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UNIFIED COUNCIL OLYMPIADS

UIMO
Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)

CLASS - 9
Question Paper Code : UM9269

KEY

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B	B	D	A	B	C	B	A	B	A
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	A	C	B	C	A	D	C	C	A
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
D	C	B	A	A	D	A	A	C	A
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B,C,D	B,D	A,C,D	A,B,C,D	B,D	D	C	B	C	D
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
A	C	B	A	C	A	A	D	B	D

EXPLANATIONS

MATHEMATICS - 1

01. (B) $\dfrac{3-\sqrt{5+x}}{(x-4)} = \dfrac{3-\sqrt{5+x}}{(x-4)} \times \dfrac{3+\sqrt{5+x}}{3+\sqrt{5+x}}$

$$= \frac{3^2-\left(\sqrt{5+x}\right)^2}{(x-4)\left(3+\sqrt{5+x}\right)}$$

$$= \frac{9-5-x}{(x-4)\left(3+\sqrt{5+x}\right)}$$

$$= \frac{4-x}{(x-4)\left(3+\sqrt{5+x}\right)}$$

$$= \frac{-1\cancel{(x-4)}}{\cancel{(x-4)}\left(3+\sqrt{5+x}\right)}$$

02. (B) Given OPQR is a rectangle

$\Rightarrow$ OQ = PR = r

$\therefore$ PR = 5 cm

03. (D) $x^2 - 3x - 4 = x^2 - 4x + x - 4$

$= x(x - 4) + 1\,(x - 4)$

$= (x + 1)\,(x - 4)$

$$\begin{array}{r|l} & \quad x^2 + x - 3 \\ \hline x + 1 & \quad x^3 + 2x^2 - 2x - 3 \\ & \quad x^3 + x^2 \\ & \quad (-)\ (-) \\ \cline{2-2} & \qquad x^2 - 2x - 3 \\ & \qquad x^2 + x \\ & \qquad (-)\ (-) \\ \cline{2-2} & \qquad\quad -3x - 3 \\ & \qquad\quad -3x - 3 \\ & \qquad\quad (+)\ (+) \\ \cline{2-2} & \qquad\qquad 0 \end{array}$$

$(x + 1)$ is a factor of $(x^3 + 2x^2 - 2x - 3)$

HCF of $(x^2 - 3x - 4)$ and $(x^3 + 2x^2 - 2x - 3)$

$= (x + 1)$

04. (A) $x^4 - 625 = (x^2)^2 - (25)^2$

$= (x^2 - 25)\,(x^2 + 25)$

$= (x^2 - 25)\,[x^2 + 5^2 + 10x - 10x]$

$= (x^2 - 25)\left[(x+5)^2 - \left(\sqrt{10x}\right)^2\right]$

$= (x^2 - 25)\,(x + \sqrt{10x} + 5)\,(x - \sqrt{10x} + 5)$

05. (B) $\dfrac{3x^2}{5} - \dfrac{11x}{5} - 4 = \dfrac{3x^2 - 11x - 20}{5}$

$= \dfrac{1}{5}\left[3x^2 - 15x + 4x - 20\right]$

$= \dfrac{1}{5}\left[3x(x-5) + 4(x-5)\right]$

$= \dfrac{1}{5} \times (3x+4)(x-5)$

$\therefore$ $(3x + 4)$ is a factor of $\dfrac{3x^2}{5} - \dfrac{11x}{5} - 4$

(or)

$\dfrac{3x^2}{5} - \dfrac{11x}{5} - 4 = \dfrac{3x^2}{5} - 3x + \dfrac{4x}{5} - 4$

$= 3x\left(\dfrac{x}{5} - 1\right) + 4\left(\dfrac{x}{5} - 1\right)$

$= \left(\dfrac{x}{5} - 1\right)(3x + 4)$

06. (C) $\sqrt{120 - 30\sqrt{15}} = \sqrt{120 - 2 \times 15\sqrt{15}}$

$= \sqrt{120 - 2\sqrt{15 \times 15 \times 15}}$

$= \sqrt{75 + 45 - 2 \times \sqrt{75 \times 45}}$

$= \sqrt{\left(\sqrt{75}\right)^2 + \left(\sqrt{45}\right)^2 - 2\sqrt{75} \times \sqrt{45}}$

$= \left(\sqrt{75} - \sqrt{45}\right)$

$= \left(5\sqrt{3} - 3\sqrt{5}\right)$

07. (B) $\dfrac{14}{\sqrt{6} - \sqrt{5} - \sqrt{11}} = \dfrac{14}{\left(\sqrt{6} - \sqrt{5}\right) - \sqrt{11}} \times$

$\dfrac{\left(\sqrt{6} - \sqrt{5}\right) + \left(\sqrt{11}\right)}{\left(\sqrt{6} - \sqrt{5}\right) + \sqrt{11}}$

$= \dfrac{14\left(\sqrt{6} - \sqrt{5} + \sqrt{11}\right)}{-\left(2\sqrt{30}\right)}$

$= \dfrac{-7\left(\sqrt{6} - \sqrt{5} + \sqrt{11}\right)}{\sqrt{30}} \times \dfrac{\sqrt{30}}{\sqrt{30}}$

$= \dfrac{-7\left(6\sqrt{5} - 5\sqrt{6} + \sqrt{330}\right)}{30}$

08. (A) $3p(x) + 7q(x) + r(x)$

$= 19x^3 - 15x^2 + 11x + 11$

09. (B) $\sqrt[3]{4}, \sqrt[4]{5}, \sqrt[4]{6}, \sqrt[3]{8}$

$= 4^{1/3}, 5^{1/4}, 6^{1/4}, 8^{1/3}$

L.C.M of 3 & 4 = 12

So, the given surds can be written as,

$= 4^{4/12}, 5^{3/12}, 6^{3/12}, 8^{4/12}$

$= (4^4)^{1/12}, (5^3)^{1/12}, (6^3)^{1/12}, (8^4)^{1/12}$

$= (256)^{1/12}, (125)^{1/12}, (216)^{1/12}, (4096)^{1/12}$

$\therefore$ The smallest one is $\sqrt[4]{5}$.

10. (A) Given (x – 2) is a factor of p(x) $\Rightarrow$ p(2) = 0

$2^3 - 3(2)^2 + p(2) + 24 = 0$

$\Rightarrow$ $8 - 12 + 2p + 24 = 0$

$\Rightarrow$ $2p = -20$

$\therefore$ $p = -10$

Given $(x - 2)$ is a factor of g(x)

$\therefore$ $g(2) = 0$

$(2)^2 - 7(2) + q = 0$

$\Rightarrow$ $4 - 14 + q = 0$

$\Rightarrow$ $-10 + q = 0 \Rightarrow q = 10$

$\therefore$ $p + q = -10 + 10 = 0$

11. (C) Let $x = 2$ & $y = \frac{-5}{2}$ then $5x - 4y$

$= 5(2) - \cancel{4}^{2}\left(\frac{-5}{\cancel{2}}\right)$

$= 10 + 10$

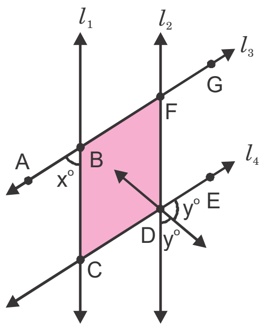
$= 20$

= RHS

$\therefore$ $\left(2, \frac{-5}{2}\right)$ lies on the line $5x - 4y = 20$

12. (A) Infinite number of lines can pass through a single point. So, the statement given in option (A) is the incorrect statement.

13. (C)



$\angle BFD = \angle ABC = x°$

[Corresponding angles]

$\angle FDE = \angle BFD = x°$ [Alternate angles]

$x + 2y = 180°$ [Linear pair]

$y = 90° - \frac{x°}{2}$

14. (B) $\angle BCD + \angle CDE = 180°$

$\Rightarrow \angle BCD + 75° = 180° \Rightarrow \angle BCD = 105°$

$\angle ABC \Rightarrow \angle BCD$

[alternate interior Δ] $= x = 105°$

15. (C) $C = \left(180 \times \frac{1}{6}\right)^{\circ} = 30°$

$\angle ACB + \angle ACD + \angle ECD = 180°$

[a straight angle]

$\Rightarrow$ $30° + 90° + \angle ECD = 180° \Rightarrow \angle ECD = 60°$

16. (A) Given CDE is an equilateral triangle

$\therefore$ $\angle CED = 60°$

In ΔAED, $\angle AED = 90° + 60° = 150°$ & AE = ED

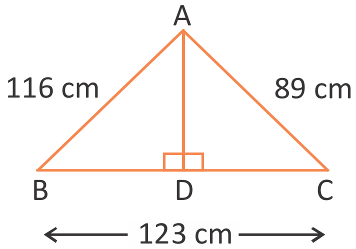
In ΔAED, $\angle EAD + \angle EDA + \angle AED = 180°$

$\angle EAD + \angle EAD + 150° = 180°$

$2\angle EAD = 180° - 150° = 30°°$

$\angle EAD = \frac{30°}{2} = 15°$

17. (D) Given C = 116 cm, a = 123 cm, b = 89 cm



$\therefore$ $S = \dfrac{a+b+c}{2} = \dfrac{(116+123+89)\,cm}{2} = \dfrac{328\,cm}{2}$

Area of DABC = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{164 \times 41 \times 75 \times 48}$ cm^2

$= \sqrt{2\times2\times41\times41\times5\times5\times3\times3\times4\times4}$ cm^2

= 2 × 41 × 5 × 3 × 4 cm^2

$= \dfrac{1}{2}$ × BC × AD = 4920 cm^2

$= \dfrac{1}{2}$ × 123 cm × AD = 4920 cm^2

AD = $\cancel{4920}^{40}$ cm^2 × $\dfrac{2}{\cancel{123}_1 \text{ cm}}$

= 80 cm

18. (C) In ΔARB, P is the mid-point of AB and PD||BR

$\Rightarrow$ D is the mid-point of AR

Since, ABCD is a parallelogram

$\Rightarrow$ DC || AB $\Rightarrow$ DQ || AB

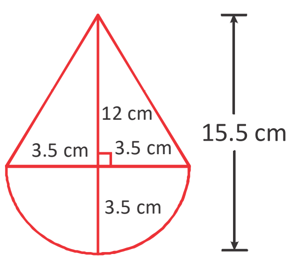
Thus, in ΔRAB, D is the mid-point of AR and DQ || AB

$\therefore$ Q is the mid-point of RB $\Rightarrow$ BR = 2BQ

19. (C) Given r = 3.5 cm and r + h = 15.5 cm

$\therefore$ h = 15.5 cm – 3.5 cm

h = 12 cm



Given height of cone (h) = 12 cm and radius = 3.5 cm

$\therefore$ Slant height of cone (l) = $\sqrt{h^2+r^2}$

$= \sqrt{12^2+3.5^2}$

$= \sqrt{144+12.25}$

$= \sqrt{156.25}$

$l = 12.5\,cm$

Total surface area of the toy = CSA of the cone + CSA of the hemisphere

$= \pi r l + 2\pi r^2$

$= \pi r(l+2r)$

$= \dfrac{22}{7}\times 3.5(12.5+2\times3.5)\,cm^2$

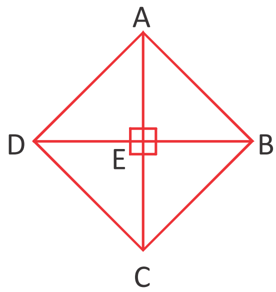
= 11 × 19.5 cm^2

= 214.5 cm^2

20. (A) Given 4s = 404 m

$\therefore$ s = $\dfrac{404}{4}$ m

s = 101 m



Given AC = 198 m

$\therefore$ AE = $\dfrac{AC}{2}$ = 99 m

In ΔAEB, $\angle$AEB = 90 $\Rightarrow$ AB^2 = AE^2 + EB^2

101^2 = 99^2 + EB^2

101^2 – 99^2 = EB^2

EB = $\sqrt{(101+99)(101-99)}$

$= \sqrt{200\times2}$ = 20

$\therefore$ BD = 2 × 20 m = 40 m

Area of the field = $\frac{1}{2}$ × AC × BD

$= \frac{1}{\cancel{2}_1} \times 198 \times \cancel{40}^{20}$ m^2

= 3960 m^2

21. (D) $\angle$POT = 2(25°) = 50°

$x° = \frac{180° + 50°}{2} = 115°$

22. (C) Volume of prism = Area of cross section × Length

$300 = \frac{1}{2}(4+6)(h) \times 12$

$= \frac{1}{2}(10)h \times 12$ = 60h $\therefore$ h = $\frac{300}{60}$ = 5 cm

23. (B) Let a = 10,000 & b = 55 then

$(a + b)^3 - (a - b)^3 = (a^3 + 3a^2b + 3ab^2 + b^3) - (a^3 - 3a^2b + 3ab^2 - b^3)$

$= a^3 + 3a^2b + 3ab^2 + b^3 - a^3 + 3a^2b - 3ab^2 + b^3$

$= 6a^2b + 2b^3$

$= 2b(3a^2 + b^2)$

$= 2 \times 55\ [3 \times (10000)^2 + (55)^2]$

= 110 [3 × 100000000 + 3025]

= 110 × 300003025

= 33000332750

24. (A) Clearly, ABCD is a cyclic quadrilateral. Then $\angle$BCD = 180° – $\angle$BAD

= 180° – 100° = 80°

In ΔBAQ, y + 100° + 25° = 180° ⇒ y = 55°

In ABCP, y + 80° + x = 180° ⇒ x = 45°

25. (A) Volume of the box =

outer volume – inner volume

$= 30 \times 25 \times 20\ cm^3 - (30 - 2 \times 1.5)(25 - 2 \times 1.5)(20 - 1.5)\ cm^3$

$= 15000\ cm^3 - 27 \times 22 \times 18.5\ cm^3$

$= 15000\ cm^3 - 10989\ cm^3$

$= 4011\ cm^3$

$= 401.1 \times 10\ cm^3$

= 401.1 × 8 g

[Given 10 cm^3 wood weight = 85]

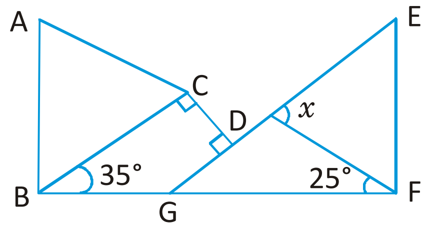
= 3208.8 g

= 3.2088 kg

= 3.209 kg

26. (D) Construction :-

Extend ED up to G



$\angle$BCD = $\angle$CDE = 90° ⇒ BC || GE

⇒ $\angle$EGF = $\angle$CBG = 35°

[$\because$ corresponding angles]

In ΔGFD, $\angle$DGF + $\angle$DFG = $\angle$FDE

$\therefore$ x = 35° + 25° = 60°

27. (A) ΔABC is a right triangle.

$\therefore$ $AC^2 = AB^2 + BC^2$

= 16 + 9 = 25

⇒ AC = 5 cm

Area of the quad. ABCD

= Area of rt. ΔABC + Area of rt. ΔACD

$= \frac{1}{2} \times 4 \times 3 + \frac{1}{2} \times 5 \times 12$

= 6 + 30 = 36 cm^2

28. (A) Area of parallelogram with base AB and attitude AM

$= 12 \times 9 = 108$ cm^2

108 cm^2 = AD × 11 cm

$\Rightarrow$ AD = $\frac{108}{11}$ cm

29. (C) A point has no dimension

30. (A) Given AB || CD $\Rightarrow \left(\frac{5x}{3} - \frac{3x}{4}\right) = 77°$

[$\because$ Exterior Alternative angles]

$\Rightarrow \frac{20x - 9x}{12} = 77°$

$\frac{11x}{12} = 77°$

$x = \cancel{77}°^{7} \times \frac{12}{\cancel{11}} = 84°$

MATHEMATICS - 2

31. (B, C, D)

Irrational numbers are part of Real numbers

$\therefore$ Sum of two irrational numbers is always a real number

$\therefore$ Option 'B' is true

$-\sqrt{3} + \sqrt{3} = 0$ which a rational number

But $\sqrt{3} + \sqrt{5}$ is an irrational number

$\therefore$ Sum of irrational numbers is some times rational number and sum times irration number

$\therefore$ Option 'B' is false

But option 'C' and 'D' are true

32. (B, D)

Given $x^2 + x(c - b) + (c - a)(a - b) = 0$

$\Rightarrow x^2 + x[c - a + a - b] + (c - a)(a - b) = 0$

$x^2 + x[(c - a) + (a - b)] + (c - a)(a - b) = 0$

$x^2 + x(c - a) + x(a - b) + (c - a)(a - b) = 0$

$x[x + c - a] + (a - b)[x + c - a] = 0$

$(x + c - a)(x + a - b) = 0$

$x + c - a = 0$

$\therefore$ $x + c - a = 0$

$x = (a - c)$

(or)

$x + a - b = 0$

$x = (b - a)$

33. (A, C, D)

For option A :-

$\sqrt[2022]{2022} \times \sqrt[2022]{2022^{2021}}$

$= \sqrt[2022]{2022 \times 2022^{2021}}$

$= \sqrt[2022]{2022^{1+2021}}$

$= \sqrt[2022]{2022^{2022}}$

= 2022 which is a rational number

For option B :-

$\sqrt[2022]{2022} \times \sqrt[2022]{2022^{2023}}$

$= \sqrt[2022]{2022^{2024}}$

$= 2022 \times \sqrt[2022]{2022^2}$ is not a rational number

For option C :-

$= \sqrt[2022]{2022} \times \sqrt[2022]{2022^{4043}}$

$= \sqrt[2022]{2022^{4044}}$

= $(2022)^2$ which is a rational number

For option D :-

$\sqrt[2022]{2022} \times \sqrt[2022]{2022^{6065}}$

$= \sqrt[2022]{2022^{6066}}$

= $(2022)^3$ which a rational number

34. (A, B, C, D)

$\sqrt{2} = 1.4142$ & $\sqrt{3} = 1.732$

$1.5,\ 1.\overline{515},\ 1.616263...$ & $\dfrac{\sqrt{2}+\sqrt{3}}{2}$ are the real numbers lie between $\sqrt{2}$ and $\sqrt{3}$

35. (B, D)

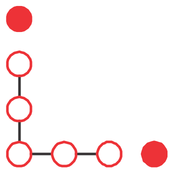
$x^2 + 2x - P^2 - 2P = x^2 - P^2 + 2x - 2p$

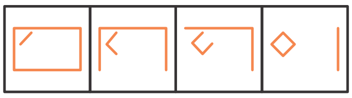
$= (x + P)(x - P) + 2(x - P)$

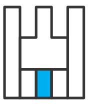
$= (x - P)(x + P + 2)$

REASONING

36. (D) The number of white dots is increased by one each time, both vertically and horizontally, and all white dots are connected.



37. (C) 

38. (B) 

39. (C) 

40. (D) First letter indicates number of lines

Second letter indicates thickness of a line .

∴ The code for right most image is ZM

41. (A) Looking across at the three circles, the number in the middle is the product of the two numbers in the same segment in the other two circles.

Thus, 3 × 2 = 6, 7 × 3 = 21 and 4 × 4 = 16.

42. (C) Anurudh > Bharath(lawyer)

Doctor > Dhanush (Engineer)

Bharath and Dhanush did not start 40 lakhs

Lawyer and Engineer did not start 40 lakhs

Cricketer earned the most at that time so, cricketer also did not start with 40 lakhs.

∴ Doctor → 40 lakhs

Doctor > Dhanush (Engineer)

Dhanush(Engineer) → 30 lakhs

Santosh (Doctor) → 40 lakhs

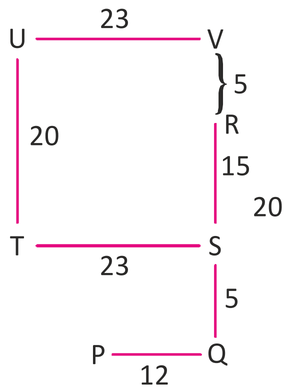
Bharath (lawyer) → 60 laksh

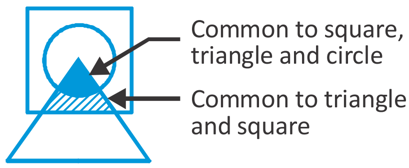
Anurudh (Cricketer) → 70

Anurudh > Bharath

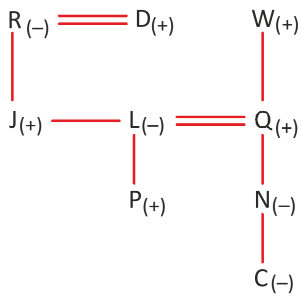
∴ Santosh profession is doctor

43. (B) 5 + 15 + 5 = 25 m



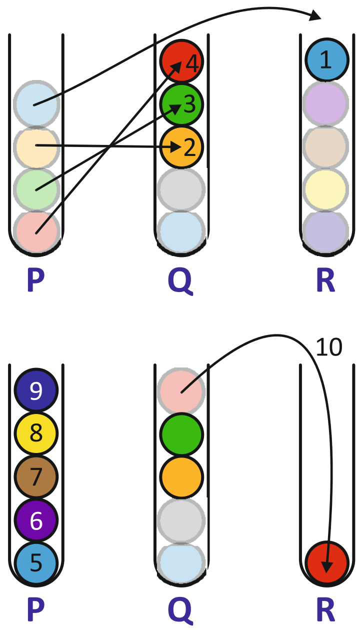
44. (A) 

45. (C) Brother



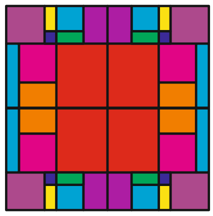
CRITICAL THINKING

46. (A)



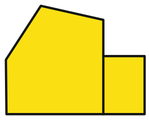
47. (A) A Devaluation is a conscious decision taken by central Bank of the country to lower the external value of domestic currency. After devaluation of the rupee Indian goods would become cheaper for foreigners.

48. (D)



49. (B) Long moment arm redues force.

50. (D) P and S



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