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UIMO
Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD

CLASS - 8

Question Paper Code : UM9267

KEY

1	2	3	4	5	6	7	8	9	10
C	C	C	B	B	D	C	A	C	A
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
D	D	D	B	A	D	A	D	C	B
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
B	B	D	A	B	C	A	B	B	D
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
B,D	A,B,D	A,C,D	A,B	A,B,D	A	D	D	C	C
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
B	A	B	C	C	A	B	C	D	C

EXPLANATIONS

MATHEMATICS - 1

01. (C) $\alpha\beta = \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$

$= \frac{(-b)^2-\left(\sqrt{b^2-4ac}\right)^2}{4a^2}$

$= \frac{b^2-\left(b^2-4ac\right)}{4a^2} = \frac{b^2-b^2+4ac}{4a^2}$

$= \frac{4ac}{4a^2} = \frac{c}{a}$

02. (C) Given $9^{-x} = 7 \Rightarrow \frac{1}{9^x} = 7$

$\therefore\ 27^{2x+1} = 27^{2x} \times 27$

$= (3^3)^{2x} \times 27$

$= (3^2)^{3x} \times 27$

$= (9^x)^3 \times 27$

$= \frac{27}{(7)^3} = \frac{27}{343}$

$= \frac{27}{343}$

03. (C) Given $a^{\frac{1}{2}} + b^{\frac{1}{2}} = c^{\frac{1}{2}}$

Squaring on both sides

$$\left(a^{\frac{1}{2}}+b^{\frac{1}{2}}\right)^2 = \left(c^{\frac{1}{2}}\right)^2$$

$$= \left(a^{\frac{1}{2}}\right)^2 + 2a^{\frac{1}{2}}b^{\frac{1}{2}} + \left(b^{\frac{1}{2}}\right)^2 = c$$

$$= a + b - c = -2a^{\frac{1}{2}}b^{\frac{1}{2}}$$

Squaring on both sides

$$(a + b - c)^2 = \left(-2a^{\frac{1}{2}}b^{\frac{1}{2}}\right)^2 = 4ab$$

04. (C) Let the sum be "P"

we have,

$$P\left(1+\frac{R}{100}\right)^3 = 13380 \quad \text{\_\_\_\_\_ (1)}$$

and $\left(1+\frac{R}{100}\right)^6 = 20070$ _____ (2)

$$\frac{(2)}{(1)} \Rightarrow \left(1+\frac{R}{100}\right)^3 = \frac{20070}{13380} = \frac{3}{2}$$

substituting in (1), we get

$$P \times \frac{3}{2} = 13380$$

$$P = 13380 \times \frac{2}{3} = ₹8920$$

05. (B) $\sqrt[3]{k(k-2)+1} = \sqrt[3]{k^2-2k+1}$

$$= \sqrt[3]{(k-1)^2}$$

$$= \sqrt[3]{(126-1)^2}$$

$$= \sqrt[3]{125^2}$$

$$= \sqrt[3]{125\times125}$$

$$= \sqrt[3]{25\times25\times25}$$

$$= 25$$

06. (D) $$\frac{\sqrt{63}\times\sqrt{48}\times\sqrt{180}}{\sqrt{405}\times\sqrt{28}\times\sqrt{192}}$$

$$= \frac{\sqrt{3\times3\times7}\times\sqrt{4\times4\times3}\times\sqrt{6\times6\times5}}{\sqrt{9\times9\times5}\times\sqrt{2\times2\times7}\times\sqrt{8\times8\times3}}$$

$$= \frac{3\sqrt{7}\times4\sqrt{3}\times6\sqrt{5}}{9\sqrt{5}\times2\sqrt{7}\times8\sqrt{3}}$$

$$= \frac{1}{2}$$

07. (C) Let the number to be multiplied be 'x'

$$-\frac{5}{48}x = 15$$

$$x = 15 \times \frac{-48}{5}$$

$$= -144$$

08. (A) Let the number to be multiple

Given answer = $\frac{3x}{7}$

Mistake answer = $\frac{3x}{17}$

Given $\frac{3x}{7} - 150 = \frac{3x}{17}$

$$\frac{3x}{7} - \frac{3x}{17} = 150$$

$$\frac{51x-21x}{119} = 150$$

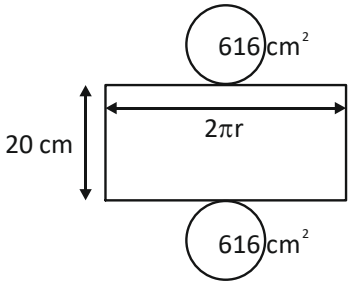
$$\frac{30x}{119} = 150$$

$$x = 150 \times \frac{119}{30} = 595$$

09. (C) Given $\pi r^2 = 616\ cm^2$

$$\frac{22}{7} \times r^2 = 616\ cm^2$$

$$r = \sqrt{2\times2\times7\times7\ cm^2}$$



r = 14 cm

Total surface area of the cylinder

$= 2\pi rh + \pi r^2 + \pi r^2$

$= 2\pi rh + \pi r^2$

$= 2\pi r \times (h + r)$

$= 2 \times \frac{22}{7} \times 14(20 + 14)\ cm^2$

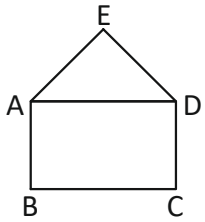
$= 88 \times 34\ cm^2$

$= 2992\ cm^2$

10. (A) The given figure has '3' lines of symmetry

11. (D) Construction: Join AD

ABCD is a quadrilateral and AED is a triangle.



∴ Sum of its five interior angles of a pentagon = sum of four angles of a quadrilateral + sum of the three

angles of a triangle

= 360° + 180°

= 540°

Let the fifth angle be 'x'

∴ 100° + 110° + 120° + 130° + x = 540°

460° + x = 540°

x = 540° – 460° = 80°

12. (D) Let q_1, q_2, q_3 and q_4 be the four required rational numbers. Then,

$$q_1 = \frac{1}{2}\left(\frac{1}{6} + \frac{1}{3}\right) = \frac{1}{2}\left(\frac{1+2}{6}\right) = \frac{1}{4}$$

$$q_2 = \frac{1}{2}\left(\frac{1}{4} + \frac{1}{3}\right) = \frac{1}{2}\left(\frac{3+4}{12}\right) = \frac{7}{24}$$

$$q_3 = \frac{1}{2}\left(\frac{1}{4} + \frac{7}{24}\right)$$

$$= \frac{1}{2}\left(\frac{6+7}{24}\right) = \frac{1}{2}\left(\frac{13}{24}\right) = \frac{13}{48}$$

and $q_4 = \frac{1}{2}\left(\frac{7}{24} + \frac{13}{48}\right)$

$$= \frac{1}{2}\left(\frac{14+13}{48}\right)$$

$$= \frac{1}{2}\left(\frac{27}{48}\right) = \frac{27}{96}$$

∴ $\frac{1}{4}, \frac{7}{24}, \frac{13}{48}$ and $\frac{27}{96}$ are the required rational numbers between $\frac{1}{6}$ and $\frac{1}{3}$

13. (D) Since angle in a semicircle is a right angle, clearly

$\angle A = \angle C = \angle B = \angle D = 90°$

The diagonals (diameters) are equal but they are not intersecting (bisecting) at right angles. Hence, it is not a square and can be only a rectangle.

14. (B) Among the given 30 numbers composite numbers are 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30

∴ Probability of getting a composite number = $\frac{19}{30}$

15. (A) Volume of the original cube = n^3

∴ volume of each small original cube

$= \frac{n^3}{k}$

∴ Edge of new small cube = $\sqrt[3]{\frac{n^3}{k}} = \frac{\sqrt[3]{n^3}}{\sqrt[3]{k}} = \frac{n}{\sqrt[3]{k}}$

16. (D) $1.9 = \frac{1.9\times1000}{1000} = \frac{1900}{1000}$

$0.57 = \frac{0.57\times1000}{1000} = \frac{570}{1000}$

$0.171 = \frac{0.171\times1000}{1000} = \frac{171}{1000}$

$171\overline{)570}\,(3$ $\quad$ $57\overline{)1900}\,(33$

513 $\quad$ 171

$57\overline{)171}\,(3$ $\quad$ 190

171 $\quad$ 171

0 $\quad$ $19\overline{)57}\,(3$

57

0

$\therefore$ HCF = $\frac{19}{1000}$ = 0.019

17. (A) Let the length of the rectangle be 'x' meters

$\therefore$ Breadth = $\frac{2}{3}x$ m

Given 2(l + b) = 180 m

l + b = $\frac{180\text{ m}}{2}$

$x + \frac{2}{3}x = 90$

$\frac{3x+2x}{3} = 90$

$5x = 90 \times 3$

$x = \frac{90\times3}{5} = 54$

$\therefore$ Breadth = $\frac{2}{3}x = \frac{2}{3} \times 54 = 36$

Area of the rectangle = $l \times$ b

= 54 × 36 m^2 = 1944 m^2

18. (D) $\sqrt{3}\,x^2 - 2x - 8\sqrt{3}$

$= \sqrt{3}\,x^2 - 6x + 4x - 8\sqrt{3}$

$= \sqrt{3}\,x\,(x - 2\sqrt{3}\,) + 4(x - 2\sqrt{3}\,)$

$= (x - 2\sqrt{3}\,)(\sqrt{3}\,x + 4)$

19. (C) Let the cost price be ₹ x

$\therefore$ Gain = 8%

SP = CP$\frac{(100+g)}{100} = ₹x \times \left(\frac{100+8}{100}\right)$

$= ₹x \times \frac{108}{100} = ₹\frac{27x}{25}$

SP = MP$\frac{(100-d)}{100}$

$₹\frac{27x}{25} = \text{MP}\frac{(100-10)}{100} = \text{MP} \times \frac{90}{100}$

$\therefore \text{MP} = ₹\frac{27x}{25} \times \frac{10}{9} = ₹\frac{6x}{5}$

More amount marked than

CP = MP – CP $= ₹\frac{6x}{5} - x$

$= ₹\frac{(6x-5x)}{5}$

$= ₹\frac{x}{5}$

$\therefore$ More percentage marked than

$\text{CP} = \frac{\left(₹\frac{x}{5}\right)}{₹x} \times 100$

$= \frac{100}{5} = 20\%$

20. (B) Given 2πr = 66 cm

$2 \times \frac{22}{7} \times r = 66$ cm

$r = 66 \times \frac{7}{22} \times \frac{1}{2}$

$= \frac{21}{2}$ cm

Volume = $\pi r^2 h = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 21$ cm

= 7,276.5 cm^3

21. (B) $36 - 25 = 48 - 37 = 64 - 53 = 11$

$\therefore$ Required number = (L.C.M. of 36, 48 and 64) – 11

2	36, 48, 64
2	18, 24, 32
2	9, 12, 16
3	9, 6, 8
2	3, 2, 8
	3, 1, 4

L.C.M. of 36, 48 and 64

$= 2 \times 2 \times 2 \times 3 \times 2 \times 3 \times 4 = 576$

$\therefore$ Required number = 576 – 11 = 565

22. (B) $\dfrac{\left(p^2 - \dfrac{1}{q^2}\right)^p \left(p - \dfrac{1}{q}\right)^{q-p}}{\left(q^2 - \dfrac{1}{p^2}\right)^q \left(q + \dfrac{1}{p}\right)^{p-q}}$

$$= \frac{\left(p + \frac{1}{q}\right)^p \left(p - \frac{1}{q}\right)^p \left(\frac{pq-1}{q}\right)^{q-p}}{\left(q - \frac{1}{p}\right)^q \left(q + \frac{1}{p}\right)^q \left(\frac{pq+1}{p}\right)^{p-q}}$$

$$= \frac{\left(\frac{pq+1}{q}\right)^p \left(\frac{pq-1}{q}\right)^p \left(\frac{pq-1}{q}\right)^{q-p}}{\left(\frac{pq-1}{p}\right)^q \left(\frac{pq+1}{p}\right)^q \left(\frac{pq+1}{p}\right)^{p-q}}$$

$$= \frac{\frac{(pq+1)^p}{q^p} \times \frac{(pq-1)^p}{q^p} \times \frac{(pq-1)^{q-p}}{q^{q-p}}}{\frac{(pq-1)^q}{p^q} \times \frac{(pq+1)^q}{p^q} \times \frac{(pq+1)^{p-q}}{p^{p-q}}}$$

$$= \frac{\frac{(pq+1)^p (pq-1)^{p+q-p}}{q^{p+p+q-p}}}{\frac{(pq-1)^q (pq+1)^{q+p-q}}{p^{q+q+p-q}}}$$

$$= \frac{(pq+1)^p \times (pq-1)^q}{q^{p+q}} \times \frac{p^{p+q}}{(pq-1)^q (pq+1)^p}$$

$$= \left(\frac{p}{q}\right)^{p+q}$$

23. (D) Let the two numbers be x & $(x - 10)$

Given $x(x - 10) = 144$

$x^2 - 10x - 144 = 0$

$x^2 - 18x + 8x - 144 = 0$

$x(x - 18) + 8(x - 18) = 0$

$(x - 18)(x + 8) = 0$

$x = 18$ (or) $x = -8$

If $x = 18$ then $x - 10 = 8$ (or) If $x = -8$ then $x - 10 = -18$ sum of two numbers

Sum of two numbers = –26

24. (A) Given $\angle A : \angle B = 2 : 3 = 2x + 3x$

$\therefore \angle A = 2x$ & $\angle B = 3x$

Given $\angle C = \angle A + \angle B = 2x + 3x = 5x$

But $\angle A + \angle B + \angle C = 180°$

$2x + 3x + 5x = 180°$

$x = 180°$

$x = \dfrac{180°}{10} = 18°$

$\therefore \angle A = 2x = 36°$, $\angle B = 3x = 54°$ & $\angle C = 90°$

25. (B) $\left(\sqrt{x} + \sqrt{y}\right)\left(\sqrt{x} - \sqrt{y}\right) = \left(\sqrt{x}\right)^2 - \left(\sqrt{y}\right)^2$

$= x - y$

26. (C) Let the speed of one train be x km/h

Then, the speed of the other train = $(x + 5)$ km/h

The distance travelled by the first train in 3 hours = $(3x)$ km

The distance travelled by the second train in 3 hours = $3(x + 5)$ km.

$\therefore 425 - [3x + 3(x + 5)] = 20$

$425 - 3x - 3x - 15 = 20$

$6x = 390$

$x = 65$

So, the speed of the first train = 65 km/h

Speed of the second train = (65 + 5) km/h = 70 km/h

27. (A) Let the sum be ₹100. Then,

$$\text{amount} = ₹\left[100\times\left(1+\frac{8}{100}\right)^2\right]$$

$$= ₹\left(100\times\frac{27}{25}\times\frac{27}{25}\right) = ₹\left(\frac{2916}{25}\right)$$

If the amount of ₹ $\frac{2916}{25}$ then the sum

= ₹100

If the amount is ₹72900 then the sum

$$= ₹\left(100\times\frac{25}{2916}\times 72900\right) = ₹62500$$

Hence, the required sum is ₹62500

28. (B) Let the measure of each of the equal angles be $x°$.

We know that the sum of all the angles of a quadrilateral is 360°.

$\therefore$ 115 + 45 + x + x = 360

160 + 2x = 360

2x = (360 – 160) = 200

x = 100

Hence, the measure of each of the equal angles in 100°

29. (B) A woman bought 3 toffees at 25 paise each. So she spent 75 paise and 3 for 65 paise, then totally for 6 toffees she spent

₹1.40 or for a dozen she spent

₹2.80 and sold the dozen at ₹3.50.

$$\therefore \text{Gain \%} = \frac{0.70}{2.80}\times 100\% = 25\%$$

30. (D) Given a^3 = 1728 cm^3

= 12 × 12 × 12 × cm^3

$a^3 = (12cm)^3$

a = 12cm

Total surface area = $6a^2 = 6\times(12\ cm)^2$

= 6 × 144 cm^2

= 864 cm^2

MATHEMATICS - 2

31. (B, D)

Given $(3^x)^2 \times 27 - 55 = 3^x \times 28 - 56$

$= 27(3^x)^2 - 28\times 3^x - 55 + 56 = 0$

Let $3^x = a$

$27a^2 - 28a + 1 = 0$

$27a^2 - 27a - a + 1 = 0$

$27a(a-1) - 1(a-1) = 0$

$(a-1)(27a-1) = 0$

$a - 1 = 0$ (or) $a = \frac{1}{27}$

$\therefore a = 1$ $3^x = 3^{-3}$

$3^x = 1 = 3°$

$\therefore x = 0$ $x = -3$

32. (A, B, D)

Let the four consecutive odd numbers be $x, x+2, x+4, x+6$

Option A : $4x + 12 = 616$

$4x = 616 - 12 = 604$

$x = \frac{604}{4}$ = 151 which is an odd number

Option A, i.e., 616 can be odd numbers.

Option A : $4x + 12 = 936$

$4x = 936 - 12 = 924$

$x = \frac{924}{4}$ = 231 which is an odd number

936 can be written as sum of four

Centsecutive odd numbers

Option C: $4x + 12 = 444$

$4x = 444 - 12 = 432$

$x = \frac{432}{4}$ = 108. which an even number.

$\therefore$ 444 can not be written as sum of four consecutive odd numbers.

Option D : $4x + 12 = 1322$

$4x = 1322 - 12 = 1332$

$x = \frac{1322}{4}$ = 333. which is an odd number.

∴ 1344 can be written as sum of four Consecutive odd numbers.

33. (A, C, D)

Given $a^2 + b^2 + c^2 = ab + bc + ca$

$2(a^2 + b^2 + c^2) = 2(ab + bc + ca)$

$2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0$

$a^2 - 2ab + b^2 + b^2 - 2bc + c^2 + c^2 - 2ca + a^2 = 0$

$(a - b)^2 + (b - c)^2 + (c - a)^2 = 0$

If sum of three perfect squares is zero, then each term is zero.

∴ $(a - b)^2 = 0$, $(b - c)^2 = 0$, $(c - a)^2 = 0$

∴ a = b, b = c, c = a

∴ a = b = c

34. (A, B)

$(x^4 + 1024) = (x^2)^2 + (32)^2$

$= (x^2)^2 + (32)^2 + 2 \times x^2 \times 32 - 2 \times x^2 \times 32$

$= (x^2 + 32)^2 - 64x^2$

$= (x^2 + 32)^2 - (8x)^2$

$= (x^2 - 8x + 32)(x^2 + 8x + 32)$

35. (A, B, D)

Except option 'C' remaining options are true because in a rectangle adjacent sides are not equal.

REASONING

36. (A) krekinblaf → workforce

drita krekin → ground work

krekinalti → workplace

Work is common in 3 words

The code for work is krekin

So, place is coded as alti

'Some' is a new word. Among the options, option (C) and option (B) are new coded word along with alti.

From the question, we observe the place is the second word.

So, 'alti' is place in second place. Option (A) is correct.

37. (D)

Each circle is divided into six portions; three pairs of identical shapes are located in opposite cells; the grey pieces rotate clockwise. All the inner parts rotate in a counter-clockwise direction. It is not easy to recognize the presence of the three pairs. However, one can reach the same conclusions using other clues. For instance, one may note that the same couple of shapes appear in the first and fourth items of the sequence, as is the second and fifth items. It is only logical that the next item of the series will include two hearts located in the right and left cells. This reasoning should lead us to narrow the possible answer to options (A) and (D). Figuring out the grey portions' movement pattern leaves us with possible options (B) and (D). Combining these two conclusions, and we're left with choice (D) as the only option to comply with this problem's reasoning.

38. (D) Here, the mathematical signs in the equations given in the options are interchanged according to the question and the option in which the equation gets satisfied is the answer.

In this example on applying the interchanges, we find that the equation given in option (D) gets satisfied.

5 × 7 + 5 = 40

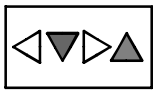
35 + 5 = 40

40 = 40

L.H.S = R.H.S

Hence, option (D) is correct

39. (C) Except for (C), the black triangles face right and left while the white triangles face up and down in all of the answer-options. Answer-option (C) exhibits the opposite trend, in which the white triangles face right and left while the black triangles face up and down.

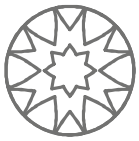


40. (C) Revanth did not go on a vacation in the months which have more than 30 days.

Means he went for vacation in Feb, April, June, November

Option (C) satisfies the given condition.

41. (B)



42. (A)

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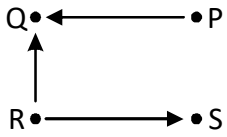
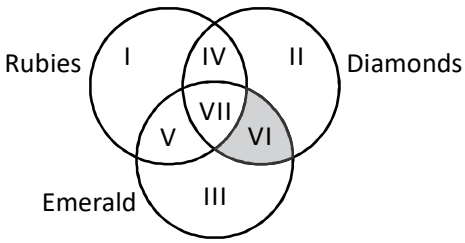
43. (B) According to P % Q + R – S



'S' is south east of Q

44. (C)



45. (C) 8:05 am to 11:15 pm

9:15, 10:20, 11:25, 12:30, 1:35, 2:40, 3:45, 4:50, 5:55, 6:00, 7:05, 8:10, 9:15, 10:20

CRITICAL THINKING

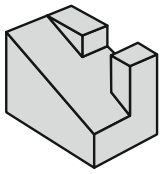
46. (A) I is a clear assumption from the statement. When the statements says that today's generation has forgotten their family values, it implies that this was not the case with the previous generations and they had their family values intact. II involves outside information and is more of an argument against the original statement of the author and is certainly not an assumption of the author. Thus II doesnot follow.

47. (B) P and R

48. (C) Both the objects will fall on the ground at the same time, because mass of the object doesn't matter when they are falling from same height and same time. So option (C) is the correct answer.

49. (D) Answer of the puzzle (Its a 5 digit number is : 21200)

50. (C)



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