# UNIFIED COUNCIL

An ISO 9001:2015 Certified Organisation

Unified International Mathematics Olympiad

## UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)

**CLASS - 8**

**Question Paper Code : UM9009**

## KEY

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| D | B | A | B | B | A | C | C | D | B |
| **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
| C | C | B | D | B | C | C | C | A | C |
| **21** | **22** | **23** | **24** | **25** | **26** | **27** | **28** | **29** | **30** |
| B | C | A | A | A | B | C | B | A | B |
| **31** | **32** | **33** | **34** | **35** | **36** | **37** | **38** | **39** | **40** |
| A,B,C | A,C,D | B,C | B,C,D | A,B,C | B | C | A, B | D | C |
| **41** | **42** | **43** | **44** | **45** | **46** | **47** | **48** | **49** | **50** |
| B | D | C | A | A | C | C | A | A | A |

## EXPLANATIONS

### MATHEMATICS - 1 (MCQ)

1. **(D)** $x^2 + 3x - x - 3 = 5$

$x^2 + 2x - 8 = 0$

$x^2 + 4x - 2x - 8 = 0$

$(x + 4)(x - 2) = 0$

$x = 2$ (OR) $-4$

2. **(B)** $36 < 42 < 49$

$\Rightarrow \sqrt{36} < \sqrt{42} < \sqrt{49}$

it $6 < \sqrt{42} < 7$

3. **(A)** $BC^2 = AB^2 + AC^2 = (20cm)^2 + (15cm)^2 = 400cm^2 + 225\ cm^2$

$BC = \sqrt{625cm^2} = 25cm$

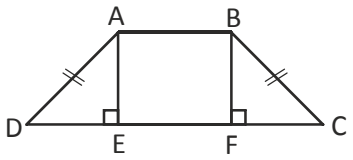
4. **(B)** Final price of computer game =

$$₹12,000\frac{(100-15)}{100}\times\left(\frac{100-20}{100}\right)$$

= ₹ 8,160

5. **(B)** Option 'B' views match with given views

6. **(A)** Const:- AE⊥CD and BF⊥CD




Proof:- $\Delta ADE \cong \Delta BFC$ [∵RHS congruency]

$\therefore$ $\angle BCD = \angle ADC$

7. **(C)** Statement '3' is false because if x = 1 then y doesn't exist

8. **(C)** Let MP be ₹ x

$$SP_1 = \frac{MP(100-10)}{100} = ₹\frac{9x}{10}$$

$$SP_2 = ₹\frac{9x}{10}\left(\frac{100-20}{100}\right) = ₹\frac{9x}{10}\times\frac{4}{5} = ₹\frac{36x}{50}$$

$$SP_3 = \frac{₹36x}{50}\left(\frac{100-40}{100}\right) = \frac{₹36}{50}\times\frac{3}{5} = ₹\frac{108x}{250}$$

final discount = MP – SP3

$$₹x - \frac{₹108x}{250} = \frac{₹250x - ₹108x}{250}$$

$$= \frac{₹142x}{250}$$

final discount % =

$$\frac{\frac{₹142x}{250}}{₹x}\times 100 = \frac{142}{5}\times 2 = 56.8\%$$

9. **(D)** There are infinite rational numbers between two rational numbers.

10. **(B)** $\sqrt{2209} = 47$

11. **(C)** Let the edge of a cube be 'a'

$\therefore$ Total length of all edges = 12 a = 9 cm

$$\therefore a = \frac{9cm}{12} = \frac{3cm}{4}$$

$$\text{Volume} = a^3\left(\frac{3}{4}cm\right)^3 = \frac{27cm^3}{64}$$

12. **(C)** $$\sqrt{1+\left(\frac{1}{x}-\frac{x}{4}\right)^2} = \sqrt{1+\frac{1}{x^2}+\frac{x^2}{16}-2\times\frac{1}{x}\times\frac{x}{4}}$$

$$= \sqrt{\frac{1}{x^2}+\frac{x^2}{16}+\frac{1}{2}}$$

$$= \sqrt{\left(\frac{1}{x}\right)^2+\left(\frac{x}{4}\right)^2+2\times\frac{1}{x}\times\frac{x}{4}}$$

$$= \sqrt{\left(\frac{1}{x}+\frac{x}{4}\right)^2} = \left(\frac{1}{x}+\frac{x}{4}\right)$$

13. **(B)** Given a = 7x + 5 & b = 7y + 3

$\therefore$ a + b = 7x + 5 + 7y + 3 = 7x + 7y + 7 + 1 = 7(x + y + 1) +1

$\therefore$ r = 1

$$\therefore \frac{3r+5}{4} = \frac{3(1)+5}{4} = \frac{8}{4} = 2$$

14. **(D)** Let salary in 2014 be ₹ x

$\therefore$ Salary in 2015 = $₹x\left(\frac{100+10}{100}\right) = ₹\frac{11x}{10}$

Salary in 2016

$$= ₹\frac{11x}{10}\frac{(100+15)}{100} = ₹\frac{11x}{10}\times\frac{23}{20} = ₹\frac{253}{200}x$$

Salary in 2017

$$= ₹\frac{253x}{200}\left(\frac{100+40}{100}\right) = ₹\frac{253x}{200}\times\frac{7}{5} = \frac{₹1771x}{1000}$$

Given $\frac{₹1771x}{1000} = ₹1,77,100$

$$\therefore x = ₹177100\times\frac{1000}{1771} = ₹1,00,000$$

15. **(B)** Given n = 9k + 7

$\therefore$ 3n – 1 = 3(9k + 7) – 1 = 27k + 21 – 1 = 27k + 20

= 27k + 18 + 2

= 9(3k + 2) + 2

$\therefore$ (3n – 1) divided 9 gives a remainder 2

16. **(C)** Maximum area grazed by houses

$$= \frac{x}{360^o}\times\pi r^2 + \frac{4}{360^o}.\pi r^2 + \frac{z}{360^o}\pi r^2$$

$$= \frac{\pi r^2}{360^\circ}[x+y+z]$$

$$= \frac{\frac{22}{7}\times 7\times 7cm^2}{360^\circ}\times 180^\circ$$

$= 77\ m^2$

17. **(C)** Given $x+\frac{9}{x}=6$

$$\Rightarrow \frac{x^2+9}{x}=6$$

$\Rightarrow x^2 - 6x + 9 = 0$

$\Rightarrow (x-3)^2 = 0$

$\Rightarrow x = 3$

$$\therefore x^2+\frac{9}{x^2}=3^2+\frac{9}{(3^2)}=9+\frac{9}{9}=9+1=10$$

18. **(C)** $x^2 - 7x - 44 = x^2 - 11x + 4x - 44$

$= (x-11)(x+4)$

19. **(A)** $$LHS=\frac{(0.35)^2+2(0.35)(1)+1^2}{2.25}+0.19$$

$$=\left(\frac{1.35}{1.5}\right)^2+0.19$$

$= (0.9)^2 + 0.19$

$= 0.81 + 0.19 = 1$

20. **(C)** $$A:B=3:4\Rightarrow \frac{A}{B}=\frac{3}{4}$$

$$\therefore A=\frac{3B}{4}$$

$$\therefore \frac{3A^2+4B^2}{3A^2-4B^2}=\frac{3\left(\frac{3B}{4}\right)^2+4B^2}{3\left(\frac{3B}{4}\right)^2-4B^2}$$

$$=\frac{3\times\frac{9B^2}{16}+4B^2}{3\times\frac{9B^2}{16}-4B^2}$$

$$=\frac{27B^2+64B^2}{27B^2-64B^2} \quad =\frac{91B^2}{-37B^2}$$

21. **(B)** $$LHS=\frac{1}{\left(6^3\right)^{-2/3}}+\frac{1}{\left(4^4\right)^{-3/4}}+\frac{1}{\left(4^5\right)^{-1/5}}$$

$$=\frac{1}{6^{-2}}+\frac{1}{4^{-3}}+\frac{1}{4^{-1}}$$

$= 6^2 + 4^3 + 4$

$= 36 + 64 + 4 = 104$

22. **(C)** Let $\sqrt{\frac{x}{y}}$ 'a'

Given $a=\frac{24}{5}+\frac{1}{a}$

$$a=\frac{24a+5}{5a}$$

$5a^2 - 24a - 5 = 0$

$5a^2 - 25a + a - 5 = 0$

$5a(a-5) + 1(a-5) = 0$

$a = 5$ (or) $a=\frac{-1}{5}$

$$\sqrt{\frac{x}{y}}=5 \text{ (or) } \sqrt{\frac{x}{y}}=\frac{-1}{5}$$

$$\frac{x}{y}=25 \text{ (or) } \frac{x}{y}=\frac{1}{25}$$

$x = 25y$ (OR) $y = 25x$

Given $x + y = 26$ (or) $25y + y = 26$

$25y + y = 26$

$x = 1$ (or) $y = 1$

If $x = 1$ then $y = 25$ (OR)

If $y = 1$ then $x = 25$

$\therefore xy = 25$

23. **(A)** Given $5\times 5^x+\frac{5}{5^x}=26$

$$\Rightarrow 5a+\frac{5}{a}=26 \text{ where } 5^x = a$$

$$\Rightarrow \frac{5a^2+5}{a}=26$$

$\Rightarrow 5a^2 - 26a + 5 = 0$

$\Rightarrow 5a^2 - 25a - a + 5 = 0$

$\Rightarrow 5a(a-5) - 1(a-5) = 0$

$\therefore$ a = 5 (or) $a = \frac{1}{5}$

$5^x = 5$ (or) $5^x = \frac{1}{5}$

x = 1 (or) –1

24. **(A)** $x^3 + 8 = (x + 2)(x^2 + 2x + 4)$

$x^2 + 5x + 6 = (x + 2)(x + 3)$

$x^3 + 4x^2 + 4x = x(x + 2)^2$

LCM = $x(x + 2)^2 (x + 3)(x^2 - 2x + 4)$

25. **(A)** $\sqrt[3]{\frac{512}{125}} = \sqrt[3]{\left(\frac{8^3}{5^3}\right)} = \frac{8}{5} = 1\frac{3}{5}$

26. **(B)** Given $P\left(1+\frac{r}{100}\right)^2 - P - \frac{p \times r \times 2}{100} = ₹150$

$P\left(1+\frac{2r}{100}+\frac{r^2}{10,000}\right) - P - \frac{2pr}{100} = ₹150$

$P + \frac{2pr}{100} + \frac{pr^2}{10,000} - P - \frac{2pr}{100} = ₹150$

$\Rightarrow \frac{P \times 5^2}{10,000} = ₹150$

$\frac{P \times 25}{10,000} = ₹150$

P = ₹ 60,000

27. **(C)** In $\Delta ABC$

$80^o + 30^o + 30^o + 2x = 180^o$

$2x = 180^o - 140^o = 40^o$

$x = 20^o$

In $\Delta BCD$ $30^o + 20^o + y = 180^o$

$y = 130^o$

28. **(B)** Let $\angle P = x$

In $\Delta PQT$, PT = TQ

$\Rightarrow \angle PQT = \angle P = x.$

$\therefore \angle TQR = 90^o - x.$

$\therefore \angle RLM = 90^o - x$

In $\Delta PQR, \angle P = x \Rightarrow \angle QRP = 90^o - x.$

$\therefore \angle MLR = \angle QRP.$ ie $\angle LRM.$

29. **(A)** Area of shaded region = $\frac{1}{2} \times AC \times CD$

= 14 $cm^2$

30. **(B)** $\sqrt{0.0144} = 0.12$

## MATHEMATICS - 2 (MAQ)

31. **(A, B, C)** Ophons A, B, C satitisfy closure property

But division doesn't satisfy closure property because p in any rational number other than zero

q is zero then $\frac{p}{q}$ is not defined

32. **(A, C, D)** $(a - b)^2 = a^2 - 2ab + b^2 = a^2 + b^2 - 2ab = (b - a)^2$

33. **(B, C)** If equal sides be a cm & unequal side be b cm

In $\Delta ABC, AD^2 = AB^2 - BD^2 = a^2 - \frac{b^2}{4} = \frac{4a^2b^2}{4}$

$AD = \sqrt{\frac{4a^2 - b^2}{4}} = \sqrt{\frac{4a^2 - b^2}{2}}$

Area of

$\Delta ABC = \frac{1}{2} \times BC \times AD = \frac{1}{2} \times b \frac{\sqrt{4a^2 - b^2}}{2} = \frac{b}{4}\sqrt{4a^2 - b^2}$

similary if equal sides are bcm and unequal side is a cm then

$\Delta ABC = \frac{1}{4} a \sqrt{4b^2 - a^2}$

34. **(B,C,D)** If k = 3 then 72 k = 216 = $6^3$

If k = 24 then 72 × 24 = 1728 = $12^3$ (or)

If k = $\frac{1}{9}$ then $72 \times \frac{1}{9} = 8 = 2^3$

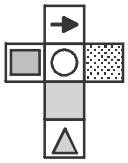
35. **(A,B,C)** Option A, B, C are true statements.

## REASONING

36. **(B)** Dividing the sequence into 4 letter groups. Every 4 letter group contain mnop letters.

mnop pmno opmn nopm

37. **(C)**



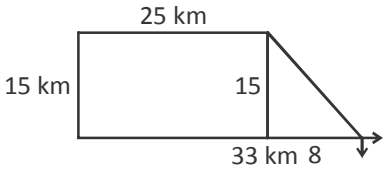
38. **(A,B)** Except: "IMQUY" remaining options has no vowel.

Alphabets succeed after regular intevals, so option (B) is also correct.

39. **(D)** ⌒ / symbols are interchange.

40. **(C)** Actual time = 23.60 – 11.20 = 12.40

41. **(B)**




$\sqrt{225 + 64} = \sqrt{289} = 17$

42. **(D)** ◯◯◯

43. **(C)** %k3, #G2, @N4

44. **(A)** PONY → TSRC

| T | I | G | E | R |
|---|---|---|---|---|
| +4 | +4 | +4 | +4 | +4 |
| X | M | K | I | V |

Similarly

| P | O | N | Y |
|---|---|---|---|
| +4 | +4 | +4 | +4 |
| T | S | R | C |

45. **(A)** 1980 is a leap year, the february will have 29 days.

26 january, 1980 → saturday
31 january, 1980 → thursday
1 february, 1980 → friday
29 february, 1980 → friday
3 march, 1980 → monday
therfore Y's birthday → thursday

## CRITICAL THINKING

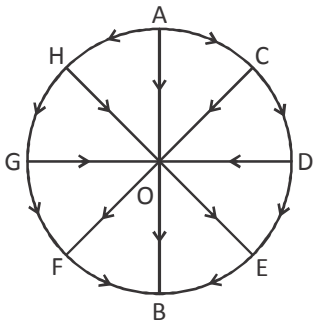
46. **(C)** A = 1, B = 4, C = 2, D = 8, E = 5, F = 7, G = 9

142857 × 7 = 999999

47. **(C)** The most appropriate reaction in this situation would be buy a gift for her daughter and help in wedding.

48. **(A)**

49. **(A)**




Ways or routes from centre (AO, OB)

(AO, OE, EB)

(AO, OF, FB)

Ways or routes from right side

(AC, CO, OB), (AC, CO, OE, EB),

(AC, CD, DO, OB), (AC, CD, DO, OE, EB)

(AC, CD, DE, EB)

(AC, CO, OF, FB) (AC, CD, DO, OF, FB)

Ways or routes from left side

(AH, HO, OB) (AH, HO, OF, FB)

(AH, HG, GF, FB) (AH, HG, GO, OB)

(AH, HG, GO, OF, FB)

(AH, HO, OE, EB), (AH, HG, GO, OE, EB)

Hence, total 17 routes or ways are there to travel from A to B.

50. **(A)** In the given figure one of the dots lies in the region common to the circle and the triangle only and the other dot lies in the region common to the square and triangle only. In the each of the figures in option (B), (C) and (D), there is no region common to the circle and the triangle only. Only option (A) consists of both the types of regions.