UNIFIED COUNCIL

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Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)

CLASS - 9

Question Paper Code : UM9009

KEY

1	2	3	4	5	6	7	8	9	10
B	C	B	D	A	D	A	B	A	**Deleted**
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
D	B	B	D	D	C	A	D	D	D
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
D	C	C	C	A	B	C	C	A	B
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
A,B,D	A,C,D	A,C	A,B,C	A,D	D	D	B	D	C
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
C	A	A,D	A	C	A	C	A	D	D

EXPLANATIONS

MATHEMATICS - 1 (MCQ)

1. **(B)** (3, 4) & (0, 0) lie on th equation $y = \frac{4}{3}x$

2. **(C)** Substitute y = x + 3 in eq 3x + y = 11

   3x + x + 3 = 11

   4x = 8

   x = 2

   If x = 2 then y = 2 + 3 = 5

   ∴ (2, 5) lies on y = x + 3 and 3x + y = 11 line

3. **(B)** Volume of the pyramid =

   $\frac{1}{3} \times \text{base area} \times \text{height}$

   $= \frac{1}{3} \times 8 \times 8 \times 12 \text{ cm}^3$

   = 256 cm^3

4. **(D)** Given each face area = 1cm^2 & volume = 1 cm^3 $\Rightarrow$ each side = 1 cm

   ∴ Total surface area = (8 + 8 + 6 + 4 + 4 + 6) × 1cm^2 = 36 cm^2

5. **(A)** $\sqrt{26}$ is neither terminates nor repeates

$\Rightarrow$ $\sqrt{26}$ is not a rational number.

6. **(D)** Given x + y = 7 $\Rightarrow$ $x^2 + y^2 + 2xy = 49$

37 + 2xy = 49

2xy = 12

xy = 6

x + y = 7 & xy = 6 $\Rightarrow$ x = 6 & y = 1 (or) x = 1 and y = 6

$\therefore$ $x^3 + y^3 = 1^3 + 6^3 = 1 + 216 = 217$

7. **(A)** Whole numbers are terminating decimals.

Irrational numbers are neither terminate nor repete

$\therefore$ Whole number are not contained in irrational numbers ie $w \not\in Q'$

8. **(B)** Given

$$x=\sqrt{3}+\sqrt{2} \Rightarrow \frac{1}{x}=\frac{1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\sqrt{3}-\sqrt{2}$$

$$\therefore x+\frac{1}{x}=\left(\sqrt{3}+\sqrt{2}\right)+\left(\sqrt{3}-\sqrt{2}\right)=2\sqrt{3}$$

Cubing on both sides

$$x^3+\frac{1}{x^3}+3x\times\frac{1}{x}\left(x+\frac{1}{x}\right)=\left(2\sqrt{3}\right)^3=8\times3\sqrt{3}=24\sqrt{3}$$

$$x^3+\frac{1}{x^3}+3\times2\sqrt{3}=24\sqrt{3}$$

$$x^3+\frac{1}{x^3}=18\sqrt{3}$$

$$\therefore x+\frac{1}{x}+x^3+\frac{1}{x^3}=2\sqrt{3}+18\sqrt{3}=20\sqrt{3}$$

9. **(A)** LCM of 2, 3, 4 is 12

$$\therefore \sqrt{2}=2^{\frac{1}{2}\times\frac{6}{6}}=2^{\frac{6}{12}}=\left(2^6\right)^{\frac{1}{12}}=\sqrt[12]{64}$$

$$\sqrt[4]{6}=6^{\frac{1}{4}\times\frac{3}{3}}=\left(6^3\right)^{\frac{1}{12}}=\sqrt[12]{216}$$

$$\sqrt[3]{4}=4^{\frac{1}{3}\times\frac{4}{4}}=\left(4^4\right)^{\frac{1}{2}}=\sqrt[12]{256}$$

$$\therefore \sqrt[2]{64}<\sqrt[12]{216}<\sqrt[12]{256} \text{ ie } \sqrt{2}<\sqrt[4]{6}<\sqrt[3]{4}$$

10. **Deleted**

11. **(D)** $f(-2) = (-2)^4 + 6(-2)^3 + 12(-2)^2 - 24(-2) + 32 = 16 + 48 + 4 + 48 + 32 = 148$

$g(-2) = (-2)^4 + 6(-2)^3 - 12(-2)^2 + 24(-2) - 32 = 16 - 48 - 48 - 48 - 32 = -120$

$h(2) = 2^4 + 6(2)^3 - 12(2)^2 + 24(2) + 32 = 16 + \cancel{48} - \cancel{48} + 48 + 32 = 96$

$k(2) = 2^4 - 6(2)^3 + 12(2)^2 - 24(2) + 32 = 16 - 48 + 48 - 48 + 32 = 0$

12. **(B)** $$\sqrt[3]{x\sqrt[3]{x\sqrt[3]{x\sqrt[3]{x}}}}=\sqrt[3]{x\sqrt[3]{x\sqrt[3]{x\times x^{\frac{1}{3}}}}}=\sqrt[3]{x\sqrt[3]{x\sqrt[3]{x^{4/3}}}}$$

$$=\sqrt[3]{x\sqrt[3]{x\times x^{\frac{4}{9}}}}=\sqrt[3]{x\sqrt[3]{x^{\frac{13}{9}}}}=\sqrt[3]{x5x^{\frac{13}{27}}}$$

$$=\sqrt[3]{x^{\frac{40}{27}}}=x^{\frac{40}{81}}$$

13. **(B)** Given ab + bc + ca = 0

bc + ca = -ab

$\therefore$ $c^2 - ab = c^2 + bc + ca = c(a + b + c)$

Similary $a^2 - bc = a(a + b + c)$

$b^2 - ac = b(a + b + c)$

$\therefore$ LHS=

$$\frac{b(a+b+c)\,c\,(a+b+c)+a(a+b+c)\,c\,(a+b+c)+ab(a+b+c)^2}{abc\ (a+b+c)^3}$$

$$=\frac{(a+b+c)^2(ab+bc+ca)}{abc\ (a+b+c)^3}=0\ \ [\because ab+bc+ca=0]$$

14. **(D)** $$\frac{5+\sqrt{10}}{5\sqrt{5}-2\sqrt{20}-\sqrt{32}+\sqrt{50}}=\frac{5+\sqrt{5}\times\sqrt{2}}{5\sqrt{5}-4\sqrt{5}-4\sqrt{2}+5\sqrt{2}}$$

$$=\frac{\sqrt{5}\left(\sqrt{5}+\sqrt{2}\right)}{\left(\sqrt{5}+\sqrt{2}\right)}=\sqrt{5}$$

15. **(D)** $$LHS=\sqrt{2}+2\sqrt{2}+3\sqrt{2}+4\sqrt{2}+5\sqrt{2}-6\sqrt{2}$$

$$=9\sqrt{2}$$

$$=\sqrt{162}$$

16. **(C)** Given $\sqrt{x+1}=19-x$

squaring on both sides

$x + 1 = 361 + x^2 - 38x$

$\Rightarrow x^2 - 39x + 360 = 0$

$x^2 - 24x - 15x + 360 = 0$

x = 24 (OR) 15

x = 15 satisfy the equation but x = 24 doesn' satisty he equation

17. **(A)**

$x^2 - 2x + 1 \,)\, x^3 \,(\, x + 2$

$x^3 - 2x^2 + x$

(–) (+) (–)

$2x^2 - x$

$2x^2 - 4x + 2$

(–) (+) (–)

$3x - 2$

18. **(D)** x = m line and y = n line intersect at (m, n)

19. **(D)** $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}=\frac{a^3+b^3+c^3}{abc}=\frac{3abc}{abc}=3$

[$\because$ Given a + b+ c = 0]

20. **(D)** Total surface area of the remaining solid

$= 6l^2-\pi r^2+2\pi r^2$

$=6l^2+\pi r^2$

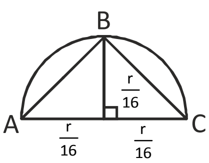
$=6l^2+\pi\left(\frac{l}{4}\right)^2 \left[\because \text{Given } d=\frac{l}{2}\right]$

$=6l^2+\frac{\pi l^2}{16}$

$=\frac{96l^2+\pi l^2}{16}$

$=\frac{l^2}{16}(\pi+96)\text{ units}^2$

21. **(D)** $AC=2\times\frac{r}{16}cm=\frac{r}{8}cm$



Area of triangle ABC =

$\frac{1}{2}AC\times BD=\frac{1}{2}\times\frac{r}{8}cm\times\frac{r}{16}cm=\frac{r^2}{256}cm^2$

22. **(C)** Given $x+\frac{1}{x}=5$

Cubing on both sides

$x^3+\frac{1}{x^3}+3x\times\frac{1}{x}\left(x+\frac{1}{x}\right)=125$

$x^3+\frac{1}{x^3}+15=125$

$x^3+\frac{1}{x^3}=110$

Cubing on both sides

$(x^3)^3+\left(\frac{1}{x^3}\right)^3+3x^3\times\frac{1}{x^3}\left(x^3+\frac{1}{x^3}\right)=(110)^3=1331000$

$x^9+\frac{1}{x^9}+$ = 1331000 - 330 = 1330670

23. **(C)** Given ABC and equilateral triangle

$\therefore AD=\frac{\sqrt{3}}{2}AB$

2AD = $\sqrt{3}$ AB

Squaring on both sides

4 AD^2 = 3 AB^2

24. **(C)** Let $x^{1/3}$ be 'a'

$\therefore$ $a^2 + a - 6$ =

$a^2 + 3a - 2a - 6 = 0$

(a + 3) (a – 2) = 0

$\therefore$ a = –3 (or) a = 2

$\therefore x^{\frac{1}{3}}=-3$ (or) $x^{\frac{1}{3}}=2$

Cubing on both sides

$\left(x^{\frac{1}{3}}\right)^3=(-3)^3$ (or) $=\left(x^{\frac{1}{3}}\right)^3=2^3$

x = – 27 (or) 8

25. **(A)** Given $\frac{4}{3}\pi r^3=\frac{1}{3}\pi(2.1cm)^2\times 8.4cm$

$r^3=4.41cm^2\times 8.4cm\times\frac{1}{4}=(2.1)cm^2\times(2.1)cm$

$r^3 = (2.1\text{ cm})^3$

$r = 2.1$ cm

26. **(B)** Given $a^3 = 335 + b^3$

$\Rightarrow (5 + b)^3 = 335 + b^3$

$125 + b^3 + 15b(5 + b) = 335 + b^3$

$75b + 15b^2 = 210$

$5b + b^2 = 14$

$b^2 + 5b - 14 = 0$

$b^2 + 7b - 2b - 14 = 0$

$b = -7$ (or) $b = 2$

If $b = 2$ then $a = 5 + b = 7$

$\therefore a + b = 9$

27. **(C)** Area of ΔABC

$$s = \frac{3cm + 2m + 4m}{2} = \frac{9}{2}m$$

$$\text{Area of } \Delta ABC = \sqrt{\frac{9}{2}\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)\left(\frac{1}{2}\right)}\ m^2$$

$$= \frac{3}{4}\sqrt{15}\ m^2$$

Area of rectangle CDEF = $2m \times 1m = 2m^2$

$$\text{Area of } \Delta BFE = \frac{1}{2} \times 3m \times 2m = 3m^2$$

Area of trapezium DEHG = $\frac{1}{2} \times 2m\ (1 + 3)$ m = $4m^2$

Total area = $\frac{3}{4}\sqrt{15}m^2 + 9\ m^2$

28. **(C)** $9^x\ 3^y = 2187$ and $3x + 2y = 2xy$ satisfy if

$x = 2$ & $y = 3$

$\therefore x + y = 2 + 3 = 5$

29. **(A)** Given $P(x) = \frac{x-1}{x+1}$

$$P(P(x)) = \frac{p(x)-1}{p(x)+1} = \frac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1} = \frac{\frac{x-1-x-1}{(x+1)}}{\frac{x-1+x+1}{(x+1)}}$$

$$= \frac{-2}{2x} = \frac{-1}{x}$$

30. **(B)** (1, 2) lies on $2x - y = 0$

MATHEMATICS - 2 (MAQ)

31. **(A, B, D)** '5' is factor of f(x)

$$\begin{array}{r|l|}
x-5 & 6x^3 - 47x^2 + 97x - 60 \\
 & 6x^3 - 30x^2 \\
 & (-)\quad (+) \\ \hline
 & -17x^2 + 97x - 60 \\
 & -17x^2 + 85x \\
 & (-)\quad (+) \\ \hline
 & 12x - 60 \\
 & 12x - 60 \\ \hline
 & 0
\end{array}$$

$\therefore 6x^2 - 17x + 12 = 6x^2 - 8x - 9x + 12$

$= 2x(3x - 4) - 3(3x - 4)$

$= (3x - 4)(2x - 3)$

$$\therefore P\left(\frac{4}{3}\right) = 0 \text{ and } P\left(\frac{3}{2}\right) = 0$$

32. **(A, C, D)** Given $x = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = 2 + \sqrt{3}$

$x - 2 = \sqrt{3}$

$x^2 - 4x + 4 = 3$

$x^2 - 4x + 1 = 0$

$$\begin{array}{r|l|l}
x^2 - 4x + 1 & x^3 - 2x^2 - 7x + 2 & x + 2 \\
 & x^2 - 4x^2 + x & \\
 & (-)\quad (+)\quad (-) & \\ \hline
 & 2x^2 - 8x + 2 & \\
 & 2x^2 - 8x + 2 & \\ \hline
 & 0 &
\end{array}$$

$\therefore x^3 - 2x^2 - 7x + 2 = (x^2 - 4x + 1)(x + 2)$
$= 0 \times (x + 2) = 0$

$$
\begin{array}{l|l|l}
x^2-4x+1 & x^4-x^3-6x^2-17x+5 & x^2+3x+5 \\
 & x^4-4x^3+x^2 & \\
 & (-)\quad(+)\quad(-) & \\
\hline
 & 3x^3-7x^2-17x+5 & \\
 & 3x^3-12x^2+3x & \\
\hline
 & 5x^2-20x+5 & \\
 & 5x^2-20x+5 & \\
\hline
 & 0 &
\end{array}
$$

33. **(A, C)** $x > 0, y > 0$ then (x, y) lies in Q_1

If $x < 0, y < 0$ then (x, y) lies in Q_3

34. **(A, B, C)** Given $\angle A + \angle B + \angle C = 180^\circ$

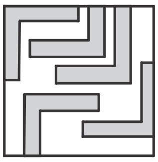
$$\angle A + \frac{2}{3}\angle A + \frac{1}{3}\angle A = 180^\circ$$

$$2\angle A = 180^\circ$$

$$\angle A = 90^\circ \ \& \ \angle B = \frac{2}{3}\angle A = 60^\circ, \angle C = \frac{1}{3}\angle A = 30^\circ$$

35. **(A, D)** Square & rhombus are parallelograms

REASONING

36. **(D)** 

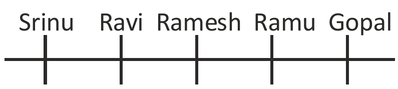
Rotate 2nd image in first pair, one step in anti-clockwise direction and insert the outer part inside.

In the same way, the 1st image in second pair, Rotate 1 step anticlockwise and folded the given figure.

37. **(D)** M is the sister of N and S.

J is the son of S

∴ M is the aunt of J

38. **(B)** 

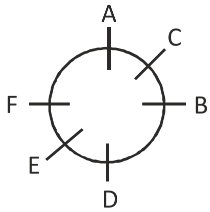
Srinu and Gopal are the extreme ends of the line.

39. **(D)** Adding numbers of alphabet letters.

H A R D W O R K =

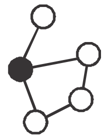
8 + 1 + 18 + 4 + 23 + 15 + 18 + 11 = 98

40. **(C)** The number in the center of each figure is the cube of the number of sides of the figure.

41. **(C)** 

'D' is the left of B.

Option (C) is correct.

42. **(A)** 

43. **(A, D)** Looking across each row of numbers, the bottom row totals 5, the fourth row totals 6, the third row totals 7, and the second row totals 8.

∴ The top row should total 9.

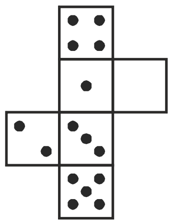
Some of the numbers of the columns form a sequence when we start from the last one. 1, 2, 4, 8, 16.

∴ 5 + 3 + 2 + 1 + ? = 16

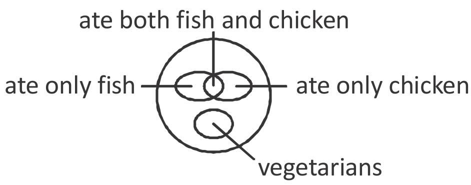
⇒ ? = 16 – 11 = 5

44. **(A)** In the rest there is the same difference between each digit

ex: 8 (–3) 5 (–3)2

45. **(C)** 

CRITICAL THINKING

46. **(A)** 

47. **(C)** DOCUMENT

48. **(A)**

Akash	Vivek	Jagat	Occupation
✗	✗	✔	Dentist
✔	✗	✗	Teacher
✗	✔	✗	Driver

Jagat > Akash > Driver

Jagat is Dentist.

49. **(D)** Both A and R false. The bulb filaments are not made of silver and silver can be melted.

50. **(D)** Data in both the statements together are not sufficient.