UNIFIED COUNCIL

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Unified International Mathematics Olympiad

## UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)

**CLASS - 10**

**Question Paper Code : UM9009**

### KEY

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| D | C | D | C | A | D | B | A | C | C |
| **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
| A, C | A | D | B | B | D | **Deleted** | B | A | B |
| **21** | **22** | **23** | **24** | **25** | **26** | **27** | **28** | **29** | **30** |
| C | C | C | A | C | A | C | B | C | A |
| **31** | **32** | **33** | **34** | **35** | **36** | **37** | **38** | **39** | **40** |
| A,B,D | B,C | A,C | A,B,D | B,C | A | A | D | B | C |
| **41** | **42** | **43** | **44** | **45** | **46** | **47** | **48** | **49** | **50** |
| C | C | C | A | A | D | D | A | D | C |

### EXPLANATIONS

**MATHEMATICS - 1 (MCQ)**

1. **(D)** LHS = $\sin^2 1° + \sin^2 2° + \sin^2 3° + \sin^2 4° + \ldots\ldots + \sin^2 88° + \sin^2 89° + \sin^2 90°$

$= \sin^2 1° + \sin^2 2° + \sin^2 3° + \sin^2 4° + \ldots\ldots + \sin^2 44° + \sin^2 45° + \sin^2(90° - 44°) + \sin^2(90° - 43°) + \ldots\ldots + \sin^2(90° - 2°) + \sin^2(90° - 1°) + 1$

$= \sin^2 1° + \sin^2 2° + \sin^2 3° + \sin^2 4° + \ldots\ldots + \sin^2 44° + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2 44° + \cos^2 43° + \ldots\ldots + \cos^2 2° + \cos^2 1°$

$= 44 + \frac{1}{2} + 1 = 45.5$

2. **(C)** Let $x = \sqrt{4 + \sqrt{4 + \sqrt{4 + \ldots\ldots\infty}}}$

$x = \sqrt{4 + x}$

$x^2 = 4 + x$

$x^2 - x - 4 = 0$

$x = \frac{-(-1) \pm \sqrt{1 + 16}}{2} = \frac{\sqrt{17} + 1}{2}$ (or) $\frac{1 - \sqrt{17}}{2}$

3. **(D)** $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{ab+bc+ca}{abc}=\frac{2}{-1}=-2$

4. **(C)** Let original speed be x kmph

Given $\frac{300\text{ km}}{x\text{ kmph}}-\frac{300\text{ km}}{(x+15)\text{ kmph}}=1$ hour

$\frac{300(x+15-x)}{x(x+15)}=1$

$4500=x^2+15x$

$x^2+15x-4500=0$

$x^2+75x-60x-4500=0$

$\therefore$ x = 60 (or) x = – 75 is rejected because speed is never negative

5. **(A)** Given $\frac{x+y}{xy}=\frac{5}{6}\Rightarrow\frac{1}{y}+\frac{1}{x}=\frac{5}{6}$

$\Rightarrow a+b=\frac{5}{6}\rightarrow(1)$

where $\frac{1}{y}=a$ & $\frac{1}{x}=b$

Given $\frac{x-y}{xy}=\frac{1}{6}\Rightarrow\frac{1}{y}-\frac{1}{x}=\frac{1}{6}$

$\Rightarrow a+b=\frac{1}{6}\rightarrow(2)$

eq(1) + (2) $\Rightarrow 2a=1\Rightarrow a=\frac{1}{2}$ & $b=\frac{1}{3}$

$\therefore$ x = 2 & y = 3

6. **(D)** Given PQRS is a rectangle

$\Rightarrow$ Mid point of PR = Mid point of QS

$\therefore\left(\frac{9+m}{2},\frac{2+13}{2}\right)=\left(\frac{15+5}{2},\frac{5+n}{2}\right)$

$\therefore$ m = 11 & n = 10

m – n = 1

7. **(B)** $xyz=(1+\cos A)(1-\cos A)(\operatorname{cosec}^2A)$

$=(1-\cos^2A)\frac{1}{\sin^2A}=\frac{\sin^2A}{\sin^2A}=1$

8. **(A)** $PQ^2+QR^2-2QR\times QS$

$=PS^2+QS^2+QR^2-2QR\times QS$

$=PS^2+(QR-QS)^2$

$=PS^2+SR^2$

$=PR^2$

9. **(C)** $\Delta POQ\sim\Delta ROS$ [ $\because$ A.A similarity]

$\therefore\frac{\text{Area of }\Delta POQ}{\text{Area of }\Delta ROS}=\frac{PQ^2}{SR^2}$

$=\frac{4(RS)^2}{(RS)^2}=\frac{4}{1}=4:1$

10. **(C)** Given [3 + 6 + 9 + 12 + .......] = 108

$\frac{n}{2}[2\times3+(n-1)(3)]=108$

$[6+3n-3]=108\times2$

$3n^2+3n=108\times2$

$n^2+n=36\times2$

$n^2+n-72=0$

$n^2+9n-8n-72=0$

n = – 9 (or) 8

11. **(A,C)** Given $\tan4\theta\times\tan5\theta=1$

$\therefore\ \tan4\theta=\frac{1}{\tan5\theta}=\cot5\theta=\tan(90°-5\theta)$

$\therefore\ 4\theta=90°-5\theta$

$9\theta=90°\Rightarrow\theta=10°$

If $\theta=30°$ also satisifies the given condition

12. **(A)** In $\Delta ABC$, if $\angle A=90°$ then $\angle B+\angle C=90°$

$\therefore\ \angle B=90°-\angle C$

$\therefore\ \sin B=\sin(90-C)=\cos C$

$\therefore\ \sin^2A+\sin^2B+\sin^2C=\sin^290°+\cos^2C+\sin^2C=1+1=2$

13. **(D)** $2+8=-a\Rightarrow a=-10$

$3\times3=b\Rightarrow b=9$

$\therefore x^2-10x+9=0$

x = 9 (or) 1

14. **(B)** $\alpha+\beta+\gamma=-\frac{b}{a}=-(-17)=17=-3+9+11$

$\alpha\beta+\beta\gamma+\gamma\alpha=39$

$=-3\times9+9\times11+(-3)(11)=39$

$\alpha+\gamma=-\frac{d}{a}=-297=(-3)(9)(11)$

$\therefore$ –3, 9, 11 are the zeros

15. **(B)** Given $\Delta ABC \sim \Delta DAC$ [ $\because$ A. A similarity]

$$\therefore \frac{BC}{AC} = \frac{AC}{DC}$$

$$\frac{12\text{ cm}}{AC} = \frac{AC}{3\text{ cm}} \Rightarrow AC = 6\text{ cm}$$

16. **(D)** Required ratio $= \left(\frac{4}{9}\right)^2 = \frac{16}{81} = 16 : 81$

17. **Deleted**

18. **(B)** $G = \left(\frac{2 \times 5 + 2}{3}, \frac{2 \times 0 + 3}{3}\right)$

$= (4, 1)$

19. **(A)** Given $\sqrt{(2 - 4)^2 + (3 - k)^2} = 8$

$4 + 9 + k^2 - 6k = 64$

$k^2 - 6k - 51 = 0$

$$\therefore \quad k = \frac{6 \pm \sqrt{36 + 204}}{2} = 3 \pm 2\sqrt{15}$$

20. **(B)** $\tan 45° + 2\tan^2 60° = 1 + 2(\sqrt{3})^2 = 1 + 6 = 7$

21. **(C)** $LHS = (\sqrt{3})^2 + (\sqrt{3})^2 - \frac{3}{4} \times \frac{1}{2} + 4$

$= 6 - \frac{3}{8} + 4$

$= 10 - \frac{3}{8}$

$= \frac{77}{8}$

22. **(C)** Let $\sin\theta - \cos\theta = k$

$\therefore a^2 + k^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta$

$a^2 + k^2 = 2$

$k^2 = 2 - a^2$

$\therefore \sin\theta - \cos\theta = k = \pm\sqrt{2 - a^2}$

23. **(C)** Given in an AP $a_4 = 3$ & $a_1 = 7.2$

$a + 3d = 3$

$7.2 + 3d = 3$

$3d = -4.2$

$d = -1.4$

$\therefore a = a_1 + d = 7.2 - 1.4 = 5.8$

$b = a + d = 5.8 - 1.4 = 4.4$

24. **(A)** $\tan 30° = \frac{AB}{BC}$

$$\frac{1}{\sqrt{3}} = \frac{AB}{100\text{ mts}}$$

$$AB = \frac{100\text{ mts}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{173.2\text{ m}}{3}$$

$= 57.73$ m

25. **(C)** Given $\Delta RSP \sim \Delta RPQ$

$$\Rightarrow \frac{RS}{RP} = \frac{SP}{PQ} = \frac{RP}{RQ}$$

$\therefore RP^2 = RS \times RQ$

26. **(A)** Given $x = 2.44888...$

$\therefore 10x = 24.4888....$

$9x = 22.04$

$$x = \frac{22.04}{9} = \frac{2204}{900} = \frac{551}{225}$$

27. **(C)** Given $x - y = 0.9$ & $x + y = \frac{11}{2} = 5.5$

$\therefore x = 3.2$ and $y = 2.3$

28. **(B)** Area of the small semicircle $= \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7\text{ m}^2 = 77\text{ m}^2$

Length of the rectangle = 14 m + 7 m = 21 m

Area of the rectangle = 21 m × 10 m = 210 $m^2$

Height of isosceles triangle

= 40.5 m – 10.5 m – 10m = 20 m

$\therefore$ Area of the triangle

$= \frac{1}{2} \times bh = \frac{1}{2} \times 21 \times 20$

$= 210\text{ m}^2$

Total area = 210 $m^2$ + 210 $m^2$ + 77 $m^2$ = 497 $m^2$

29. **(C)** Given 4P – 1, 4P + 1.5, 5P are in AP

$\therefore$ 4P + 1.5 – 4P + 1 = 5P – 4P – 1.5

2.5 = P – 1.5

P = 4

$\therefore$ 4P + 1.5 = 4 × 4 + 1.5 = 17.5

30. **(A)** $\dfrac{2\tan A}{1+\tan^2 A} = \dfrac{2\tan 45°}{1+\tan^2 45°} = \dfrac{2}{1+1} = \dfrac{2}{2} = 1$

sin 2 × 45° = sin90° = 1

$\therefore \sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$

### MATHEMATICS - 2 (MAQ)

31. **(A, B, D)** Options A, B & D are correct

32. **(B, C)** $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2} \Rightarrow$ consistent but unique solution

If $\dfrac{a_1}{a_2} = \dfrac{c_1}{c_2}$ and either

$\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$ or $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

then the given lines are consistent

33. **(A, C)** $x^2 - 4\sqrt{2}\,x + 6 = 0$

$x^2 - 3\sqrt{2}\,x - \sqrt{2}\,x + 6 = 0$

$x(x - 3\sqrt{2}) - \sqrt{2}(x - 3\sqrt{2}) = 0$

$(x - 3\sqrt{2})(x - \sqrt{2}) = 0$

$\therefore$ $x = 3\sqrt{2}$ (or) $x = \sqrt{2}$

34. **(A, B, D)** $AC = \sqrt{AB^2 + BC^2} = 25$

A) $\sin\theta + \cos\theta = \dfrac{7}{25} + \dfrac{24}{25} = \dfrac{31}{25}$

B) $\sin\theta - \cos\theta = \dfrac{7}{25} - \dfrac{24}{25} = \dfrac{-17}{25}$

D) $\sec\theta \times \operatorname{cosec}\theta = \dfrac{25}{7} \times \dfrac{25}{24} = \dfrac{225}{168}$

35. **(B, C)** LSA of the cube = $4a^2$ = 4 × 49 $cm^2$ = 196 $cm^2$

volume of the cube = $a^3$ = $(7\ cm)^3$ = 343 $cm^3$

### REASONING

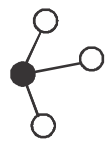
36. **(A)**

| mechanism, | mechanic, | manoeuvre, | machine, | machinate |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |

is the reverse alphabetical order.

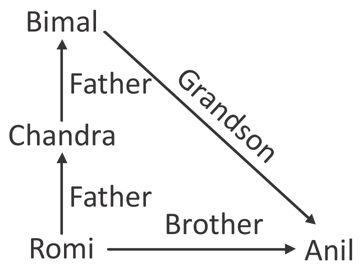
2nd word is ‘mechanic’.

37. **(A)** Option (A) is different from others.

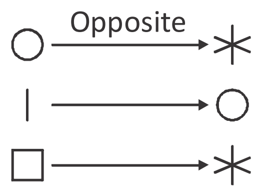


38. **(D)** US91Q4M5W3

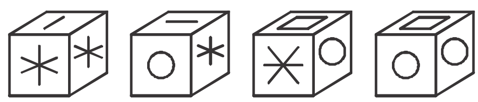
39. **(B)** Anil is the brother of Romi and Romi is the son of Chandra. So, Anil is the son of Chandra. Now, Bimal is the father of Chandra. So, Anil is the grandson of Bimal. Hence, option (B) is correct. The pictorial representation is




40. **(C)** According to the net of the cube,

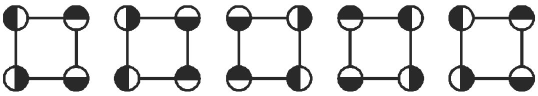



The different views of the cube



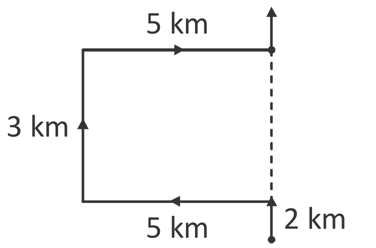
So, option (C) is incorrect view of the cube.

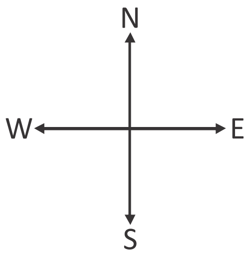
41. **(C)**



42. **(C)** In the second and third statement, the common code is '9a' and the common word is 'not'. So, '9a' means not'. In the first and second statements, the common codes are '7c' and '3a' and the common words are 'is' and 'eternal'. The second statement, '8b' means 'enmity.

43. **(C)** The direction diagram of Mr. Booshi is as follows:




So, the venue was towards the North direction of the point, where Mr. Booshi met the passerby.

44. **(A)** I returned on Tuesday.

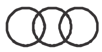
Given, today is Friday.

Then, tomorrow is Saturday.

Now, day before Saturday is again Friday.

Day after Friday is Saturday, then four days before Saturday is Tuesday.

Hence, I returned on Tuesday from the trip.

45. **(A)** 

**CRITICAL THINKING**

46. **(D)** A nucleus is an essential part of a cell. It is located at the center of a cell. Chlorophyll is not produced by cells, and not all cells are neurons. All living entities, not just animals are made up of cells.

47. **(D)** From statements I and II, we conclude that out of 30 students 10 students scored less than Geeta. It means, 19 students scored more than Geeta. So, Geeta's rank in the class is 20th. Thus, both the statements are needed to answer the question.

48. **(A)** The need of self-working engine to pump out the water from the flooded mines led James Watt to invent the steam engine.

49. **(D)** tm = of.

gv = detail.

mk = either ‘results’ or ‘first’.

50. **(C)**