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UNIFIED COUNCIL OLYMPIADS

UIM
Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD

CLASS - 9
Question Paper Code : UM9267

KEY

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B	C	C	C	C	C	A	C	C	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	C	A	B	C	D	D	D	C	B
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
D	A	D	B	A	D	C	A	D	C
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B,D	B,C	A,B,C,D	A,B,C,D	A,B,C,D	B	D	D	C	B
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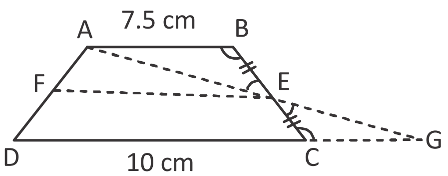
B	D	B	B	D	D	D	A	A	A

EXPLANATIONS

MATHEMATICS - 1

01. (B) Construction :- Extend DC and AE to meet at G



Proot :- $\Delta ABE \cong \Delta GCE$

[$\because$ ASA congraency]

$\therefore$ CG = AB and AE = EG

In ΔADG, F & E are midpoint of AD and AG respectively

$\therefore$ EF = $\frac{1}{2}$ DG = $\frac{1}{2}$ (DC + CG)

= $\frac{1}{2}$ (10 cm + 7.5 cm)

= 8.75 cm

02. (C)

$$\left(6\sqrt{x}-21\right)\left(4\sqrt{x}-13\right)=\left(8\sqrt{x}-11\right)\left(3\sqrt{x}-14\right)$$

$$\Rightarrow 24x-78\sqrt{x}-84\sqrt{x}+273=24x-112\sqrt{x}-33\sqrt{x}+154$$

$$-162\sqrt{x}+145\sqrt{x}=154-273$$

$$-17\sqrt{x}=-119$$

$$\sqrt{x}=\frac{119}{17}=7$$

$\therefore x = 49$

03. (C) Option 'C' ie $\sqrt{7}$ is irrational number

04. (C) $15-\frac{4}{x}-\frac{32}{x^2}=\left(\frac{15x^2-4x-32}{x^2}\right)$

$$=\frac{1}{x^2}\left(15x^2-24x+20x-32\right)$$

$$=\frac{1}{x^2}\left[3x\left(5x-8\right)+4\left(5x-8\right)\right]$$

$$=\frac{1}{x^2}\left(5x-8\right)\left(3x+4\right)$$

$$=\left(\frac{5x-8}{x}\right)\left(\frac{3x+4}{x}\right)$$

$$=\left(5-\frac{8}{x}\right)\left(3+\frac{4}{x}\right)$$

05. (C) $\frac{\text{a}\sqrt{\text{a}}+\text{b}\sqrt{\text{b}}}{\text{a}-\sqrt{\text{ab}}+\text{b}}=\frac{\left(\sqrt{\text{a}}\right)^3+\left(\sqrt{\text{b}}\right)^3}{\left(\sqrt{\text{a}}\right)^2-\sqrt{\text{a}}\times\sqrt{\text{b}}+\left(\sqrt{\text{b}}\right)^2}$

$$=\frac{\left(\sqrt{\text{a}}+\sqrt{\text{b}}\right)\left[\left(\sqrt{\text{a}}\right)^2-\sqrt{\text{a}}\times\sqrt{\text{b}}+\left(\sqrt{\text{b}}\right)^2\right]}{\left[\left(\sqrt{\text{a}}\right)^2-\sqrt{\text{a}}\times\sqrt{\text{b}}+\left(\sqrt{\text{b}}\right)^2\right]}$$

$$=\left(\sqrt{\text{a}}+\sqrt{\text{b}}\right)$$

06. (C) Given $2\pi rh = 176\ cm^2$ &

$2\pi rh + 2\pi r^2 = 253\ cm^2$

$\therefore 176\ cm^2 + 2\pi r^2 = 253\ cm^2$

$2\pi r^2 = 253\ cm^2 - 176\ cm^2$

$$2\times\frac{22}{7}\times r^2 = 77\ cm^2$$

$$r^2=77cm^2\times\frac{7}{2\times22}=\left(\frac{7}{2}cm\right)^2$$

$$\therefore r=\frac{7}{2}cm$$

$$2\times\frac{22}{7}\times\frac{7}{2}\times h=176\ cm^2$$

$$h=\frac{176\ cm^2}{22\ cm}$$

h = 8 cm

Volume = πr^2h

$$=\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times8\ cm^3 = 308\ cm^3$$

07. (A) Let $x=\sqrt[3]{27+\sqrt{756}}+\sqrt[3]{27-\sqrt{756}}$

$$\therefore x^3=\left(\sqrt[3]{27+\sqrt{756}}+\sqrt[3]{27-\sqrt{756}}\right)^3$$

$$=\left(\sqrt[3]{27+\sqrt{756}}\right)^3+\left(\sqrt[3]{27-\sqrt{756}}\right)^3$$

$$+3\times\sqrt[3]{27+\sqrt{756}}\times\sqrt[3]{27-\sqrt{756}}$$

$$\left(\sqrt[3]{27+\sqrt{756}}+\sqrt[3]{27-\sqrt{756}}\right)$$

$$=27+\sqrt{756}+27-\sqrt{756}+3\times\sqrt[3]{\left(27\right)^2-\left(\sqrt{756}\right)^2}\left(x\right)$$

$$=54+3\times\sqrt[3]{-27}\,x$$

$$=54+3\left(-3x\right)$$

$x^3 + 9x - 54 = 0$

$\therefore x = 3$

08. (C) Sum of the exterior angles of a polygon is 360°

Hence, $\angle EAB + \angle ACD + \angle FBC$

= 360°

09. (C)

$$s = \frac{a+b+c}{2} = \frac{\left(\frac{y}{z}+\frac{z}{x}+\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+\frac{x}{y}\right)\text{ cm}}{2}$$

$$= \frac{2\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)}{2}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)\left(\frac{x}{y}+\frac{z}{x}+\frac{y}{z}\right)}$$

$$= \sqrt{\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)}\text{ cm}^2$$

10. (B) $60^2 + 11^2 = 3600 + 121 = 3721 = 61^2$

$\therefore$ Given sides form a right angled triangle

$\therefore$ Mid point of hypotenuse is circumradius

$\therefore$ Radius = $\frac{61\text{ cm}}{2} = 30.5\text{ cm}$

11. (B) $\angle BOC = 2\angle BAC \Rightarrow \angle BOC + \angle BAC = 144°$

$2\angle BAC + \angle BAC = 144°$

$\angle BAC = \frac{144°}{3} = 48°$

$\angle BOC = 2\angle BAC = 96°$

12. (C) $2x^2 - 7x - 4 = 2x^2 - 8x + x - 4$

$= 2x(x - 4) + 1(x - 4)$

$= (2x + 1)(x - 4)$

$$\therefore \sqrt{3x-3+2\sqrt{2x^2-7x-4}}$$

$$= \sqrt{2x+x+1-4+2\sqrt{2x+1}\sqrt{x-4}}$$

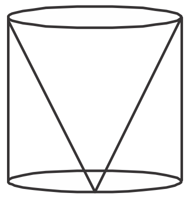
$$= \sqrt{\sqrt{(2x+1)^2}+\sqrt{(x-4)^2}+2\sqrt{2x+1}\sqrt{x-4}}$$

$$= \sqrt{\left(\sqrt{(2x+1)}+\sqrt{(x-4)}\right)^2}$$

$$= \left(\sqrt{2x+1}+\sqrt{x-4}\right)$$

13. (A) (A) Given radius of cone = 7 cm &

height of cone = 24 cm



$\therefore l = \sqrt{h^2+r^2} = \sqrt{24^2+7^2} = 25\text{ cm}$

TSA of the remaining solid = CSA of the cylinder + Base area of the cylinder + CSA of the cone

$= 2\pi rh + \pi r^2 + \pi rl$

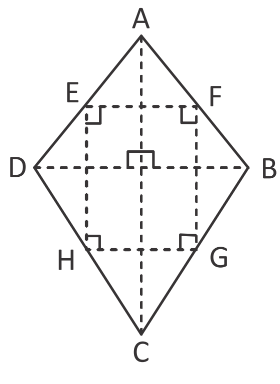
$= \pi r(2h + r + l)$

$= \frac{22}{7} \times 7\text{ cm }(48 + 7 + 25)\text{ cm}$

$= 22 \times 80\text{ cm}^2$

$= 1760\text{ cm}^2$

14. (B) Mid point of a kite are pined then we get a rectangle



15. (C) LHS =

$$\frac{(x+y+z)(x+y-z)}{(x+y+z)^2} \div \left\{\frac{(x-z+y)(x-y-z)}{x(x+y+z)} \div \frac{(x-y-z)(x-y+z)}{x(x-y+z)}\right\}$$

$$= \left(\frac{x+y-z}{x+y+z}\right) \div \left\{\frac{(x+y-z)(x-y-z)}{x(x+y+z)} \times \frac{x}{(x-y-z)}\right\}$$

$$= \frac{(x+y-z)}{(x+y+z)} \times \frac{(x+y+z)}{(x+y-z)} = 1$$

16. (D) Let $x+\frac{1}{x}=a$

C.O.B.S

$\left(x+\frac{1}{x}\right)^3=a^3$

$\Rightarrow x^3+\frac{1}{x^3}+3x\times\frac{1}{x}\left(x+\frac{1}{x}\right)=a^3$

$\Rightarrow 1298 + 3a = a^3$

$\Rightarrow a^3 - 3a - 1298 = 0$

$\therefore$ a = 11 satisfies $a^3 - 3a - 1298 = 0$

$\therefore a=x+\frac{1}{x}=11$

17. (D) $x^3 - 3x + 2 = x^2 - 2x - x + 2$

$= x(x-2) - 1(x-2)$

$= (x-2)(x-1)$

Let $x = 2$ then

$x^{2022} = (x^2 - 3x + 2)q(x) + (ax + b)$

$\Rightarrow 2^{2022} = [2^2 - 3(2) + 2]\, q(x) + (2a + b)$

$2^{2022} = 0 + 2a + b$

$\therefore 2a + b = 2^{2022} \rightarrow (1)$

Let $x = 1 \Rightarrow 1^{2022} = 0 + a(1) + b$

$\therefore a + b = 1 \rightarrow (2)$

equ (1) – (2)

$\Rightarrow (2a + b) - (a + b) = (2^{2022} - 1)$

$2a + b - a - b = 2^{2022} - 1$

$a = 2^{2022} - 1$

$2^{2022} - 1 + b = 1 \rightarrow (2)$

$b = 1 + 1 - 2^{2022} = 2 - 2^{2022}$

$\therefore$ The remainder $R = ax + b$

$= (2^{2022} - 1)x + (2 - 2^{2022})$

18. (D) $\frac{\sqrt{72}}{5\sqrt{72}+3\sqrt{288}-2\sqrt{648}}$

$=\frac{\sqrt{72}}{5\sqrt{72}+3\sqrt{4\times72}-2\sqrt{9\times72}}$

$=\frac{\sqrt{72}}{5\sqrt{72}+6\sqrt{72}-6\sqrt{72}}$

$=\frac{\sqrt{72}}{5\sqrt{72}}=\frac{1}{5}$

19. (C) Given $\pi r l = 2\pi r^2$

$l = 2r$

But $l^2 = h^2 + r^2$

$4r^2 = h^2 + r^2$

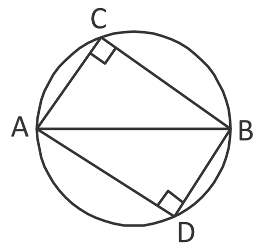
$h^2 = 3r^2$

$h=\sqrt{3r^2}=\sqrt{3}r$

$\therefore \frac{r}{h}=\frac{1}{\sqrt{3}}$

$\therefore r : h = 1 : \sqrt{3}$

20. (B) In a quadrilateral ACBD, $\angle ADB + \angle ACB = 90° + 90° = 180°$



$\therefore$ ACBD is a cyclic quadrilateral

$\therefore \angle BAC = \angle BDC = 70°$

[$\because$ angles in the same segment]

$\therefore \angle ADC = 90° - \angle BDC = 20°$

21. (D) In ΔADE, $\angle E = 90° \Rightarrow AD^2 = AE^2 + ED^2$

[$\because$ pythagorus theorem]

$29^2 = AE^2 + 21^2$

$841 = AE^2 + 441$

$AE = \sqrt{841-441} = \sqrt{400} = 20$

$\therefore$ BF = 20 cm

In ΔBCF, $\angle F = 90° \Rightarrow BC^2 = BF^2 + FC^2$

$25^2 = 20^2 + FC^2$

$FC=\sqrt{(20+25)(25-20)}$

$=\sqrt{45\times5}$

= 15 cm

$\therefore$ CD = CF + FE + ED

= 15 cm + 32 cm + 21 cm = 68 cm

$\therefore$ Area of the trapezium ABCD

$= \frac{1}{2} \times AE(AB + CD)$

$= \frac{1}{2} \times 20\text{ cm}(32\text{ cm} + 68\text{ cm})$

$= 10 \times 100\text{ cm}^2$

$= 1000\text{ cm}^2$

22. (A) Angles in the same segment are equal.

$\Rightarrow \angle BAD = \angle BCD = 30°$

In $\triangle CBP$, $\angle C + \angle B + \angle P = 180°$

$\Rightarrow 30° + \angle B + 45° = 180°$

$\Rightarrow \angle B = 180° - 75° = 105°$

$\therefore \angle CBP = 105°$

23. (D) Area of the (parallelogram ABCD)

$= BC \times h$

$432\text{ cm}^2 = BC \times 20\text{ cm}$

$BC = \frac{432\text{ cm}^2}{20\text{ cm}^2} = 21.6\text{ cm}$

24. (B) In ΔAOB, $OA = OB \Rightarrow \angle OBA = \angle OAB = 54°$

$\therefore \angle BOD = \angle OBA + \angle OAB = 54° + 54° = 108°$

ABCD is a cyclic quadrilateral

$\therefore \angle DAB + \angle BCD = 180°$

$54° + \angle BCD = 180°$

$\therefore \angle BCD = 180° - 54° = 126°$

$\therefore \angle BOD + \angle DCB = 108° + 126° = 234°$

25. (A) Given in a quadrilateral ABCD

$\angle A + \angle C + \angle D = 360° - 90° = 270° \rightarrow (1)$

and

$\angle A - \angle C - \angle D = 10° \qquad \rightarrow (2)$

equ (1) + (2)

$\Rightarrow 2\angle A = 270° + 10° = 280°$

$\therefore \angle A = \frac{280°}{2} = 140°$

$\angle C + \angle D = 130°$ & $\angle C - \angle D = 60°$

$\therefore \angle C = 95°$ & $\angle D = 35°$

(OR) Verify from options

26. (D) Given $\angle B - \angle A = 12° \Rightarrow \angle B = 12° + \angle A$

Given $\angle C - \angle A = 24° \Rightarrow \angle C = 24° + \angle A$

In ΔABC, $\angle A + \angle B + \angle C = 180°$

$\angle A + 12° + \angle A + 24° + \angle A = 180°$

$3\angle A = 180° - 36° = 144°$

$\angle A = \frac{144°}{3} = 48°$

$\angle C = 24° + \angle A = 72°$

27. (C) $7x^2 - 2x + \frac{1}{7} = 7x^2 - x - x + \frac{1}{7}$

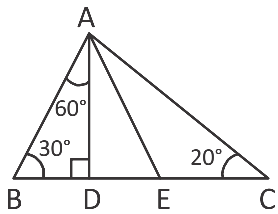
$= 7x^2 - \frac{7x}{7} - x + \frac{1}{7}$

$= 7x\left(x - \frac{1}{7}\right) - 1\left(x - \frac{1}{7}\right)$

$= \left(x - \frac{1}{7}\right)(7x - 1)$

28. (A) In ΔABC, given $\angle B = 30°$ & $\angle C = 20°$

$30° + 20° + \angle BAC = 180°$



$\angle BAC = 180° - 50° = 130°$

$\therefore \angle BAE = \frac{\angle BAC}{2} = \frac{130°}{2} = 65°$

In ΔABD, $\angle B = 30°$ & $\angle D = 90°$

$\therefore \angle BAD = 180° - 90° - 30° = 60°$

But $\angle BAD + \angle DAE = \angle BAE$

$60° + \angle DAE = 65°$

$\angle DAE = 65° - 60° = 5°$

29. (D) $\angle ACB = \angle DAC = 40°$

[$\because$ Alternative angles]

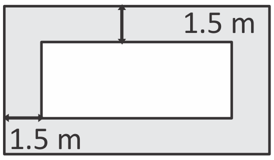
In ΔBOC, $\angle BOA = \angle OCB + \angle OBC$

$72° = 40° + \angle OBC$

$\therefore \angle OBC = 72° - 40° = 32°$

$\therefore \angle DBC = \angle OBC = 32°$

30. (C) Let the length and breadth of the park be l & b



Given l : b = 8 : 5 = $8x$: $5x$

Outer length (L) = l + 1.5 m + 1.5 m

Outer breadth (B) = b + 1.5 m + 1.5 m

= b + 3

Total area = (l + 3)(b + 3)

Area of the path = 594 m^2

$\Rightarrow$ Total area – area of the park = 594 m^2

$\therefore$ (l + 3)(b + 3) – lb = 594

lb + 3l + 3b + 9 – lb = 594

3(l + b) + 9 = 594

3(l + b) = 594 – 9

$(l + b) = \frac{585}{3} = 195$

$8x + 5x = 195$

$13x = 195$

$x = \frac{195}{15} = 15$

$\therefore$ Area of the park = $(8x)(5x)$

= $40x^2 = 40 \times 15^2$

= 40 × 225

= 9000 m^2

MATHEMATICS - 2

31. (B, D)

$\sqrt{2} \times \sqrt{3} = \sqrt{6}$ & $\sqrt{2} \times \sqrt{8} = \sqrt{16} = 4$

32. (B, C)

$(x + y)^2 = (x + y)(x + y)$

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$\therefore$ LCM of $(x + y)^2$ and $(x^3 + y^3)$

$= (x + y)^2(x^2 - xy + y^2)$

$= (x + y)(x + y)(x^2 - xy + y^2)$

$= (x + y)(x^3 + y^3)$

33. (A, B, C, D)

All options are true.

34. (A, B, C, D)

Given $\frac{(x^2 - 6)^2 + 5x^2}{x(x^2 - 6)} = 6$

$\Rightarrow (x^2 - 6)^2 + 5x^2 = 6x^3 - 36x$

$\Rightarrow x^4 - 12x^2 + 36 + 5x^2 - 6x^3 + 36x = 0$

$\Rightarrow x^4 - 7x^2 - 6x^3 + 36x + 36 = 0$

$\Rightarrow x^2(x^2 - 6x - 7) + 36(x + 1) = 0$

$\Rightarrow x^2(x + 1)(x - 7) + 36(x + 1) = 0$

$(x + 1)[x^2(x - 7) + 36] = 0$

$x + 1 = 0$ (or) $x^3 - 7x^2 + 36 = 0$

$x = -1$ (or)

$$
\begin{array}{l|l|l}
x - 6 & x^3 - 7x^2 + 36 & x^2 - x - 6 \\
 & x^3 - 6x^2 & \\
 & (-)\quad(+) & \\
\hline
 & -x^2 + 36 & \\
 & -x^2 + 6x & \\
 & (+)\quad(-) & \\
\hline
 & -6x + 36 & \\
 & -6x + 36 & \\
 & (+)\quad(-) & \\
\hline
 & 0 &
\end{array}
$$

$\therefore x^3 - 7x^2 + 36 = 0$

$\Rightarrow (x - 6)(x - 3)(x + 2) = 0$

$\therefore x = -1, 6, 3, -2$

35. (A, B, C, D)

If $\left(5+2\sqrt{6}\right)^{x^2-3}+\left(5-2\sqrt{6}\right)^{x^2-3}=10$, then

$x^2 - 3 = \pm 1$

$\therefore\ x^2 - 3 = 1$ (or) $x^2 - 3 = -1$

$x^2 = 1 + 3$ $\quad$ $x^2 = -1 + 3$

$x^2 = 4$ $\quad$ $x^2 = 2$

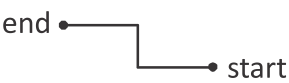
$x = \pm\sqrt{4} = \pm 2$ $\quad$ $x = \pm\sqrt{2}$

REASONING

36. (B) The series is aabbcc/aabbcc/aabbcc. Thus the pattern aabbcc is repeated.

37. (D) Cube: It is a three-dimensional figure. The rest are all two-dimensional figures.

38. (D) North - West

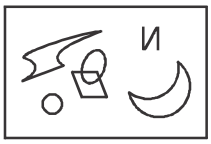


39. (C) In the question figure, we see that dot is placed in a triangle.

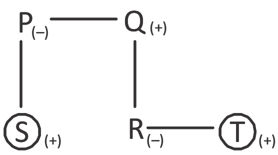
Now from the answer figures, we see that in option (C) dot is placed in a triangle.

Hence, option (C) is correct.

40. (B)



41. (B)



S and T are sons of 'Q'

42. (D) First letter – Colour of are second letter – type of star

43. (B) looking at lines of numbers from the top: 9 × 8 = 72; 72 × 8 = 576; 576 × 8 = 4608.

44. (B) Clearly, the child moves from A 90 m east-wards upto B, then turns right and moves 20 m upto C, then turns right and moves 30 m upto D. Finally, he turns right and moves 100 m upto E. Clearly, AB = 90 m, BF CD = 30 m. So, AF = AB – BF = 60 m Also, DE = 100 m, DF = BC = 20 m

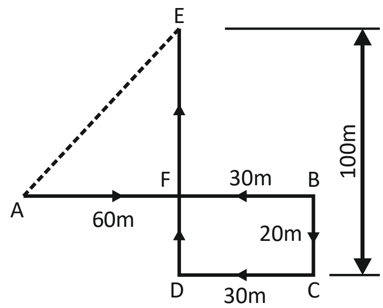
$\therefore$ His distance from starting point

So EF = DE – DF = 80 m

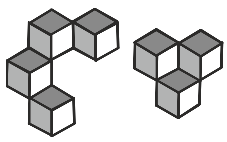
$A = AE = \sqrt{AF^2 + EF^2}$

$\Rightarrow \sqrt{(60)^2 + (80)^2} \Rightarrow \sqrt{3600 + 6400}$

$\Rightarrow \sqrt{10000} = 100$ m



45. (D)



CRITICAL THINKING

46. (D) The input and operation are known, whereas the output is unknown.

The operation that is applied is: all symbols rotate 90° clockwise.

Rotation by 90° clockwise has to be performed on each of the three shapes:

- The black square in the L shape moves from the bottom left position to the top left position, with one white square being to the right from it and the second white square being under it. This results in possible answers being B or D.
- The diagonal line in the round shape moves from a bottom-left to top-right direction to a top-left to bottom-right direction. This results in possible answers being A, C or D.
- The black triangle that is at the top rotates and moves to the right side, while the white triangle that is at the bottom rotates and moves to the left. This results in possible answers being B or D.
- There is only one answer (D) in common for each shape.

47. (D) From the conclusion 1 and 2 are logical conclusions for the given paragraph.

48. (A) The bird in picture P flies with her wings backwards, minimized the contact surface with the wind and creating less resistance. The bird in picture Q flies with her wings in the wind direction, creating much more resistance. The same phenomenon makes a crumpled piece of paper fall faster than an open sheet of paper, which has more surface and therefore more resistance.

Remember the physical principle: The larger the surface of contact with air/wind, the more resistance (force) is created.

49. (A) 1A , 4D , 6D

50. (A)

The End