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UNIFIED COUNCIL OLYMPIADS

UIMO
Unified International Mathematics Olympiad

# UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)

***CLASS - 8***

***Question Paper Code : UM9269***

## KEY

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| D | D | B | A | B | C | B | B | A | C |
| **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
| D | A | B | C | C | B | D | B | B | B |
| **21** | **22** | **23** | **24** | **25** | **26** | **27** | **28** | **29** | **30** |
| C | A | Delete | C | B | D | D | B | B | C |
| **31** | **32** | **33** | **34** | **35** | **36** | **37** | **38** | **39** | **40** |
| A,D | A,B,D | A,C | A,B | A,C | D | B | B | A | C |
| **41** | **42** | **43** | **44** | **45** | **46** | **47** | **48** | **49** | **50** |
| C | C | B | D | B | A | C | B | D | A |

## EXPLANATIONS

### MATHEMATICS - 1

01. (D) $(x^2 - 5)^2 = 16$

$(x^2 - 5) = \pm \sqrt{16}$

$x^2 - 5 = \pm 4$

$\therefore x^2 - 5 = 4$

$x^2 = 4 + 5$

$x^2 = 9$

$x = \pm \sqrt{9}$

$x = \pm 3$

(or)

$x^2 - 5 = -4$

$x^2 = -4 + 5$

$x^2 = 1$

$x = \pm \sqrt{1}$

$x = \pm 1$

Hence four integers satisfy this equation

02. (D) $2^8 + 1 = 256 + 1 = 257$

$\therefore$ $2^{18} + 1 = (2^6)^3 + 1 = 64^3 + 1$

$257 < 7^3 < 64^3 < 64^3 + 1$

$\therefore$ No. of perfect cubes = 64 – 7 + 1 = 58

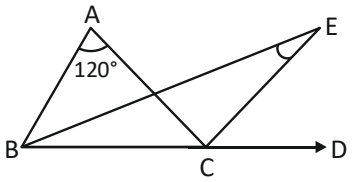
03. (B) $\angle ACD = \angle A + \angle ABC$

$\therefore \quad \angle ACE = \frac{\angle ACD}{2} = \frac{\angle A + \angle ABC}{2}$

In $\Delta BCE$, $\angle EBC + \angle E + \angle BCE = 180°$

$\frac{\angle ABC}{2} + \angle ACB + \frac{\angle A}{2} + \frac{\angle ABC}{2} + \angle E\ 180°$

$\frac{\angle A}{2} + \angle ABC + \angle ACB + \angle E = 180°$




$\angle E = 180° - \angle ABC - \angle ACB - \frac{\angle A}{2}$

$= \angle A + \angle ABC + \angle ACB - \angle ABC - \angle ACB - \frac{\angle A}{2}$

[$\because$ In $\Delta ABC$ $\angle A + \angle ABC + \angle ACB = 180°$]

$= \angle A - \frac{\angle A}{2}$

$= \frac{2\angle A - \angle A}{2}$

$= \frac{\angle A}{2} \Rightarrow \frac{120°}{2} = 60°$

04. (A) $\sqrt[3]{-2\times2\times2\times7\times7} \times \sqrt[3]{4\times4\times4\times7}$

$\Rightarrow \sqrt[3]{-2\times2\times2\times7\times7\times7\times4\times4\times4}$

$\Rightarrow -2 \times 7 \times 4$

$\Rightarrow -56$

05. (B) First term

$= \sqrt{1+1+\frac{1}{4}} = \sqrt{\frac{9}{4}} = \frac{3}{2} = 2 - \frac{1}{2}$

Sum of first two terms $= \frac{3}{2} + \sqrt{1+\frac{1}{4}+\frac{1}{9}}$

$= \frac{3}{2} + \sqrt{\frac{36+9+4}{36}} = \frac{3}{2} + \frac{7}{6} = \frac{9+7}{6} = \frac{\cancel{16}^{8}}{\cancel{6}_{3}}$

$= 3 - \frac{1}{3}$

Sum of first three terms

$= \frac{8}{3} + \sqrt{1+\frac{1}{9}+\frac{1}{16}} = \frac{8}{3} + \frac{13}{12}$

$\frac{45}{12} = \frac{15}{4} = 4 - \frac{1}{4}$

$\therefore$ Sum of all terms $= 2021 - \frac{1}{2021}$

06. (C) Given $a - x = \sqrt{x^2+1}$

squaring on both sides

$(a - x)^2 = (\sqrt{x^2+1})^2$

$a^2 - 2ax + \cancel{x^2} = \cancel{x^2} + 1$

$a^2 - 1 = 2ax$

$2x = \frac{a^2-1}{a} = a - \frac{1}{a} = (a - a^{-1})$

$\therefore \quad x = \frac{1}{2}(a - a^{-1})$

07. (B) $\left[\frac{1}{x-2} - \frac{4}{x^2-4}\right] =$

$\left[\frac{x+2-4}{x^2-4}\right] = \frac{(x-2)}{(x+2)(x-2)} = \frac{1}{(x+2)}$

08. (B) $(3x + 8x^2)^2 + (3x^2 + 8x)^2$

$= 9x^2 + 48x^3 + 64x^4 + 9x^4 + 48x^3 + 64x^2$

$= 9(x^2 + x^4) + 96x^3 + 64(x^4 + x^2)$

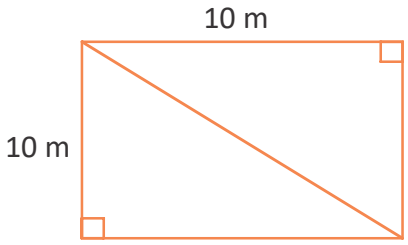
$= 9(x^2 + x^3 \times x) + 96(1) + 64[x \times x^3 + x^2]$

$= 9(x^2 + x) + 96 + 64(x + x^2)$

$= 9(-1) + 96(1) + 64(-1)$

$= 96 - 73 = 23$

09. (A)




Diagonal of the (base) rectangle

$= \sqrt{l^2 + b^2} = \sqrt{10^2 + 10^2}$

$= \sqrt{200}$

$\therefore$ length of the longest pole

$= \sqrt{(\text{base diagonal})^2 + h^2}$

$= \sqrt{(\sqrt{200})^2 + (5\text{m})^2}$

$= \sqrt{225}$ m

= 15 m

10. (C) Given $x + \frac{9}{x} = 6 \Rightarrow \frac{x^2 + 9}{x} = 6$

$\Rightarrow x^2 - 6x + 9 = 0$

$\Rightarrow x^2 - 3x - 3x + 9 = 0$

$\Rightarrow x = 3$

$\therefore x^2 + \frac{9}{x^2} = 9 + \frac{9}{9} = 9 + 1 = 10$

11. (D) Area of the shaded region = Total area – inner rectangle area

$= (2x + 3)(2x - 3)\ \text{cm}^2 - (x + 1)(x + 5)\ \text{cm}^2$

$= [4x^2 - 9 - (x^2 + 6x + 5)]\ \text{cm}^2$

$= [4x^2 - 9 - (x^2 + 6x + 5)]\ \text{cm}^2$

$= (3x^2 - 6x - 14)\ \text{cm}^2$

12. (A) Given QRS is divisible by 5 $\Rightarrow$ $\boxed{S = 5}$

Given RST is divisible by 3

$\Rightarrow$ R + T = 4, R + T = 7

Given PQR is divisible by 4

$\Rightarrow$ QR may be 12, 24, 32, 52

If R = 2 then R + T = 4 $\Rightarrow$ T = 2 But T $\neq$ 2

If R = 2 then R + T = 7 $\Rightarrow$ T = 5 But T $\neq$ 5

$\therefore$ $\boxed{R = 4} \Rightarrow R + T = 7 \Rightarrow \boxed{T = 3}$

If R = 4 then Q must be 2 $\Rightarrow$ $\boxed{Q = 2}$

$\therefore$ P = 1

$\therefore$ The required number = 12453

13. (B) $\frac{23}{2^5 \times 5^6} = \frac{23}{10^5 \times 5} = \frac{4.6}{10^5} = \frac{4.6}{100000}$

= 0.000046

14. (C) Given $\frac{x^2 + 1}{x} = 2$

$\Rightarrow x^2 + 1 = 2x$

$\Rightarrow x^2 - 2x + 1 = 0$

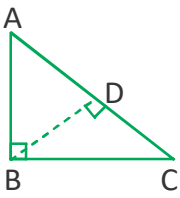
$\Rightarrow (x - 1)^2 = 0$

$x - 1 = 0$

$x = 1$

$\therefore x^2 + \frac{1}{x^8} = (1)^2 + \frac{1}{(1^8)} = 1 + 1 = 2$

15. (C)




In $\Delta ABC$, $\angle ABC = 90° \Rightarrow AC^2 = AB^2 + BC^2$

$= 12^2 + 16^2$

$= 144 + 256$

$AC = \sqrt{400}$

AC = 20 cm

Area of ABC = $\frac{1}{2} \times AC \times BD = \frac{1}{2} \times AB \times BC \Rightarrow 20\ \text{cm} \times BD = 12 \times 16\ \text{cm}^2$

$BD = \frac{12 \times 16^{4}\ \text{cm}^2}{20_{5}\ \text{cm}} = 9.6$ cm

16. (B) $3x^2 + \sqrt{60}\,xy + 5y^2 = (\sqrt{3}\,x)^2 + 2\sqrt{15}\,xy + (\sqrt{5}\,y)^2$

$= (\sqrt{3}\,x)^2 + 2\sqrt{3}\,x + \sqrt{5}\,y + (\sqrt{5}\,y)^2$

$= (\sqrt{3}\,x + \sqrt{5}\,y)^2$

$\therefore (\sqrt{3}\,x + \sqrt{5}\,y)$ is a factor of $(3x^2 + \sqrt{60}\,xy + 5y^2)$

17. (D) Given AB||DC and AD is not parallel to BC

$\therefore$ $\angle DAC \neq \angle ACB$

$\therefore$ we cannot find $\angle ACB$

18. (B) Given $\pi R^2 = \pi r_1^2 + \pi r_2^2$

$\Rightarrow$ $\cancel{\pi} R^2 = \cancel{\pi}(63^2 + 16^2)\text{cm}^2$

$R^2 = (3969 + 256)\text{ cm}^2$

$R = \sqrt{4225}\text{ cm}^2$

$R = 65$ cm

$\therefore$ Diameter $= 2R = 130$ cm

19. (B) Given $a^3 = 1728\text{ cm}^3 = (12\text{ cm})^3$

$\therefore$ $a = 12$ cm

$\therefore$ $l = a + a = 24$ cm, $b = 12$ cm and $h = 12$ cm

surface area $= 2(lb + bh + bl)$

$= 2(24 \times 12 + 12 \times 12 + 12 \times 24)\text{ cm}^2$

$= 2 \times 12 \times 12 (2 + 1 + 2)\text{ cm}^2$

$= 1440\text{ cm}^2$

20. (B) Total balls $= 9 + 12 = 21$

$\therefore$ Probability of drawing a black ball

$$= \frac{\text{No. of black balls}}{\text{Total balls}} = \frac{9}{21} = \frac{3}{7}$$

21. (C) The number of revolutions is directly proportional to the number of hours.

$\therefore 19500 : x = 3 : 7$

$$\Rightarrow x = \frac{\cancel{19500}^{6500} \times 7}{3} = 45500$$

$\therefore$ The wheel revolves 45,500 times in 7 hours

22. (A) Top view of a cube is square

23. (Delete)

24. (C) $8^3 = (2^3)^3 = 2^9$

$4^{4.5} = (2^2)^{4.5} = 2^{2 \times 4.5} = 2^9$

$\therefore$ $$\frac{8^3}{(4^{4.5})} = \frac{2^9}{2^9} = 2^{9-9} = 2^0 = 1$$

25. (B) $$\frac{3^{2023} - 3^{2022} + 3^{2021}}{3^{2022} + 3^{2021} - 3^{2020}} = \frac{3^{2021}(3^2 - 3 + 1)}{3^{2020}(3^2 + 3 - 1)}$$

$$= 3^{2021-2020}\frac{(9 - 3 + 1)}{(9 + 3 - 1)}$$

$$= 3 \times \frac{7}{11} = \frac{21}{11}$$

26. (D) $\sqrt{(1234567)^2 - 2469133}$

$= \sqrt{(1234567)^2 - 2469133 - 1 + 1}$

$= \sqrt{(1234567)^2 - 2469134 + 1}$

$= \sqrt{(1234567)^2 - 2(1234567)(1) + 1^2}$

$= \sqrt{(1234567 - 1)^2}$

$= 1234566$

27. (D) Given QR = PR

$\Rightarrow 4x - 7 = 2x + 5$

$4x - 2x = 5 + 7$

$2x = 12$

$$x = \frac{12}{2} = 6$$

$\therefore$ $PR = 2x + 5 = 2 \times 6 + 5 = 17$ cm

$\therefore$ $PQ = 17$ cm

28. (B) $$\left(\frac{x^a}{x^b}\right)^{a^2 + ab + b^2} = \left(x^{a-b}\right)^{(a^2 + ab + b^2)}$$

$$= x^{(a-b)(a^2 + ab + b^2)}$$

$$= x^{a^3 + \cancel{a^2b} + \cancel{ab^2} - \cancel{a^2b} - \cancel{ab^2} - b^3}$$

$$= x^{a^3 - b^3}$$

Similarly $\left(\frac{x^b}{x^c}\right)^{(b^2 + bc + ca)} = x^{b^3 - c^3}$ and

$\left(\frac{x^c}{x^a}\right)^{(c^2 + ac + a^2)} = x^{c^3 - a^3}$

$\therefore \quad \left(\frac{x^a}{x^b}\right)^{(a^2+ab+a^2)} \left(\frac{x^b}{x^c}\right)^{(b^2+bc+c^2)}$

$\left(\frac{x^c}{x^a}\right)^{(c^2+ca+a^2)} = x^{a^3-b^3} x^{b^3-c^3} x^{c^3-a^3}$

$= x^{a^3-b^3+b^3-c^3+c^3-a^3}$

$= x^0$

$= 1$

29. (B) $90° \_\_\_\_ 180$

$? \_\_\_\_ 400$

$\Rightarrow \quad 400 \times \frac{90°}{180°} = 200°$

Angle of sector showing oranges = 200°

$\therefore \quad 90° + 200° + x° = 360°$

$\Rightarrow \quad x = 360° - 290° = 70°$

30. (C) It is in inverse proportion.

$\therefore \quad x_1 y_1 = x_2 y_2$

$18 \times 10 = x_2 \times 6$

$x_2 = \frac{\cancel{18}^{3} \times 10}{\cancel{6}_{1}} = 30$

$\therefore$ More men required = 30 – 18 = 12

## MATHEMATICS - 2

31. (A, D)

Given $\frac{3^x}{3^2} + \frac{3^3}{3^x} = 4 \Rightarrow \frac{(3^x)^2 + 3^5}{3^x \times 9} = 4$

$\Rightarrow \quad (3^x)^2 + 3^5 = 36 + 3^x$

$\Rightarrow \quad (3^x)^2 - 36 \times 3^x + 243 = 0$

let $3^x = a \Rightarrow a^2 - 36a + 243 = 0$

$\Rightarrow \quad a^2 - 27a - 9a + 243 = 0$

$\Rightarrow \quad a(a - 27) - 9(a - 27) = 0$

$\Rightarrow \quad (a - 27)(a - 9) = 0$

$\Rightarrow \quad a = 27 \quad$ (or) $\quad a = 9$

$\Rightarrow \quad 3^x = 3^3 \quad$ (or) $\quad 3^x = 3^2$

$\therefore \quad x = 3 \quad$ (or) $\quad 2$

32. (A, B, D)

LHS = $\sqrt{(x^2+x-12)(x^2-x-20)(x^2-8x+15)}$

$= \sqrt{(x+4)(x-3)(x+4)(x-5)(x-3)(x-5)}$

$= \sqrt{(x-3)^2(x+4)^2(x-5)^2}$

$= (x-3)(x+4)(x-5) = (x+4)(x^2-8x+15)$

$= (x-5)(x^2+x-12)$

$= x(x^2+x-12) - 5(x^2+x-12)$

$= x^3 + x^2 - 12x - 5x^2 - 5x + 60$

$= (x^3 - 4x^2 - 17x + 60)$

33. (A, C)

$6 - \frac{7}{x} - \frac{20}{x^2} = \frac{6x^2 - 7x - 20}{x^2}$

$= \frac{1}{x^2}(6x^2 - 15x + 8x - 20)$

$= \frac{1}{x^2}[3x(2x-5) + 4(2x-5)]$

$= \frac{1}{x^2}(2x-5)(3x+4)$

$= \left(\frac{2x-5}{x}\right)\left(\frac{3x+4}{x}\right)$

$= \left(2 - \frac{5}{x}\right)\left(3 + \frac{4}{x}\right)$

34. (A, B)

In a square & rectangle diagonals are equal because SAS congruency.

In a parallelogram adjacent angles need not be equal.

$\therefore$ In a parallelogram diagonals need not be equal.

35. (A, C)

Given $\sqrt[3]{5x} = \sqrt{2x}$

Rising 6th power on both sides

$\left(\sqrt[3]{5x}\right)^6 = \left(\sqrt{2x}\right)^6$

$(5x)^2 = (2x)^3$

$\therefore \quad 25x^2 = 8x^3$

$\Rightarrow$ $8x^3 - 25x^2 = 0$

$x^2 (8x - 25) = 0$

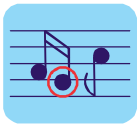
$x^2 = 0$ (OR) $8x - 25 = 0$

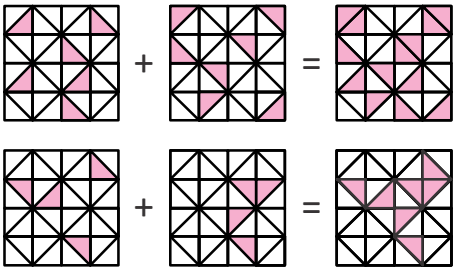
$x = 0$ (OR) $x = \frac{25}{8}$

## REASONING

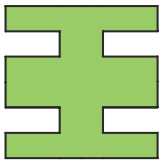
36. (D) Missing number is 16. Alphabets are starting from A, B, C, ..... K.

Every time the alphabet are place before and after the number. Every time difference between two numbers is increased by 0, 1, 2, 3, 4, 5, 6, 7, 8 ...

37. (B) 

38. (B) 

39. (A) The cut-out is apparent in more than one quadrant.



40. (C) dionot = oak tree

blyonot = oak leaf

blycrin = maple leaf

oak = onot

leaf = bly

maple = crin

Hence maple syrup = patricrin

41. (C) Option (A) : $3 \div 4 \times 2 = 9 \div 3 - 3$

$3 / 2 = 0$ (wrong)

Option (B) : $5 + 3 - 7 > 8 \div 4 \div 1$

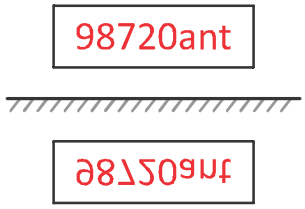
$1 > 2$ (wrong)

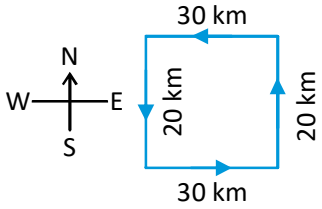
Option (C) : $5 \times 2 \div 2 < 10 - 4 + 8$

$5 < 14$ (correct)

Option (D) : $3 + 2 - 4 > 16 \times 2 \div 4$

$1 > 8$ (wrong)

42. (C) 


43. (B) 


44. (D) All students are men, some men are students, some men are sportsperson are all correct statements according to the given diagram.

45. (B) $6^{th} \rightarrow$ Tuesday

(2 days preceding Thursday)

$13^{th} \rightarrow$ Tuesday

$20^{th} \rightarrow$ Tuesday

$21^{st} \rightarrow$ Wednesday

$22^{nd} \rightarrow$ Thursday

$23^{rd} \rightarrow$ Friday

$24^{th} \rightarrow$ Saturday

$25^{th} \rightarrow$ Sunday

$26^{th} \rightarrow$ Monday

Hence $25^{th}$ of the month will be sunday and is followed by $26^{th}$ on monday.

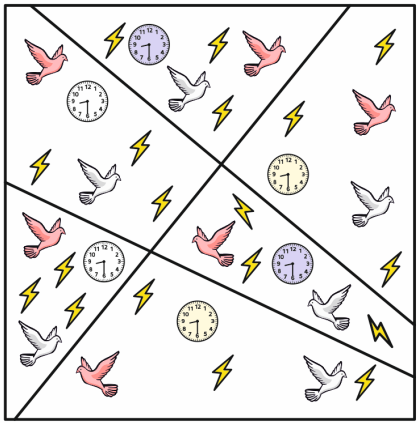
## CRITICAL THINKING

46. (A) If Statement I is the 'Cause' and the Statement II is the 'Effect'.

47. (C) Rope X and Z

50 kg presses in the downward direction. So the rope Y is needed to counter it and press in upward direction. Now the forces are balanced and the ropes X and Z are not needed.

48. (B)  is not possible, same colour cubes in two different places.

49. (D) 

50. (A) 

The End