UNIFIED COUNCIL

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Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)

CLASS - 11

Question Paper Code : UM9009

KEY

1	2	3	4	5	6	7	8	9	10
D	A	A	A	B	B	C	B	C	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A	B	A	C	C	D	B	C	B	D
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
C	A	B	C	A	B	B	A	A	D
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
A,B,D	B,C	A,D	C,D	A,B,D	D	D	D	D	C
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
D	B	D	C	B	B	B	C	A	B

EXPLANATIONS

MATHEMATICS - 1 (MCQ)

1. **(D)** f is not a function as both (a, 1) and (a, 4) $\in$ f i.e. f (a) is not uniquely defined

2. **(A)** When n = 1, we have : $3^{2^n} - 1 = 3^2 - 1 = 8$

   Also, when n = 1, then :

   $2^{n+2} = 2^3 = 8$; $3^{n+1} = 3^2 = 9$; $2^{n+3} = 2^4 = 16$; $5^n = 5^1 = 5$

   Clearly, $3^{2^n} - 1$ is divisible by 2^{n+2} only (amongst the given alternatives) for n = 1

   We shall show that $3^{2^n} - 1$ is divisible by 2^{n+2} for all integers $n \geq 1$

   Let P(n) : $3^{2^n} - 1$ is divisible by 2^{n+2}

   Then, P (1) is true (Proved above)

   Let P (m) be true

   Then, $3^{2^m} - 1$ is divisible by 2^{m+2}

   Let $3^{2^m} - 1 = k.2^{m+2} \Rightarrow 3^{2^m} = k.2^{m+2} + 1$ ...(i)

Now, $3^{2^m} - 1 = (3^{2^m})2 - 1 = (k.2^{m+2} + 1)2 - 1$

[Using (i)]

$\{k^2 (2^{m+2})^2 + 2k.2^{m+2} + 1\} - 1$

2^{m+2} {k 2.2^{m+2} +2 k}, which is divisible by 2^{m+2}

P (m+1) is true for all integers $n \geq 1$

Hence, $3^{2^m} - 1$ is divisible by 2^{n+2} for all integers $n \geq 1$

3. **(A)** Since $\frac{z-1}{z+1}$ is purely imaginary,

$$\frac{z-1}{z+1} = iy, \forall \in R$$

$$\Rightarrow \quad \frac{z+1}{z-1} = \frac{1}{iy}$$

Applying componendo and dividendo, we get

$$\frac{(z+1)+(z-1)}{(z+1)-(z-1)} = \frac{1+iy}{1-iy} \Rightarrow \frac{2z}{2} = \frac{1+iy}{1-iy}$$

$$\Rightarrow \quad |z| = \left|\frac{1+iy}{1-iy}\right| = \frac{\sqrt{1^2+y^2}}{1^2+(-y)^2} = 1$$

4. **(A)** $\frac{SP}{PM} = e = \frac{1}{2}$

$\Rightarrow$ $4SP^2 = PM^2$

$\Rightarrow$ $PM = \frac{x-y+3}{\sqrt{2}}$

$$\Rightarrow \quad 4\left\{(x+1)^2 + (y-1)^2\right\} = \left(\frac{x-y+3}{\sqrt{2}}\right)^2$$

$\Rightarrow$ $8(x^2 + y^2 + 2x - 2y + 2) = x^2 + y^2 + 9 - 2xy + 6x - 6y$

$\therefore$ $\boxed{7(x^2+y^2)+2xy+10x-10y+7=0}$

5. **(B)** A × B is empty when at least one of A and B is an empty set.

6. **(B)** $x^2 + 6x - 27 > 0 \Rightarrow (x + 9)(x - 3) > 0 \Rightarrow x < -9$ or $x > 3$

Also, $-x^2 + 3x + 4 > 0$

$\Rightarrow$ $-x^2 - 3x - 4 < 0$

[$\because a > b \Rightarrow am < bm \ \forall \ m < 0$]

$\Rightarrow$ $(x - 4)(x + 1) < 0 \Rightarrow 1 < x < 4$

$\therefore$ $x^2 + 6x - 27 > 0$ and $-x^2 + 3x + 4 > 0$

$\Rightarrow$ $(x < -9$ or $x > 3)$ and $(-1 < x < 4) \Rightarrow x > 3$ and $x < 4 \Rightarrow x \in (3, 4)$.

7. **(C)** G, being the geometric mean between 'a' and 'b', we have $G = \sqrt{ab}$. Hence,

$$\frac{1}{G-a} + \frac{1}{G-b} = \frac{1}{\sqrt{ab}-a} + \frac{1}{\sqrt{ab}-b}$$

$$= \frac{1}{\sqrt{a}(\sqrt{b}-\sqrt{a})} + \frac{1}{\sqrt{b}(\sqrt{a}-\sqrt{b})}$$

$$= \frac{1}{\sqrt{a}(\sqrt{b}-\sqrt{a})} - \frac{1}{\sqrt{b}(\sqrt{b}-\sqrt{a})}$$

$$= \frac{\sqrt{b}-\sqrt{a}}{\sqrt{a}\sqrt{b}(\sqrt{b}-\sqrt{a})}$$

$$= \frac{1}{\sqrt{a}\sqrt{b}} = \frac{1}{\sqrt{ab}} = \frac{1}{G}$$

8. **(B)** The exponent of 3 in 35! is given by

$$E_3(35!) = \left[\frac{35}{3}\right] + \left[\frac{35}{3^2}\right] + \left[\frac{35}{3^3}\right]$$

$= [11.67] + [3.89] + [1.296] = 11 + 3 + 1 = 15$

Clearly, $35! = 3^{15} \times 2k \times ....$

$\therefore$ n = 15 is the largest positive integral value of n such that 35! is divisible by 3^n

9. **(C)**

q	p	q→p	p→(q→p)	p∨q	p→(p ∨ q)
T	T	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	F	T	T	F	T

So, statement $p \to (q \to p)$ is equivalent to $p \to (p \vee q)$

10. **(B)** $a^x + a^y = a^{x+y}$

$\Rightarrow$ $a^{-y} + a^{-x} = 1$

$\Rightarrow$ $-a^{-y} \log a. \frac{dy}{dx} - a^{-x} \log a = 0$

$\Rightarrow$ $\frac{1}{a^y} \cdot \frac{dy}{dx} + \frac{1}{a^x} = 0$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{a^y}{a^x} = -a^{(y-x)}$

$\Rightarrow \quad \left.\frac{dy}{dx}\right]_{(1,1)} = -a^0 = -1$

$\therefore$ The correct answer is (B)

11. **(A)** $\{(x + 4y)^3 (x - 4y)^3\}^2 = [\{(x + 4y)(x - 4y)\}^3]^2$

$= (x^2 - 16y^2)^6$, which contains 6 + 1 = 7 terms in its expansion

12. **(B)**

13. **(A)** $R^{-1} = \{(y, x) : (x, y) \in R\} = \{(1, a), (2, a), (1, b)\}$

14. **(C)** Given limit

$$\lim_{x\to 0} 6\left(\frac{e^{2\sqrt{x}}-1}{2\sqrt{x}}\right)\cdot\left(\frac{\tan 3\sqrt{x}}{3\sqrt{x}}\right)\cdot\left(\frac{x}{\sin x}\right)$$

$= (6 \times 1 \times 1 \times 1) = 6$

Hence, the correct answer is (C)

15. **(C)** $x^2 + y^2 > 0$

$\Rightarrow \quad x^2 + y^2 \geq 0$

and $x^2 + y^2 \neq 0$.

Observe that $x^2 + y^2 \geq 0$ is true for all points in XOY-plane

Also $x^2 + y^2 = 0$

$\Leftrightarrow \quad x = 0, y = 0$

$\Leftrightarrow (x, y) = (0, 0)$

Hence, the given inequality represents all the points in XOY-plane except the origin

16. **(D)** **Case 1.** When $x - 3 \geq 0$ i.e. when $x \geq 3$

In this case: $|x - 3| = (x - 3)$

$\therefore \quad 2 \leq |x - 3| < 4 \Rightarrow 2 \leq x - 3 < 4$

$\Rightarrow \quad 2 + 3 \leq x < 4 + 3$

$[\because a > b \Rightarrow a + m > b + m \ \forall \ m \in N]$

$\Rightarrow \quad 5 \leq x < 7$

Thus, in this case, we have: $x \in [5, 7)$

Case 2. when $x - 3 < 0$ i.e. when $x < 3$

In this case: $|x - 3| = -(x - 3)$

$\therefore 2 \leq |x - 3| < 4 \Rightarrow 2 \leq -(x - 3) < 4$

$\Rightarrow \quad 2 \leq -x + 3 < 4$

$\Rightarrow \quad 2 - 3 \leq -x < 4 - 3$

$[\because a > b \Rightarrow a - m > b - m \ \forall \ m \in N]$

$\Rightarrow \quad -1 \leq -x < 1$

$\Rightarrow \quad -1 < x \leq 1$

[Multiplying throughout by -1]

[Note: $a > b \Rightarrow am < bm \ \forall \ m < 0$]

Thus, in this case, we have : $x \in (-1, 1]$

Hence, from both the cases, we get:

$x \in (-1, 1] \cup [5, 7)$

17. **(B)** The product of an odd number of terms in a G.P.

$= (\text{middle term})^{\text{number of terms}}$

If the number of terms is 2k + 1 and middle term is 'm', then the terms are

$\frac{m}{r^k}, \frac{m}{r^{k-1}}, \frac{m}{r^{k-2}}, \ldots\ldots\ldots\ldots, \frac{m}{r}, m, mr, mr^2, \ldots\ldots mr^k$

where 'r' is the common ratio

Hence, the required product is m^n

18. **(C)** We know that, in centre is the point which is equidistant from the three sides of the triangle

19. **(B)** Here $P(n) \Rightarrow P(n + 1)$ and $P(3)$ is true. Therefore, by the principle of induction, $P(n)$ is true for all $n \geq 3$

20. **(D)** The general term in the expansion of $\left(1+2\sqrt{x}\right)^{40}$ is

$t_{r+1} = {}^{40}C_r \left(2\sqrt{x}\right)^r = {}^{40}C_r 2^r . x^{r/2}$

Clearly, this term contains an integral power of x if $\frac{r}{2}$ is an integer and $0 \leq r \leq 40$

i.e. if r = 0, 2, 4, 6,...,40

Sum of the coefficients of all the integral powers if x is

$S = {}^{40}C_0 + 2^2\,{}^{40}C_2 + 2^4\,{}^{40}C_4 + \ldots + 2^{40}\,{}^{40}C_{40}$ ...(i)

Now, by Binominal Expansion :

$= (1 + 2)^{40} = {}^{40}C_0 + {}^{40}C_1 .2 + {}^{40}C_2 .2^2 + {}^{40}C_3 .2^3 + ...+ {}^{40}C_{40} .2^{40}$ ...(ii)

and $(1 - 2)^{40} = {}^{40}C_0 - {}^{40}C_1 .2 + {}^{40}C_2 .2^2 - {}^{40}C_3 .2^3 + ...+ {}^{40}C_{40} .2^{40}$ ...(iii)

Adding (ii) and (iii), we get:

$(1 + 2)^{40} + (1 - 2)^{40} = 2\{{}^{40}C_0 + 2^2.{}^{40}C_2 + 2^4.{}^{40}C_4 + ...+ {}^{40}C_{40} .2^{40}\}$

$\Rightarrow$ ${}^{40}C_0 + 2^2.{}^{40}C_2 + 2^4.{}^{40}C_4 + ...+ {}^{40}C_{40} . 2^{40}$

$= \frac{1}{2}(3^{40} + 1)$ ....(iv)

From (i) and (iv) we get: $S = \frac{1}{2}(3^{40} + 1)$

21. **(C)** The given expression is equal to

$$\frac{\sin^2 A}{\cos A(\sin A - \cos A)} + \frac{\cos^2 A}{\sin A(\cos A - \sin A)}$$

$$-\sec A \operatorname{cosec} A = \frac{\sin^3 A - \cos^3 A}{\sin A \cos A(\sin A - \cos A)}$$

$-\sec A \operatorname{cosec} A$

$$= \frac{\sin^2 A + \cos^2 A + \sin A \cos A}{\sin A \cos A}$$

$-\sec A \operatorname{cosec} A$

$= \sec A \operatorname{cosec} A + 1 - \sec A \operatorname{cosec} A = 1.$

22. **(A)** From $\sin x + \sin^2 x = 1$, we get $\sin x = \cos^2 x$. Now the given expression is equal to

$\cos^6 x (\cos^6 x + 3\cos^4 x + 3 \cos^2 x + 1) -1$

$= \cos^6 x (\cos^2 x + 1)^3 - 1$

$= \sin^3 x (\sin x + 1)^3 -1$

$= (\sin^2 x + \sin x)^3 - 1 = 1 - 1 = 0$

23. **(B)** $\underline{|n + 2} = 210 \underline{|n - 1}$

$\Rightarrow$ $(n + 2)(n + 1)n \underline{|n - 1} = 210 \underline{|n - 1}$

$\Rightarrow$ $(n + 2)(n + 1)n = 7 \times 6 \times 5 \Rightarrow n = 5$

24. **(C)** Since, P is one end of the focal chord of the parabola $y^2 = x$

Let coordinates of point P are $\left(\frac{1}{4}t^2, \frac{2}{4}t\right)$

$\therefore$ Coordinates of other vetex of focal chord

i.e. Q is $\left(\frac{1}{4t^2}, \frac{2}{4t}\right)$ $\{\because t_1 t_2 = -1\}$

Since, coordinates of P are (4, –2)

$\Rightarrow$ $\frac{1}{4}t^2 = 4$ or $\frac{2}{4}t = -2$

$\Rightarrow$ $\boxed{t = \pm 4}$ or $\boxed{t = -4}$

$\therefore$ $\boxed{t = -4}$

Hence, coordinates of Q are $\left(\frac{1}{64}, \frac{1}{8}\right)$

Now, equation of tangent at Q is

$$yy_1 = \frac{1}{2}(x + x_1)$$

$$\Rightarrow \frac{1}{8}y = \frac{1}{2}\left(x + \frac{1}{64}\right)$$

$$\Rightarrow y = 4x + \frac{1}{16}$$

$\therefore$ Slope to triangle at Q is 4

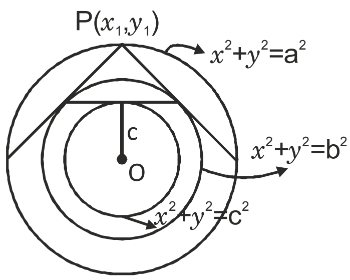
25. **(A)** If the angle opposite to $5x + 5y$ is θ then

$$\cos\theta = \frac{(3x+4y)^2 + (4x+3y)^2 - (5x+5y)^2}{2(3x+4y)(4x+3y)}$$

$$= \frac{-2xy}{2(3x+4y)(4x+3y)} < 0$$

$\Rightarrow$ θ is obtuse $\Rightarrow$ The triangle formed is obtuse angled

26. **(B)** Let (x_1, y_1) be any point on the circle $x^2 + y^2 = a^2$ then $x_1^2 + y_1^2 = a^2$



Equation of chord of contact of tangents from (x_1, y_1) to the circle $x^2 + y^2 = b^2$ is $xx_1 + yy_1 = b^2$

$\because$ (2) touches the circle $x^2 + y^2 = c^2$,

$\therefore$ the length of $\perp$ from centre (0, 0) on (2) is = radius c.

$$\Rightarrow \frac{b^2}{\sqrt{x_1^2+y_1^2}} = c \quad \text{or} \quad \frac{b^2}{\sqrt{a^2}} = c \quad \text{[Using (1)]}$$

or $b^2 = ac$, Hence, a, b, c are in G.P

27. **(B)** If α, β, γ are the angles made by the line with x, y, and z-axis respectively, then

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Given $\alpha = \beta = \gamma$, $\therefore$ $3\cos^2\alpha = 1$

or $\cos\alpha = \pm\frac{1}{\sqrt{3}}$

Possible direction cosines are

$$\left(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}\right)$$

Different sets of Dc's are

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right),$$

$$\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \text{ and } \left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

Thus four lines are equally inclined to axes

28. **(A)** Minimum value of $3\sin\theta - 4\cos\theta + 7$

= maximum value of

$3\sin\theta - 4\cos\theta + 7$

$= \sqrt{3^2+4^2} + 7$

$= \sqrt{25} + 7$

$= 5 + 7 = 12$

$\therefore$ Minimum value of $\frac{1}{3\sin\theta - 4\cos\theta + 7}$ is $1/12$

29. **(A)** From the given relation, we have

$$\frac{\tan(x+100°)}{\tan(x-50°)} = \tan(x+50°)\tan x$$

$$\Rightarrow \frac{\sin(x+100°)\cos(x-50°)}{\cos(x+100°)\sin(x-50°)} = \frac{\sin(x+50°)\sin x}{\cos(x+50°)\cos x}$$

$$\Rightarrow \frac{\sin(2x+50°)}{\sin 150°} = \frac{\cos 50°}{-\cos(2x+50°)}$$

[Applying componendo and dividendo]

$\Rightarrow$ $2\sin(2x+50°)\cos(2x+50°) = -\cos 50°$

$\Rightarrow$ $\sin(4x+100°) = -\sin 40°$

$\Rightarrow$ $\sin(4x+100°) = \sin(180°+40°)$

or $\sin(4x+100°) = \sin(360°-40°)$

$\Rightarrow$ $4x + 100° = 220°$ or $4x + 100° = 320°$

$\Rightarrow$ $x = 30°$ or $55°$

30. **(D)** $\log_{10}(x-9)(x) = 1$

$\Rightarrow$ $x^2 - 9x - 10 = 0$

$x = 10$ (or) $x = -1$

MATHEMATICS - 2 (MAQ)

31. **(A, B, D)** Let the variable line be

$ax + by + c = 0$ ...(1)

Given : $\frac{[(2a+c)+(2b+c)+(a+b+c)]}{\sqrt{a^2+b^2+c^2}} = 0$

$\Rightarrow$ $3a + 3b + 3c = 0$ or $a + b + c = 0$.

So, the equation of the line becomes

$ax + by - a - b = 0$

or $a(x-1) + b(y-1) = 0$

$\Rightarrow$ the line passes through the point of intersection of lines $x - 1 = 0$ and $y - 1 = 0$ i.e. the fixed point (1, 1)

So, all such lines are concurrent. Also, (1, 1) is the centroid of the ΔABC

32. **(B,C)**

$$\text{L.H.S.} = \left\{\frac{\cancel{2}\cancel{\cos\left(\frac{A+B}{2}\right)}\cos\left(\frac{A-B}{2}\right)}{\cancel{2}\cancel{\cos\left(\frac{A+B}{2}\right)}\sin\left(\frac{A-B}{2}\right)}\right\}^n + \left\{\frac{\cancel{2}\cancel{\sin\left(\frac{A+B}{2}\right)}\cos\left(\frac{A-B}{2}\right)}{\cancel{2}\cancel{\sin\left(\frac{A+B}{2}\right)}\sin\left(\frac{B-A}{2}\right)}\right\}^n$$

$$\Rightarrow \text{LHS} = \cot^n\left(\frac{A-B}{2}\right) + \left(-\cot\frac{A-B}{2}\right)^n$$

$$\Rightarrow \quad \text{LHS} = \cot^n\left(\frac{A-B}{2}\right) + (-1)^n \cot^n \frac{A-B}{2}$$

$$\therefore \quad \text{LHS} = \begin{Bmatrix} 2\cot^n \dfrac{A-B}{2} & \text{when n is even} \\ 0 & \text{when n is odd} \end{Bmatrix}$$

33. **(A, D)** Since, the line $\frac{x}{3}+\frac{y}{4}=1$ touches the circle $x^2 + y^2 - 2cx - 2cy + c^2 = 0$

$\therefore$ r = d

$$\Rightarrow \left| \frac{\frac{c}{3} + \frac{c}{4} - 1}{\sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{4}\right)^2}} \right| = c$$

$$\Rightarrow \left| \frac{7c-12}{5} \right| = c$$

$$\Rightarrow \frac{7c-12}{5} = \pm c$$

$$\therefore \frac{7c-12}{5} = c$$

$\Rightarrow$ 7c – 12 = 5c

$\therefore$ $\boxed{c = 6}$

or $\frac{7c-12}{5} = -c$

$\Rightarrow$ 7c – 12 = –5c

$\Rightarrow$ $\boxed{c = 1}$

34. **(C,D)** Let A, B, C be the events that the student is successful in tests I, II and III respectively. Then the probability that the student is successful is

$$\Rightarrow P(E) = P\{(A \cap B \cap \vec{C}) \cup (A \cap \vec{B} \cap C) \cup (A \cap B \cap C)\}$$

$$\Rightarrow P(E) = P(A \cap B \cap \vec{C}) + P(A \cap \vec{B} \cap C) + P(A \cap B \cap C)\}$$

$\Rightarrow$ +P(A)P(B)P(C)

{$\because$ A, B, C are independent}

$$\therefore \quad P(E) = pq\left(1 - \frac{1}{2}\right) + p(1-q)\left(\frac{1}{2}\right) + pq\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad P(E) = \frac{pq + p - pq + pq}{2} = \frac{p(1+q)}{2}$$

Also, given that the probability for the student to be successful = $\frac{1}{2}$

$$\therefore \quad \frac{p(1+q)}{\cancel{2}} = \frac{1}{\cancel{2}}$$

This is satisfied by p = 1, q = 0 and also by $p = \frac{n}{n+1}$, $q = \frac{1}{n}$, where $n \in N$

Hence, there are infinite values of p and q

35. **(A, B, D)** We have x + |y| = 2y

$$\Rightarrow \begin{cases} x - y = 2y & y < 0 \\ x + y = 2y & y \geq 0 \end{cases}$$

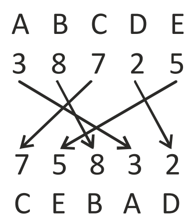
$$\Rightarrow y = \begin{cases} \dfrac{x}{3} & x < 0 \\ x & x \geq 0 \end{cases}$$

Clearly, y = f(x) is continuous for all x but it is not differentiable at x = 0.

Because Lf'(0) = $\frac{1}{3}$ and Rf'(0) = 1.

REASONING

36. **(D)** 42198

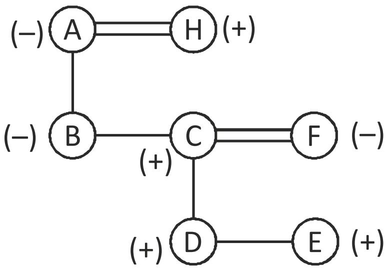


Similarly

A	B	C	D	E
9	1	4	8	2
C	E	B	A	D
4	2	1	9	8

37. **(D)** F = jagged inner line, G = straight inner line. M = one black dot, N = two black dots. X = six big rectangles, Y = five big rectangles.

38. **(D)** In this question, first we will drew the generations tree:



'═══' represent wife and husband

'_____' represent brother and sister

'|' represent children

(–) represents female, (+) represents male

So, A is grand mother to E.

39. **(D)** All the others contain three consecutive digits.

40. **(C)** Looking into the alphabets there are five such pairs namely ON, HONE, ST, TRAPHO, RAP.

1. ON - NO　　2. HONE - EFGH
3. ST - ST　　4. TRAPHO - OPQRST
5. RAP - PQR

41. **(D)** adb_ac_da_cddcb_dbc_cbda

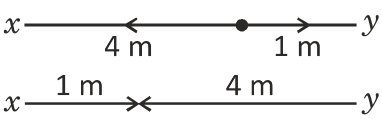
Fill in the missing letter out of a, b, c, d and the pattern will be as follows: adbc/acbd/abcd/dcba/dbca/cbda

So, the missing letters are cbbaa.

42. **(B)**

So, the correct number of the car is 5 6 7 9 S L 1 R B

43. **(D)**



44. **(C)** The different symbols used by the mathematician to denote the operations are

> ⟶ '+'　　< ⟶ '–'

\+ ⟶ '÷'　　^ ⟶ 'x'

– ⟶ '='　　× ⟶ '>'

= ⟶ '<'

Now, consider

$8 < 4 + 2 = 6 > 3$

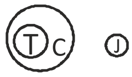
$8 - 4 \div 2 < 6 + 3$

$= 6 < 9$ (true)

Hence, option (C) follows the symbols correctly.

45. **(B)** All thieves are criminals.

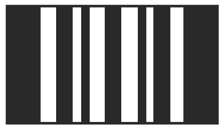
Judges are different from thieves and criminals.



Option (B) is correct.

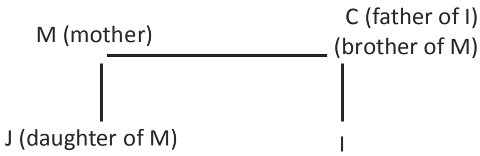
CRITICAL THINKING

46. **(B)**



47. **(B)** Some artists may be drug addicts.

48. **(C)**



49. **(A)** (1) Line 1 states that the Large Silver Watch displays the time as 15:50.

(2) Line 4 states that the Gold Watch is ten minutes behind the Large Silver Watch, hence 15:40.

(3) Line 2 states that the Gold Watch has the same time as the Small Silver Watch, which line 5 states is five minutes ahead of the Bronze Watch.

(4) Therefore, the Bronze Watch is five minutes behind the Large Silver Watch, displaying a time of 15:50, thus, it is true that the Bronze Watch displays the time as 15:45.

50. **(B)** As the craze for learning English is growing among people, the institutes which teach English are mushrooming. So (B) is the cause and (A) is the effect.