D)

$13^4 - 11^4 = (13^2)^2 - (11^2)^2$

$= (13^2 + 11^2)(13^2 - 11^2)$

$= (169 + 121)(13 - 11)(13 + 11)$

$= 290 \times 2 \times 24$

$87 = 29 \times 3$ & $696 = 29 \times 24$

$\therefore$ 29,87 & 696 are the factors of $(13^4 - 11^4)$

33. (A, D)

$x^2 + x - 6 = x^2 + 3x - 2x - 6 = (x + 3)(x - 2)$

$x^2 - 2x - 15 = x^2 - 5x + 3x - 15$

$= (x - 5)(x + 3)$

$\therefore$ LCM $= (x - 2)(x + 3)(x - 5)$

34. (A, B, C, D)

If $l = 12$ cm & b = 4 cm then side of square $= l$ + b

ABCD = 16 cm

$\therefore$ Possible Area = $(16 \text{ cm})^2 = 256 \text{ cm}^2$

If $l = 24$ cm & b = 2 cm then side of ABCD = 26 cm

Possible area = $(26 \text{ cm})^2 = 676 \text{ cm}^2$

If $l = 8$ cm and b = 6 cm then side of ABCD = 14 cm

$\therefore$ Possible area = $(14 \text{ cm})^2 = 196 \text{ cm}^2$

If the side of a square ABCD = 25 cm,

then area of square ABCD = $(25 \text{ cm})^2 = 625 \text{ cm}^2$

35. (B, C, D)

A square has 4 lines of symmetry

A rectangle has 2 lines of symmetry

A rhombus has 2 lines of symmetry

An isosceles triangle has one line of symmetry

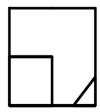
$\therefore$ Option B, C & D have almost three lines of symmetry

REASONING

36. (D) By examining the series right to left, we can see that the third number is the product of the first two: i.e. 5 x 4 = 20. If we continue to apply that rule, we see that if we multiply the third and fourth numbers, we get the fifth number: 20 x 3 = 60. So, the final number must be a product of the fifth and sixth numbers: i.e. 60 × ? = 240; and this resolves as 240 / 60 = 4

37. (C) Simple : The rest have alternate consonant/ vowel arrangement.

38. (A) The relationship between figure 1 and figure 2 is as follows:



1) Figure 2 represents figure 1 rotated 90° clockwise

2) The dark triangle becomes white.

The correct answer must have the same relationship with figure 3.

Answers (C) and (D) can be eliminated, as they represent the shapes in figure 3 in new locations; however, they are not rotations of the entire figure.

Option (B) can be eliminated, as it is a 90° counterclockwise rotation of figure 3.

We are left with answer (A), which the correct answer portrays figure 3 rotated 90° clockwise, while the dark triangle becomes white.

39. (A) Hint : Correct option is rotated at a 45 degree CCW angle to the shape.

40. (B) Four vertical lines = G

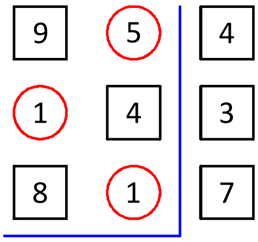
Three vertical lines = H

Two horizontal lines = I

One horizontal line = J

So we need to find the code for three vertical lines (H) followed by one horizontal line (J). Only option (B) is correct.

41. (A) The intention of the problem is that squares are positive and circles are negative. You can then sum rows and columns, and everything adds up. The line helps to show that the top left 2 × 2 array is the thing being summed.



42. (C) 4 - Saturdays 3 - Fridays

2 - Thursdays 1 - Wednesday

30 - Tuesdays

30, 23, 16, 9, 2 dates of June is Tuesday.

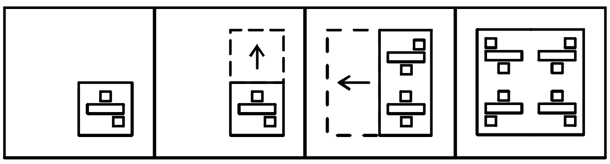
43. (B) Pairs of opposite faces are (1 and 4), (2 and 6) and (3 and 5), in option (A) 2 is shown adjacent to 6, in option (C), 3 is shown adjacent to 5 and in option (D), 1 is shown adjacent to 4. Hence, these alternatives are not possible. Only the cube given in option (B) can be formed because 6, 5 and 4 can be on adjacent faces. Hence, option (B) is correct.

44. (D) Prithvi reached 1:30

Harsha reached hour and a half before Prithvi means = 1:30 – 1:30 = 12:00

Ram reached 15 after Harsha means 12:15

45. (D)

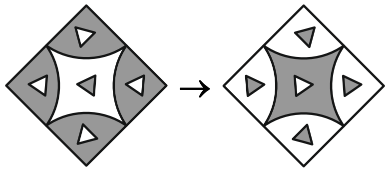


CRITICAL THINKING

46. (A) (A) is true, (R) is true and (R) is the correct explanation of (A)

47. (C) This is a simple physics question. Higher density liquids will settle down, while the lower density liquid will be on top of that; if both the liquids are not mixing and at the same time water occupies less space than ice i.e., water expands when it freezes.

48. (C)



49. (D) If Advik plays basketball, then Arya sits on the bench.

If Arya sits on the bench, then Akash starts at center.

50. (B)