UNIFIED COUNCIL
Foundation for Success

UNIFIED COUNCIL OLYMPIADS

UIMO
Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)

CLASS - 9
Question Paper Code : UM9264

KEY

1	2	3	4	5	6	7	8	9	10
B	B	C	C	B	C	C	B	C	D
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A	C	B	B	A	D	A	C & D	C	C
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
D	D	D	A	B	A	A	B	C	A
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
A,D	A,B,C,D	A,B	A,B,D	A,C,D	D	C	A	C	B
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
A	B	C	B	D	C	A	A	B	B

EXPLANATIONS

MATHEMATICS - 1

01. (B) $\sqrt{\left(3+\sqrt{2}\right)\left(12-\sqrt{32}\right)}$

$=\sqrt{\left(3+\sqrt{2}\right)\left(12-4\sqrt{2}\right)}$

$=\sqrt{\left(3+\sqrt{2}\right)(4)\left(3-\sqrt{2}\right)}$

$=\sqrt{4\times\left[3^2-\left(\sqrt{2}\right)^2\right]}$

$=2\sqrt{7}$

02. (B) LHS = $\left[\frac{1}{(x-5)(x-3)}+\frac{1}{(x-3)(x-1)}-\frac{2}{(x-5)(x-1)}\right]$

$\left[\frac{(x-1)+(x-5)-2(x-3)}{(x-1)(x-3)(x-5)}\right]$

$=\frac{2x-6-2x+6}{(x-1)(x-3)(x-5)}$

= 0

03. (C) Given $3\pi r^2 = 115.5\ cm^2$

$$3\times\frac{22}{7}\times r^2=\frac{231}{2}\ cm^2$$

$$r^2=\frac{231}{2}cm^2\times\frac{7}{22}\times\frac{1}{3}$$

$$r=\sqrt{\frac{49}{4}}\ cm$$

$$r=\frac{7}{2}\ cm$$

$$\text{Volume}=\frac{2}{3}\pi r^3$$

$$=\frac{2}{3}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times\frac{7}{2}\ cm^3$$

$$=\frac{539}{6}cm^3$$

$$=89\frac{5}{6}cm^3$$

04. (C) Given

s – a = 60 cm, s – b = 15 cm & s – c = 5 cm

∴ s – a + s – b + s – c = (60 + 15 + 5) cm

3s – 2s = 80 cm

s = 80 cm

Area of ΔABC

$$=\sqrt{s(s-a)(s-b)(s-c)}$$

$$\sqrt{80\times60\times15\times5}$$

$$\sqrt{20\times4\times20\times3\times5\times3\times5}$$

$= 20 \times 2 \times 3 \times 5\ cm^2$

$= 600\ cm^2$

05. (B) $\left(2\sqrt{2}+3\sqrt{3}\right)^2$

$$\left(2\sqrt{2}\right)^2+2\times2\sqrt{2}+3\sqrt{3}+\left(3\sqrt{3}\right)^2$$

$$=8+12\sqrt{6}+27$$

$$=35+12\sqrt{6}$$

06. (C) $\dfrac{1}{\sqrt{7}+\sqrt{6}-\sqrt{13}}$

$$=\frac{1}{\left(\sqrt{7}+\sqrt{6}\right)-\left(\sqrt{13}\right)}\times\frac{\left(\sqrt{7}+\sqrt{6}\right)+\sqrt{13}}{\left(\sqrt{7}+\sqrt{6}\right)+\left(\sqrt{13}\right)}$$

$$=\frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{\left(\sqrt{7}+\sqrt{6}\right)^2-\left(\sqrt{13}\right)^2}$$

$$=\frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{7+6+2\sqrt{42}-13}$$

$$=\frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{13+2\sqrt{42}-13}$$

$$=\frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{2\sqrt{42}}$$

$$=\frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{2\sqrt{42}}\times\frac{\sqrt{42}}{\sqrt{42}}$$

$$=\frac{\sqrt{7}\times\sqrt{42}+\sqrt{6}\times\sqrt{42}+\sqrt{13}\times\sqrt{42}}{2\left(\sqrt{42}\right)^2}$$

$$=\frac{7\sqrt{6}+6\sqrt{7}+\sqrt{546}}{2\times42}$$

$$=\frac{7\sqrt{6}+6\sqrt{7}+\sqrt{546}}{84}$$

07. (C) Given $3x + 7° + 2x – 19° + x = 180°$

$6x = 180° + 12°$

$$x=\frac{192°}{6}=32°$$

∴ $\angle COD = 2x – 19° = 64° – 19° = 45°$

08. (B) $x^2 – y^2 + z^2 – p^2 – 2yp – 2zx$

$= (x^2 + z^2 – 2zx) – (y^2 + p^2 + 2yp)$

$= (x – z)^2 – (y + p)^2$

$= (x – z + y + p)(x – z – y – p)$

09. (C) LHS = $6x(x^2 - 4y^2) - 3y(x^2 - 4y^2)$

$= (x^2 - 4y^2)(6x - 3y)$

$= (2 - 2y)(x + 2y)(3)(2x - y)$

$= 3(2x - y)(x + 2y)(x - 2y)$

10. (D) Given $(x + 1)$ is a factor of $p(x)$

$= x^{2023} - 3x^{2022} + k$

$p(-1) = 0$

$p(-1) = (-1)^{2023} - 3(-1)^{2022} + k = 0$

$-1 - 3 + k = 0$

$k = 4$

11. (A) Given $(2x - 3)$ is a factor of $p(x)$

$= 2x^3 - x^2 + mx + n$

$\therefore p\left(\frac{3}{2}\right) = 0$

$2\left(\frac{3}{2}\right)^3 - \left(\frac{3}{2}\right)^2 + m\left(\frac{3}{2}\right) + n = 0$

$\Rightarrow 2 \times \frac{27}{8} - \frac{9}{4} + \frac{3m}{2} + n = 0$

$\frac{27 - 9 + 6m + 4n}{4} = 0$

$18 + 6m + 4n = 0 \times 4$

$6m + 4n = -18$

$2(3m + 2n) = -18$

$3m + 2n = -9$

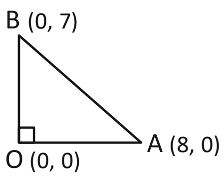
12. (C) Given $x = -3$ and $y = 4$ is the solution of $5x + 3y = k$

$\therefore$ $5(-3) + 3(4) = k$

$-15 + 12 = k$

$k = -3$

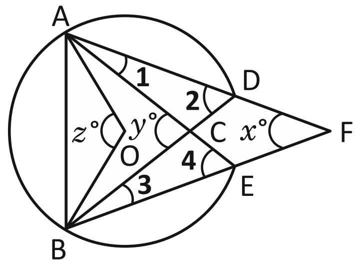
13. (B) It is a right angled triangle of base 8 units and height 7 units



$\therefore$ Area of $\Delta AOB = \frac{1}{2} \times bh = \frac{1}{2} \times 8 \times 7$ sq. units

= 28 square units

14. (B) Consider the following figure



Using the exterior angle theorem, we get

$\angle y = \angle 1 + \angle 2$ .... (1)

$\angle 1 + \angle x = \angle 4$ .... (2)

Add equations (1) and (2)

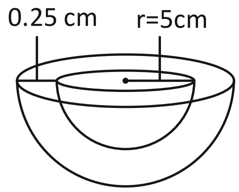
$\angle y + \angle 1 + \angle x = \angle 1 + \angle 2 + \angle 4$

$\Rightarrow \angle x + \angle y = \angle 2 + \angle 4$

Now, since $\angle 2 = \angle 4 = \frac{z}{2}$, $x + y = z$

$\Rightarrow x = z - y$

15. (A) Consider the figure



Since thickness of the bowl is 0.25 cm, the outer radius R of the bowl is 5 + 0.25 = 5.25 cm

$V_{inner} = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(5)^3 cm^3$

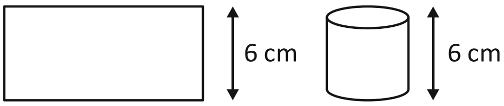
$\therefore$ Volume of steel used = $V_{outer} - V_{inner}$

$= \frac{2}{3}\pi(5.25)^3 - \frac{2}{3}\pi(5)^3$

$= \frac{2}{3} \times 3.14 \times (5.25^3 - 5^3)$

$= 41.25\ cm^3$

16. (D) Since the height of the cylinder is 6 cm, the circumference of the base becomes 22 cm.



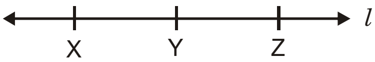
Let r be the radius of its base. Then $2\pi r$ = 22 cm

$\Rightarrow r = \frac{7}{2}$cm

∴ Volume of cylinder $= \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 6$

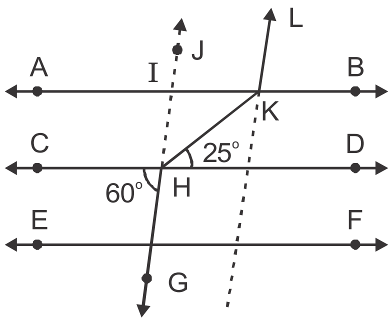
= 231 cm^3

17. (A) Given



In the figure, XZ coincides with XY + YZ. Also, Euclid's axiom (11) states that things which coincide with one another are equal to one another. So it is evident that XY + YZ = XZ

18. (C and D)



∠HKI = 25°

[Alternate angles since AB||CD]

∠AIH = ∠CHG = 60°

[Corresponding angles]

⇒ ∠JIA = 180° – 160° = 120° [Linear pair]

∠LKI = ∠JIA = 120°

[Corresponding angles since, GJ||KL]

∠HKL = ∠HKI + ∠LKI = 145°

Reflex ∠HKL = 360° – 145° = 215°

19. (C) $a = \sqrt{5}$, $b = \sqrt[3]{7}$ and $c = \sqrt[4]{36}$

$a = 5^{\frac{1}{2}}, b = 7^{\frac{1}{3}}$ and $c = (6^2)^{\frac{1}{4}} = 6^{\frac{1}{2}}$

$a = (5^3)^{\frac{1}{6}}, b = ((7)^2)^{\frac{1}{6}}$ and $c = (6^3)^{\frac{1}{6}}$

$a = (125)^{\frac{1}{6}}, b = (49)^{\frac{1}{6}}$ and $c = (216)^{\frac{1}{6}}$

∴ b < a < c

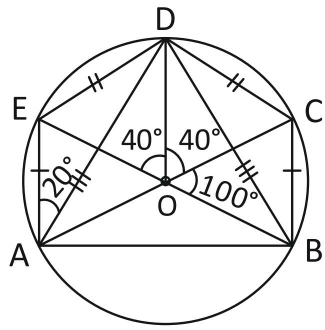
20. (C) $\sqrt{4a^2 + 25b^2 + 49c^2 - 20ab + 70bc - 28ca}$

$= \sqrt{(2a)^2 + (-5b)^2 + (-7c)^2 + 2(2a)(-5b) + 2(-5b)(7c) + 2(-7c)(2a)}$

$= \sqrt{(2a - 5b - 7c)^2}$

= (2a – 5b – 7c)

21. (D) Given $\angle EAD = 20° \Rightarrow \angle EOD = 2\angle EAD = 40°$



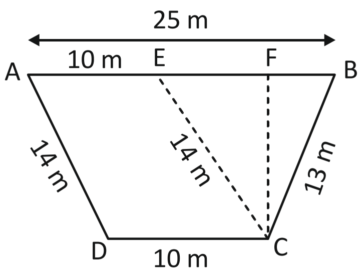
But ΔEOD ≅ ΔCOD [∵ Given DE = DC]

⇒ ∠COD = ΔEOD = 40°

∴ $\angle BOD = \angle BOD + \angle COD = 100° + 40° = 140°$

$\therefore \angle BAD = \frac{\angle BOD}{2} = \frac{140°}{2} = 70°$

22. (D) Let ABCD be the field in the form of a trapezium in which AB || CD such that



AB =25 m, BC = 13 cm, CD = 10 cm and DA = 14 m

Draw CE || DA and CF ⊥ EB

Clearly, ABCE is a parallelogram.

∴ CE = DA = 14 m and AE = CD = 10 m

∴ EB = AB – AE = (25 – 10) m = 15 m

In ΔEBC, we have

EB = 15 m, BC = 13 m and CE = 14 m

∴ a = 15 m, b = 13 m and c = 14 m

∴ $s = \frac{1}{2}(15 + 13 + 14)$ m = 21 m

∴ (s – a) = (21 – 15) m = 6 m

(s – b) = (21 – 13) m = 8 m and

(s – c) = (21 – 14) m = 7 m

∴ area (ΔEBC) = $\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{21\times6\times8\times7}\ m^2$

$=\sqrt{7\times3\times3\times2\times2\times4\times7}\ m^2$

= (7 × 3 × 2 × 2) m^2 = 84 m^2

Also area (ΔEBC)

$=\left(\frac{1}{2}\times EB\times CF\right)=\left(\frac{1}{2}\times 15m\times CF\right)$

∴ $\frac{1}{2}$ × 15 m × CF = 84 m^2

$\Rightarrow CF=\frac{84\times2}{15}m=\frac{56}{5}m=11.2\ m$

∴ CF = 11.2 m

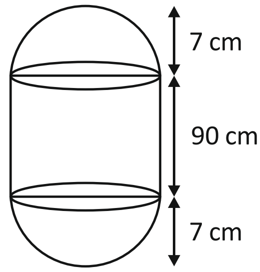
Area (trap. ABCD) = $\frac{1}{2}$ × (AB + CD) × CF

$=\left\{\frac{1}{2}\times(25+10)\times11.2\right\}m^2$

= (35 × 5.6) m^2 = 196 m^2

Hence, area of trapezium ABCD is 196 m^2

23. (D) Radius of each hemispherical end = 7 cm



Hence of each hemispherical part = its radius = 7 cm

Height of the cylindrical part

= (104 – 2 × 7) cm = 90 cm

Area of surface to be polished

= 2(curved surface area of the hemisphere) + (curved surface area of the cylinder)

= [2(2πr^2) + 2πrh] sq. units

$=\left[\left(4\times\frac{22}{7}\times7\times7\right)+\left(2\times\frac{22}{7}\times7\times90\right)\right]cm^2$

= (616 + 3960) cm^2 = 4576 cm^2

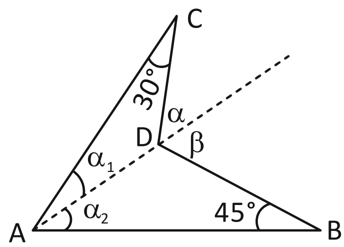
$=\left(\frac{4576}{10\times10}\right)dm^2 = 45.76\ dm^2$

[∵ 10 cm = 1 dm]

∴ cost of polishing the surface of the solid

= ₹ (45.76 × 10) = ₹ 457.60

24. (A) 30° + α_1 = α, 45° + α_2 = β



Adding (30° + 45°) + (α_1 + α_2) = α + β

$\Rightarrow$ 75° + 55°

= x° $\Rightarrow$ x = 130°

25. (B) $\angle$ACD + 80° = 180°

$\Rightarrow$ $\angle$ACD = 100°

$\angle$ECF = $\angle$ACD = 100°

100° + 25° + $\angle$CEF = 180°

$\Rightarrow$ $\angle$CEF = 55°

26. (A) We know that the perpendicular from the centre of a circle to a chord bisects the chord.

$$\therefore AP = \frac{1}{2}AB = \left(\frac{1}{2}\times 8\right)cm = 4\ cm$$

$$CQ = \frac{1}{2}CD = \left(\frac{1}{2}\times 6\right)cm = 3\ cm$$

Join OA and OC

Then, OA = OC = 5 cm

From the right-angled ΔOPA, we have

$OP^2 = OA^2 - AP^2 = [(5)^2 - (4)^2]\ cm^2 = 9\ cm^2$

$\Rightarrow$ OP = 3 cm

From the right-angled ΔOQC, we have

$OQ^2 = OC^2 - CQ^2 = [(5)^2 - (3)^2]\ cm^2 = 16\ cm^2$

$\Rightarrow$ OQ = 4 cm

Since OP ⊥ AB, OQ ⊥ CD and AB || CD, the points P, O, Q are collinear.

$\therefore$ PQ = OP + OQ = (3 + 4) cm = 7 cm

27. (A) Since ACE is a striaght line, we have

$\angle ACB + \angle BCE = 180°$

$\Rightarrow 90° + \angle BCE = 180°$

[$\because$ ∠ACB is in a semicircle]

$\Rightarrow \angle BCE = 90°$

Also, $\angle DBC = \frac{1}{2}\angle COD = \left(\frac{1}{2}\times 40°\right) = 20°$

[angle at centre = 2 × angle at a point on a circle]

$\Rightarrow \angle EBC = \angle DBC = 20°$

Now, in ∠EBC, we have

$\angle EBC + \angle BCE + \angle CEB = 180°$

$\Rightarrow 20° + 90° + \angle CED = 180°$

[$\because \angle CEB = \angle CED$]

$\Rightarrow \angle CED = 180° - 110° = 70°$

Hence, $\angle CED = 70°$

28. (B) Given a = 169 cm, b = 174 cm & c = 245 cm

$$\therefore s = \frac{a+b+c}{2} = \frac{588}{2}cm = 294\ cm$$

Area of the triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{294\times 125\times 120\times 49}\ cm^2$$

$$= \sqrt{7\times 7\times 6\times 5\times 5\times 5\times 5\times 6\times 4\times 49}\ cm^2$$

$= 7 \times 6 \times 5 \times 5 \times 2 \times 7\ cm^2$

$= 14700\ cm^2$

29. (C) Given ∠BAD + ∠BCD = 180°

[$\because$ Given ABCD is a cyclic quadrilateral]

$x - y + x + y = 180°$

$2x = 180°$

$x = 90°$

30. (A) Given $x = \frac{5-\sqrt{21}}{2}$

$$\therefore \frac{1}{x} = \frac{2}{5-\sqrt{21}} = \frac{2}{\left(5-\sqrt{21}\right)}\times\frac{\left(5+\sqrt{21}\right)}{\left(5+\sqrt{21}\right)}$$

$$= \frac{2\left(5+\sqrt{21}\right)}{(5)^2 - \left(\sqrt{21}\right)^2}$$

$$= \frac{2\left(5+\sqrt{21}\right)}{(25-21)} = \frac{2\left(5+\sqrt{21}\right)}{4} = \frac{5+\sqrt{21}}{2}$$

$$\therefore x + \frac{1}{x} = \left(\frac{5-\sqrt{21}}{2}\right) + \left(\frac{5+\sqrt{21}}{2}\right)$$

$$= \frac{5-\sqrt{21}+5+\sqrt{21}}{2} = \frac{10}{2} = 5$$

$$\therefore \left(x + \frac{1}{x}\right)^2 = 5^2 = 25$$

$$\Rightarrow x^2 + \frac{1}{x^2} + 2\times x\times\frac{1}{x} = 25$$

$$\Rightarrow\left(x^2+\frac{1}{x^2}\right)+2=25\Rightarrow\left(x^2+\frac{1}{x^2}\right)=25-2=23$$

$$\text{And, } x+\frac{1}{x}=5\Rightarrow\left(x+\frac{1}{x}\right)^3=(5)^3=125$$

$$x^3+\frac{1}{x^3}+3\times x\times\frac{1}{x}\left(x+\frac{1}{x}\right)=125$$

$$\Rightarrow\left(x^3+\frac{1}{x^3}\right)+3\times5=125$$

$$\Rightarrow\left(x^3+\frac{1}{x^3}\right)=125-15=110$$

$$\therefore\left(x^3+\frac{1}{x^3}\right)-5\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)$$

= 110 – 5 × 23 + 5 = 110 – 115 + 5

= 115 – 115 = 0

MATHEMATICS - 2

31. (A, D)

π is irrational

$\Rightarrow$ Surface area and volume of the sphere also irrational numbers

32. (A, B, C, D)

Option (A) : If a + b + c = 0, then $a^3 + b^3 + c^3 = 3abc$

Option (B) : Given a + b + c = 0

squaring on both sides $(a + b + c)^2 = 0$

$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 0$

$\therefore a^2 + b^2 + c^2 = -2(ab + bc + ca)$

Option (C) : Given a + b + c = 0

$\therefore c = -(a + b)$

If a + b + c = 0, then

$a^3 + b^3 + c^3 = 3abc = -3ab(a + b)$

Option (D) : Given a + b + c = 0

$\therefore (a + b)^2 = (-c)$

squaring on both sides

$(a + b)^2 = (-c)^2$

$a^2 + b^2 + 2ab = c^2$

$\therefore a^2 + b^2 - c^2 = -2ab$

33. (A, B)

Given $f(x) = 2x^{100} - 19x^{99} + 8x^{98} + 19x^{95} - 10x^{92}$

If $f(x)$ is divided by $(x - 1)$ then the remainder is f(1)

$\therefore f(1) = 2\times1^{100} - 19(1)^{99} + 8(1)^{98} + 19(1)^{95} - 10(1)^{92}$

= 2 – 19 + 8 + 19 – 10

= 0

$f(1) = 0 \Rightarrow (x - 1)$ is a factor of $f(x)$

$f(-1) = 2(-1)^{100} - 19(-1)^{99} + 8(-1)^{98} + 19(-1)^{95} - 10(-1)^{92}$

2 × 1 – 19 × –1 + 8(1) + 19(–1) – 10(1)

= 2 + 19 + 8 – 19 – 10

$f(1) = 0 \Rightarrow (x + 1)$ is a factor of $f(x)$

$f(-2) = 2(-2)^{100} - 19(-2)^{99} + 8(-2)^{98} + 19(-2)^{95} - 10(-2)^{92}$

$= 2\times2^{100} + 19\times2^{99} + 8\times2^{98} - 19\times2^{95} - 10\times2^{92}$

$\therefore f(-2) \neq 0 \Rightarrow (x + 2)$ is not a factor of $f(x)$

Similarly $(x - 2)$ is not a factor of $f(x)$

34. (A, B, D)

Every square is also a parallelogram

Every rectangle is also a parallelogram

Every rhombus is also a parallelogram

35. (A, C, D)

Option A × $\sqrt[5]{a^8b^7c^4} = \sqrt[5]{a^7b^3c}\times\sqrt[5]{a^8b^7c^4}$

$\sqrt[5]{a^7\times a^8\times b^3\times b^7\times c\times c^4}$

$\sqrt[5]{a^{15}b^{10}c^5}$

= a^3b^2c which is a rational number

$\therefore \sqrt[5]{a^7b^3c}$ is the RF of $\sqrt[5]{a^8b^7c^4}$

Option B × $\sqrt[5]{a^8b^7c^4} = \sqrt{a^2b^3c}\times\sqrt[5]{a^8b^7c^4}$

Orders of both surds are not same.

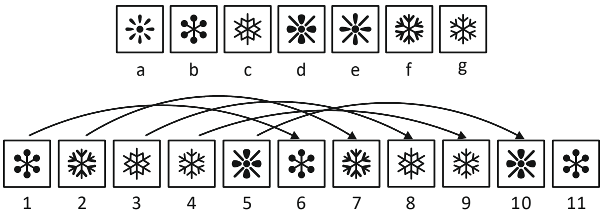
If we made orders are same also we cannot get rational number.

$\therefore \sqrt{a^2b^3c}$ is not a RF of $\sqrt[5]{a^8b^7c^4}$

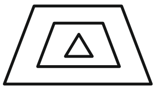
Similarly we can prove $\sqrt[5]{a^2b^8c^6}$ and $\sqrt[5]{a^7b^8c^{11}}$ are also RF of $\sqrt[5]{a^8b^7c^4}$

REASONING

36. (D) 1 = d, 2 = c



37. (C) In all the others the outer figure is repeated in the middle.



38. (A) Silk (Mohair is type of wool, where as shanting is type of silk.

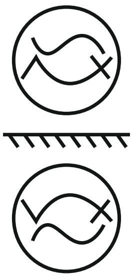
39. (C) We have

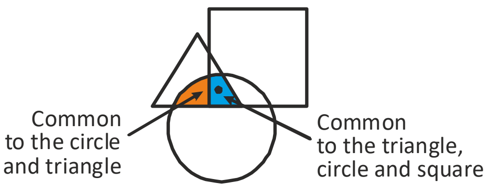
15×2 = 30, 2×7 = 14, 7×9 = 63

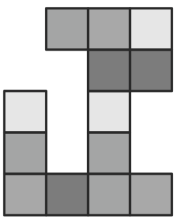
So, missing number = 9×15 = 135

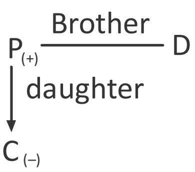
40. (B) Odds are successor and even are predecessor.

41. (A) 

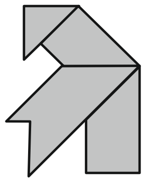
42. (B) 

43. (C) 

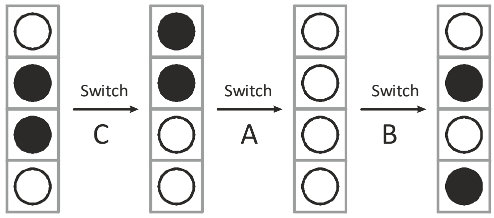
44. (B) 

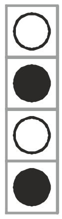
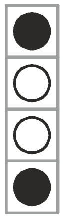
45. (D) 

CRITICAL THINKING

46. (C) 

47. (A) Both the koala and the weight go up with equal acceleration.

48. (A) 

If the pattern follows C, A and B switches one by one the result figure  but figure 2 in question is  lights 1 and 2 are in reverse order. So, switch (A) is fault.

49. (B) Option (B) is correct because the argument 1 states that banning pesticides is the only way to save underground water but we know that it is not the only way. Other measures can also be taken to reduce the pollution.

50. (B) The steepest lines on the graph represent the thinnest part of the container, as this is a part of the container that would become fuller quicker. This graph represents a container that starts thin, gradually becomes wider and then becomes thin again at the top.