Foundation for success

Unified International
Mathematics Olympiad

## UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)

## CLASS - 10 Question Paper Code : UM9269

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## EXPLANATIONS

## MATHEMATICS - 1

1. (C) $A O B$ is a right angled triangle

$\therefore$ Circumcentre is mid point of hypotenuse
$\therefore$ Mid point of $A B$
$=\left(\frac{x+0}{2}, \frac{0+y}{2}\right)=\left(\frac{x}{2}, \frac{y}{2}\right)$
2. (D) Given $\mathrm{a}_{2}=7$ and $\mathrm{a}_{4}=12$

$$
\begin{align*}
& \therefore a+3 d=23  \tag{1}\\
& a+d=7 \tag{2}
\end{align*}
$$

eq. $(2)-(1) \Rightarrow 2 d=16$
$d=8$
$\therefore c=23+d=23+8=31$
03. (B) $\Delta \mathrm{ADG} \sim \Delta \mathrm{BDF} \sim \Delta \mathrm{CDE}[\because$ A.A similarlily $]$

$\therefore \frac{\mathrm{DB}}{\mathrm{DC}}=\frac{\mathrm{BF}}{\mathrm{CE}} \Rightarrow \frac{\mathrm{h}+2}{\mathrm{~h}}=\left(\frac{3}{2}\right)$
$\Rightarrow 2 h+4=3 h \Rightarrow h=4$
$\frac{(h+2)}{(h+5)}=\frac{\left(\frac{3}{2}\right)}{A G}$
$\Rightarrow A G=(h+5) \times \frac{3}{2} \times \frac{1}{(h+2)}[\because h=4]$
$=9 \times \frac{3}{2} \times \frac{1}{6}$
$\mathrm{AG}=\frac{9}{4} \Rightarrow 2 \mathrm{AG}=\frac{9}{2}$
Area of the total triangle $=$
$\frac{1}{2} \times(4+3+2) \times \frac{9}{2}=\frac{81}{4}=20 \frac{1}{4} \mathrm{~cm}^{2}$
04. (D) Given $\beta=\alpha^{2}$

Given $\alpha+\beta=-(-12)$
$\Rightarrow \alpha^{2}+\alpha=12$
$\Rightarrow \alpha^{2}+\alpha-12=0$
$\Rightarrow \alpha^{2}+4 \alpha-3 \alpha-12=0$
$\Rightarrow \alpha=-4$ (or) $\alpha=3$
$\therefore \alpha \beta=3 \mathrm{k}$
$\Rightarrow(-4)(-4)^{2}=3 k \quad$ (or) $3 \times 3^{2}=3 k$
$\therefore \mathrm{k}=\frac{-64}{3} \quad \mathrm{k}=9$
05. (D) Given $6 \mathrm{a}^{2}=294 \mathrm{~cm}^{2}$
$\mathrm{a}^{2}=\frac{294}{6} \mathrm{~cm}^{2}$
$a^{2}=(7 \mathrm{~cm})^{2}$
$\therefore$ Volume of cube $=\mathrm{a}^{3}=343 \mathrm{~cm}^{3}$
Given $\pi r^{2} h=343 \mathrm{~cm}^{3}$
$\Rightarrow \frac{22}{7} \times \mathrm{r}^{2} \times \frac{49}{22}=343 \mathrm{~cm}^{3}$
$r^{2}=\frac{343}{7} \mathrm{~cm}^{2}=49 \mathrm{~cm}$
$r=7 \mathrm{~cm}$
SA of cylinder $=2 \pi \mathrm{rh}$
$=2 \times \frac{22}{7} \times 7 \times \frac{49}{22}=98 \mathrm{~cm}^{2}$
06. (C) Area of $\triangle A B D$
$=\frac{1}{2}|-1(1-9)+4(9-2)+3(2-1)|$
$=\frac{1}{2}|8+28+3|$
$=\frac{39}{2}$ units $^{2}$
$\therefore$ Area of $\triangle A B C=2$ Area of $\triangle A B D$
$=2 \times \frac{39}{2}$ units $^{2}$
$=39$ units $^{2}$
Area of $\triangle A B G=\frac{1}{3}$ of area of $\triangle A B C$
$=\frac{1}{3} \times 39$ units $^{2}$
$=13$ square units
07. (A) Given $\alpha+\beta=\frac{-b}{a}=-4$
$\therefore 5 \alpha+5 \beta=-20 \rightarrow(1)$
Given $5 \alpha+2 \beta=1 \rightarrow(2)$
(-) (-) (-)
$3 \beta=-21$
$\beta=-7$
$\alpha-7=-4$
$\alpha=-4+7=3$
$\therefore \alpha \beta=\mathrm{k} \Rightarrow \mathrm{k}=3 \times-7=-21$
08. (D) $\tan A+\tan B=\frac{a}{b}+\frac{b}{a}=\frac{a^{2}+b^{2}}{a b}=\frac{c^{2}}{a b}$

$$
\left[\because a^{2}+b^{2}=c^{2}\right]
$$


09. (C) If $\cos \theta=\frac{1}{2}$ then $\sec \theta=2$

$$
\begin{aligned}
& \Rightarrow \sec \theta+\cos \theta=2+\frac{1}{2}=\frac{5}{2} \\
& \therefore \cos \theta=\frac{1}{2}=\cos 60^{\circ} \Rightarrow \theta=60^{\circ} \\
& \therefore \sin ^{2} \theta=\sin ^{2} 60^{\circ}=\left(\frac{\sqrt{3}}{2}\right)^{2}=\frac{3}{4}
\end{aligned}
$$

(or)
Given $\frac{1}{\cos \theta}+\cos \theta=\frac{5}{2}$
$\Rightarrow \frac{1+\cos ^{2} \theta}{\cos \theta}=\frac{5}{2}$
$\Rightarrow 2 \cos ^{2} \theta-5 \cos \theta+2=0$
$\Rightarrow 2 \cos ^{2} \theta-4 \cos \theta-\cos \theta+2=0$
$\Rightarrow 2 \cos \theta(\cos \theta-2)-1(\cos \theta-2)=0$
$\therefore \cos \theta=2$ (or) $\cos \theta=\frac{1}{2}$

But $\cos \theta$ never be greater than 1
$\therefore \cos \theta=2$ is refected
$\therefore \cos \theta=\frac{1}{2}=\cos 60^{\circ}$
$\therefore \theta=60^{\circ}$
$\sin ^{2} \theta=\left(\sin 60^{\circ}\right)^{2}$
$=\left(\frac{\sqrt{3}}{2}\right)^{2}=\frac{3}{4}$
10. (D) $\quad \triangle \mathrm{BDA} \sim \triangle \mathrm{ADC}[\because$ A.A Similarlily]

$\therefore \frac{\mathrm{BD}}{\mathrm{AD}}=\frac{\mathrm{DA}}{\mathrm{DC}}$
$\mathrm{AD}^{2}=\mathrm{BD} \times \mathrm{DC}=8 \times 12.5 \mathrm{~cm}^{2}$
$A D=\sqrt{100 \mathrm{~cm}^{2}}=10 \mathrm{~cm}$
11. (B) Given $\angle X A D=30^{\circ} \Rightarrow \angle A D E=30^{\circ}$
[ $\because$ Alternative angles ]

$\angle \mathrm{XAC}=\angle \mathrm{ACB}=60^{\circ}$
[ $\because$ Alternative angles]
In $\triangle \mathrm{ABC}, \angle \mathrm{B}=90^{\circ} \& \angle \mathrm{ACB}=60^{\circ}$
$\Rightarrow \tan 60^{\circ}=\frac{A B}{B C}$
$\sqrt{3}=\frac{30 \mathrm{~m}}{\mathrm{BC}}$
$B C=\frac{30 \mathrm{~m}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$
$B C=10 \sqrt{3}$ metres
In $\triangle A D E \tan 60^{\circ}=\frac{A E}{E D}$
$\frac{1}{\sqrt{3}}=\frac{\mathrm{AE}}{10 \sqrt{3}}$
$A E=\frac{10 \sqrt{3}}{\sqrt{3}}$
$\therefore C D=B E=30-10=20 \mathrm{~cm}$
12. (D) Let $\angle \mathrm{CQB}=x$

$\Rightarrow \angle \mathrm{QBR}=90^{\circ}-x$ and $\angle \mathrm{QCB}=\left(90^{\circ}-x\right)$
$\therefore \triangle \mathrm{BAP} \cong \triangle \mathrm{CBQ}[\because$ ASA congruency $]$
$\therefore C Q=13 \mathrm{~cm}$
Let $\mathrm{RQ}=y \Rightarrow \mathrm{CR}=(13-y)$
In $\triangle \mathrm{BCQ}, \angle \mathrm{QBC}=90^{\circ} \& \mathrm{BR} \perp \mathrm{QC}$
$\therefore \mathrm{BR}^{2}=\mathrm{QR} \times \mathrm{RC}$
$36 \mathrm{~cm}^{2}=y(13-y)$
$y^{2}-13 y+36=0$
$y=4$ (or) 9
$\therefore y=4$ is selected because $\mathrm{QB}<\mathrm{BC}$
$\therefore \mathrm{RC}=9$
In $\triangle B R C, B C^{2}=B R^{2}+R C^{2}=117$
13. (B) Given
$(\sec A-\tan A)(\sec B+\tan B)(\sec C-\tan C)$
$=(\sec A+\tan A)(\sec B-\tan B)(\sec C+\tan C)$
Multiplify
$(\sec A+\tan A)(\sec B-\tan B)(\sec C+\tan C)$ on both sides
$\therefore(\sec A+\tan A)^{2}(\sec B-\tan B)^{2}(\sec C+\tan C)^{2}$
$=\left(\sec ^{2} A-\tan ^{2} A\right)\left(\sec ^{2} B-\tan ^{2} B\right)\left(\sec ^{2} C-\tan ^{2} C\right)$
= 1
$\therefore(\sec A+\tan A)(\sec B-\tan B)(\sec C+\tan C)$
$= \pm \sqrt{1}= \pm 1$
14. (A) Const:- Join OC \& OB

$\angle O C Q=90^{\circ} \Rightarrow \angle O C B=90^{\circ}-30^{\circ}=60^{\circ}$
$\therefore \angle \mathrm{OBC}=\angle \mathrm{OCB}=60^{\circ}$
$\therefore \angle \mathrm{BOC}=60^{\circ}$
$\therefore \angle \mathrm{CAB}=\frac{\angle \mathrm{BOC}}{2}=30^{\circ}$
15. (C) Given $\alpha+\beta=\frac{-b}{a}=\frac{-1}{6}$,

$$
\begin{aligned}
& \alpha \beta=\frac{c}{a}=\frac{-12}{6}=-2 \\
& \alpha^{2}+\beta^{2}=(\alpha+\beta)^{2}-2 \alpha \beta \\
& =\left(\frac{-1}{6}\right)^{2}-2 \times-2 \\
& =\frac{1}{36}+4=\frac{145}{36} \\
& \alpha^{4}+\beta^{4}=\left(\alpha^{2}+\beta^{2}\right)^{2}-2 \alpha^{2} \beta^{2} \\
& =\left(\frac{145}{36}\right)^{2}-2(-2)^{2} \\
& =\frac{21025}{1296}-8 \\
& =\frac{21025-10368}{1296}=\frac{10657}{1296}
\end{aligned}
$$

16. (A) Let $\frac{1}{\sqrt{x}}=\mathrm{a} \& \frac{1}{\sqrt{y}}=\mathrm{b}$
$\Rightarrow 2 a+3 b=\frac{2}{3}$
$6 \mathrm{a}+9 \mathrm{~b}=2 \quad \rightarrow(1)$
$4 a-9 b=\frac{-1}{3}$
$12 \mathrm{a}-27 \mathrm{~b}=-1 \quad \rightarrow(2)$
equ $(1) \times 2 \Rightarrow 12 a+18 b=4$

$$
12 a-27 b=-1 \quad \rightarrow(2
$$

$$
\begin{aligned}
& (-) \quad(+) \quad(+) \\
& \hline 45 b=5
\end{aligned}
$$

$b=\frac{5}{45}=\frac{1}{9}$
$6 a+9 \times \frac{1}{9}=2 \rightarrow(1)$
$6 a=1$
$a=\frac{1}{6}$
$\therefore a=\frac{1}{\sqrt{x}}=\frac{1}{6} \Rightarrow x=36$
$\mathrm{b}=\frac{1}{\sqrt{y}}=\frac{1}{9} \Rightarrow y=81$
$\therefore x+y=36+81=117$
17. (A) Let the original number be $(10 x+y)$

Given $10 x+y=4(x+y)$
$10 x+y=4 x+4 y$
$6 x-3 y=0$
$2 x-y=0 \quad \rightarrow(1)$
Given $10 x+y=2 x y \quad \rightarrow(2)$
equ (1) $\times 5 \Rightarrow 10 x-5 y=0$

$$
10 x+y=2 x y
$$

$$
(-) \quad(-) \quad(-)
$$

$$
-6 y=-2 x y
$$

$x=3$
$\therefore 2(3)-y=0 \rightarrow(1)$
$y=6$
$\therefore$ Required number $=10 \times 3+6=36$
$\therefore$ Difference the digits $=6-3=3$
18. (D) $2 x+y=6 \& y=0$ lines intersect at $(3,0)$ $2 x-y+2=0 \& y=0$ lines intersect at $(-1,0)$ $2 x+y=6$ and $2 x-y=-2$ lines intersect at $(1,4)$

$A(1,4) B(-1,0) C(3,0)$
Area of the $\triangle \mathrm{ABC}$
$=\frac{1}{2}|1(0-0)-1(0-4)+3(4-0)|$
$=\frac{1}{2}|4+12|$
$=8$ sq units
19. (C) Let the age of swathi 7 years back be ' $x$ ' years

Given varun's 7 years back age $=5 x^{2}$
$\therefore$ Present age of swathi $=(x+7)$ years
Age of swathi after 3 years
$=(x+10)$ years
Age of varun after 3 years
$=\left(5 x^{2}+10\right)$ years
Given $(x+10)=\frac{2}{5}\left(5 x^{2}+10\right)$
$=\frac{2}{5} \times 5\left(x^{2}+2\right)$
$=x+10=2 x^{2}+4$
$=2 x^{2}-x-6=0$
$=2 x^{2}-4 x+3 x-6=0$
$=2 x(x-2)+3(x-2)=0$
$x=2$
Present age of swathi $=x+7=9$ years
Present age of varun $=5 x^{2}+7=20+7=27$
$\therefore$ Sum of their present ages
$=9 y+27 y=36$ years
20. (A) Let the width of the path be ' $x$ ' $m$

$\therefore$ Outer length $=(19+2 x)$ metres
Outer breadth $=(16+2 x)$ metres
$\therefore$ Given area of the path $=200$ sq metres
$\Rightarrow(19+2 x)(16+2 x)-19 \times 16=200$
$\Rightarrow 19 \times 16+38 x+32 x+4 x^{2}-19 \times 16=200$
$4 x^{2}+70 x=200$
$2 x^{2}+35 x=100$
$2 x^{2}+35 x-100=0$
$2 x^{2}+40 x-5 x-100=0$
$2 x(x+20)-5(x+20)=0$
$x=-20$ (or) $x=\frac{5}{2}=2.5 \mathrm{~m}$
21. (B) Given $a+7 d=40 \rightarrow(1)$

$$
a+13 d=73 \rightarrow(2)
$$

equ (2) - (1)
$\Rightarrow 6 d=73-40=33$
$d=\frac{33}{6}=5.5$
$\therefore \mathrm{a}+7 \times 5.5=40$
$a+38.5=40$
$\mathrm{a}=1.5$
$\therefore a_{20}=a+19 d$
$=1.5+19 \times 5.5=1.5+104.5=106$
22. (D) Given 100, 102, 104, 1232 aren in AP
$\therefore a=100, d=102-100=2 \& a_{n}=1232$
$a+(n-1) d=1232$
$100+(n-1)(2)=1232$
$(n-1)(2)=1132$
$n-1=\frac{1132}{2}=566$
$n=567$

$$
\begin{aligned}
& S_{n}=\frac{n}{2}\left(a+a_{n}\right)=\frac{567}{2}(100+1232) \\
& =\frac{567}{2} \times 1332=377622
\end{aligned}
$$

23. (C) LHS

$$
=\left[\frac{1}{\frac{1}{\cos ^{2} \theta}-\cos ^{2} \theta}+\frac{1}{\frac{1}{\sin ^{2} \theta}-\sin ^{2} \theta}\right] \times \sin ^{2} \theta \cos ^{2} \theta
$$

$$
=\left[\frac{1}{\frac{1-\cos ^{4} \theta}{\cos ^{2} \theta}}+\frac{1}{\frac{1-\sin ^{4} \theta}{\sin ^{2} \theta}}\right] \times \sin ^{2} \theta \cos ^{2} \theta
$$

$$
=\left[\frac{\cos ^{2} \theta}{1-\cos ^{4} \theta}+\frac{\sin ^{2} \theta}{1-\sin ^{4} \theta}\right] \times \sin ^{2} \theta \cos ^{2} \theta
$$

$$
=\left[\frac{\cos ^{2} \theta}{\left(1-\cos ^{2} \theta\right)\left(1+\cos ^{2} \theta\right)}+\frac{\sin ^{2} \theta}{\left(1-\sin ^{2} \theta\right)\left(1+\sin ^{2} \theta\right)}\right] \times \sin ^{2} \theta \cos ^{2} \theta
$$

$$
=\left[\frac{\cos ^{2} \theta}{\sin ^{2} \theta\left(1+\cos ^{2} \theta\right)}+\frac{\sin ^{2} \theta}{\cos ^{2} \theta\left(1+\sin ^{2} \theta\right)}\right] \times \sin ^{2} \theta \cos ^{2} \theta
$$

$$
=\left[\frac{\cos ^{2} \theta \times \cos ^{2} \theta\left(1+\sin ^{2} \theta\right)+\sin ^{2} \theta \times \sin ^{2} \theta\left(1+\cos ^{2} \theta\right)}{\sin ^{2} \theta\left(1+\cos ^{2} \theta\right) \cos ^{2} \theta\left(1+\sin ^{2} \theta\right)}\right] \times \sin ^{2} \theta \cos ^{2} \theta
$$

$$
=\left[\frac{\cos ^{4} \theta+\cos ^{4} \theta \sin ^{2} \theta+\sin ^{4} \theta+\sin ^{4} \theta \cos ^{2} \theta}{1+\sin ^{2} \theta+\cos ^{2} \theta+\sin ^{2} \theta \cos ^{2} \theta}\right]
$$

$$
=\left[\frac{\sin ^{4} \theta+\cos ^{4} \theta+\sin ^{2} \theta \cos ^{2} \theta\left(\sin ^{2} \theta+\cos ^{2} \theta\right)}{1+1+\sin ^{2} \theta \cos ^{2} \theta}\right]
$$

$$
=\left[\frac{\left(\sin ^{2} \theta+\cos ^{2} \theta\right)^{2}-2 \sin ^{2} \theta \cos ^{2} \theta+\sin ^{2} \theta \cos ^{2} \theta}{2+\sin ^{2} \theta \cos ^{2} \theta}\right]
$$

$$
=\left[\frac{1-\sin ^{2} \theta \cos ^{2} \theta}{2+\sin ^{2} \theta \cos ^{2} \theta}\right]
$$

24. (A) We know that $3\left(A B^{2}+B C^{2}+C A^{2}\right)$

$$
\begin{aligned}
& =4\left(A D^{2}+B E^{2}+C F^{2}\right) \\
& \therefore A D^{2}+B E^{2}+C F^{2} \\
& =\frac{3}{4}\left(6^{2}+8^{2}+10^{2}\right) \mathrm{cm}^{2} \\
& =\frac{3}{4} \times 200=150 \mathrm{~cm}^{2}
\end{aligned}
$$

25. (D) Let $\mathrm{BD}=x \Rightarrow \mathrm{DC}=(12-x) \mathrm{cm}$
$\therefore \mathrm{CE}=(12-x) \mathrm{cm}$
$\therefore \mathrm{AE}=8-(12-x)=(x-4) \mathrm{cm}$
$\mathrm{BF}=x \& \mathrm{AF}=(10-x) \mathrm{cm}$
But $\mathrm{AE}=\mathrm{AF} \Rightarrow x-4=10-x$
$2 x=14 \mathrm{~cm}$
$x=7 \mathrm{~cm}$
26. (C) Given $\mathrm{OP}=\mathrm{PQ}=\mathrm{QR}=\mathrm{RO} \& \mathrm{OQ}=\mathrm{OP}$

$$
\text { [ } \because \text { radii] }
$$

$\therefore \mathrm{POQ}$ is an equilateral triangle
$\therefore$ Area of an equilateral triangle
$=\frac{\sqrt{3}}{4} \mathrm{r}^{2}=\frac{32 \sqrt{3}}{2} \mathrm{~cm}^{2}$
$\therefore r^{2}=\frac{16 \times 4 \sqrt{3}}{\sqrt{3}} \mathrm{~cm}^{2}$
$r=\sqrt{64 \mathrm{~cm}^{2}}=8 \mathrm{~cm}$
27. (C) The denominator of option ' $C$ ' is having 7 which a prime number other than 2 and 5
$\therefore$ Option ' C ' is non terminating but repeating decimal
28. (A) Given $B D=61 \sqrt{2} \mathrm{~m}$


In $\triangle C D E \tan (90-\theta)=\frac{2 x}{30.5 \sqrt{2} m}$
$\cot \theta=\frac{2 x}{30.5 \sqrt{2} \mathrm{~m}}$
$\therefore \tan \theta=\frac{30.5 \sqrt{2} \mathrm{~m}}{2 x}$
From (1) \& (2)
$\frac{x}{30.5 \sqrt{2}}=\frac{30.5 \sqrt{2}}{2 x}$
$2 x^{2}=(30.5)^{2} \times 2$
$x=30.5 \mathrm{~m}$
$\therefore \quad$ Height of poles are $30.5 \mathrm{~m} \& 61 \mathrm{~m}$
29. (C) Const: $C E \perp A D$
$A D \perp A B$ and $C B \perp A B$

$\therefore$ ABCE is a rectangle
$\therefore A E=B C=r \mathrm{~cm}$
But DA $=\mathrm{Rcm}$
$A E+E D=R \mathrm{~cm}$
$\mathrm{rcm}+\mathrm{ED}=\mathrm{Rcm}$
$E D=(R-r) c m$
In $\triangle C D E, \angle E=90^{\circ}$
$\Rightarrow \mathrm{CD}^{2}=\mathrm{DE}^{2}+\mathrm{EC}^{2}$
$d^{2}=(R-r)^{2}+E C^{2}$
$E C^{2}=d^{2}-(R-r)^{2}$
$E C=\sqrt{d^{2}-(R-r)^{2}}$
$\therefore A B=E C=\sqrt{d^{2}-(R-r)^{2}}$
$[\because E C=A B]$
30. (B) Given $2 \pi R=132 \mathrm{~cm}$
$\Rightarrow 2 \times \frac{22}{7} \times \mathrm{R}=132 \mathrm{~cm}$
$R=132 \times \frac{7 \mathrm{~cm}}{44}=21 \mathrm{~cm}$
$2 \pi r=\frac{440}{7} \mathrm{~cm}$

$$
\begin{aligned}
& \frac{44}{7} r=\frac{440}{7} \mathrm{~cm} \\
& r=10 \mathrm{~cm} \\
& h=\sqrt{l^{2}-(R-r)^{2}}=\sqrt{61^{2}-(21-10)^{2}} \\
& =\sqrt{3721-121} \\
& =\sqrt{3600}=60 \mathrm{~cm}
\end{aligned}
$$

Volume of the frustum of a cone
$=\frac{\pi h}{3}\left(R^{2}+R r+r^{2}\right)$
$=\frac{22}{7} \times \frac{60}{3} \mathrm{~cm}\left(21^{2}+21 \times 10+10^{2}\right)$
$=\frac{440}{7}(441+210+100)$
$=\frac{440}{7} \times 751 \mathrm{~cm}^{3}$
$=\frac{330440}{7} \mathrm{~cm}^{3}=47,205 \frac{5}{7} \mathrm{~cm}^{3}$

## MATHEMATICS - 2

31. (A, D)

Sum of first 60 natural numbers
$=\frac{n(n+1)}{2}=\frac{60 \times 61}{2}=30 \times 61$
$\therefore 30$ and 61 are the factors of the sum of first 60 natural numbers
32. (A, B, C)

Options $A, B, \& C$ are true but all rational numbers are real but converse is not true.
33. (A,B,C,D)

## Option A

$\Delta=16^{2}-4 \times 2 \times 3=256-24=232>0$
$\therefore$ Option A has real roots

## Option B

$\Delta=10^{2}-4 \times 2(-1)=100+8=108>0$
$\therefore$ Option B has real roots

Option C
$\Delta=(-8)^{2}-4 \times 1=64-4=0$
$\Delta>0 \Rightarrow$ It has real roots
Option D
$\Delta=9^{2}-4 \times 4 \times(-6)$
$=81+96$
$\Delta>0$
It has real roots
34. (A, B, C, D)

Option A,
$\cos 50^{\circ}=\cos \left(90^{\circ}-40^{\circ}\right)=\sin 40^{\circ}$
$\therefore \sin 57^{\circ}>\sin 40^{\circ}$ ie $\sin 57^{\circ}>\cos 50^{\circ}$
Option B,
$\cos 80^{\circ}=\cos \left(90^{\circ}-10^{\circ}\right)$
$\cos 20^{\circ}=\cos \left(90^{\circ}-70^{\circ}\right)$

$$
=\sin 70^{\circ}
$$

$\therefore \sin 10^{\circ}<\sin 70^{\circ}$
$\therefore \cos 80^{\circ}<\cos 20^{\circ}$
Option C,
$\operatorname{cosec} 63^{\circ}=\frac{1}{\sin 63^{\circ}} \&$
$\sec 5^{\circ}=\frac{1}{\cos 5^{\circ}}=\frac{1}{\cos \left(90^{\circ}-5^{\circ}\right)}=\frac{1}{\sin 85^{\circ}}$
$\sin 85^{\circ}>\sin 63^{\circ} \Rightarrow \frac{1}{\sin 85^{\circ}}<\frac{1}{\sin 63^{\circ}}$
i.e., $\sec 5^{\circ}<\operatorname{cosec} 63^{\circ}$
$\cot 40^{\circ}=\cot \left(90^{\circ}-50^{\circ}\right)=\tan 50^{\circ}$
$\therefore \tan 50^{\circ}>\tan 40^{\circ}$ ie $\cot 40^{\circ}>\tan 40^{\circ}$
35. (B, C, D)

Regular polygons with equal number of sides are similar

Hence option ' $A$ ' is false
Remaining all options are true.

## REASONING

36. (B) In order to solve these kinds of series, we should fill the given blanks by taking each option one by one and see where it forms a logical pattern. When you try to fill the first option, it becomes
cabbabcabcacbabcac
It doesnot result into any particular logical pattern. If you fill the second option you get

## cabbaccabbaccabbac

This becomes a pattern of writing 'cabbac' again and continuously. So, option (B) is correct.
37. (C) Both $A B$ and $C D$
38. (C)

1. $G$ is to the immediate right of $D$ and to the immediate left of $B=D G B$
2. $A$ is on the immediate right of $C=C A$
3. $A$ and $D$ have one child between them

- Combining statements 1,2 and $3, \mathrm{CA}$ has to be left of DGB. Therefore CA_DGB (since $\mathrm{D}, \mathrm{G}$ and B are together, the only way $A$ and $D$ can have one child between them is if $A$ is to the left of $D$ )

4. $E$ and $B$ have two children between them

- From conclusion in point 3 and statement 4 we can say that E will come between CA and DGB. Therefore CAEDGB

5. D and $F$ have two children between them

- From conclusion in point 4 and statement 5 we can say that $F$ will come on extreme right as if on extreme left it will lead to 3 children between $D$ and $F$.

Hence correct order would be CAEDGBF.
Therefore F is on extreme right.
39. (A) Let the name of the woman showing the photograph as $X$ and the name of the man in the photograph as Y .

Y's brother means Y's father and he is the only son of X 's grandfather. This clearly indicates that $Y$ 's father and $X$ 's father is one and the same. So, X is Y 's sister.
40. (C)

41. (A) $5: 70 \div 14=5 ; 91 \div 13=7,120 \div 24=5$;
42. (D) Given 3 colours Red, blue and white wear by Sachin, Ravi and Ajay. We don't know the $4^{\text {th }}$ colour. So, we can't say which colour Sohan wear.

|  | Red | Blue | White |
| :---: | :---: | :---: | :---: |
| Ravi | $X$ | $\checkmark$ | $X$ |
| Ajay | $X$ | $X$ | $\checkmark$ |
| Sohan | $X$ | $X$ |  |
| Sachin | $\checkmark$ | $X$ | $X$ |

43. (D) Harika started from A , moved 75 m upto B, turned left and waked 25 m upto C. She then turned left again and moved 80 m upto $D$. Turning to the right at an angle of $45^{\circ}$, she was finally moving in the direction DE. i.e., south west.

44. (C) Clearly, all cows ( X ) are animals ( Y ). This is represented by two concentric circles. But some cows and some animals can be white in colour (Z).

So, the circle (Z) which represents white intersects the other two concentric circles.

45. (C) The pattern is as follows


## CRITICAL THINKING

46. (A)


Set 1

The out put when the switches $R, Q, S$


Set 2


Hence 3 and 4 lights are in reverse order. So, switch (S) is fault.
47. (B) Intermittently anti-clockwise
48. (C) The prices of petroleum products being stagnant in the domestic market and the increase in the same in the international market.

So, they both are causes of different events.
49. (A) Number of surfaces does triangular have 5 Number of surfaces does triangle have 3 3 triangles means $3+3+3=9$

Total no. of surfaces are $=9+5=14$
50. (A) It is mentioned in the statement that India's economy depends mainly on forests. This means that forests should be preserved. So, I follows. But, that only preservation of forests can improve the economy, cannot be said. So, II does not follow.

