UNIFIED COUNCIL

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UIMO
Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)

CLASS - 12

Question Paper Code : UM9009

KEY

1	2	3	4	5	6	7	8	9	10
C	D	B	B	D	A	B	A	B	D
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	A	B	A	B	C	C	D	A	C
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
B	B	D	B	A	B	D	B	A	D
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
A,D	A,B,C	A,D	A,C	A,C,D	B	B	A	B	B
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
C	C	C	A	C	A	D	A	A	D

EXPLANATIONS

MATHEMATICS - 1 (MCQ)

1. **(C)** R is reflexive since (x, x) R $\cancel{\forall}$ x $\in$ A.

   R is not symmetric since $(3, 6) \in$ R but $(6, 3) \notin$ R.

   Now, if $x, y, z \in$ A, then $(x, y) \in$ R, $(y, z) \in$ R $\Rightarrow (x, z) \in$ R.

   $\therefore$ R is transitive.

2. **(D)** Let $\tan^{-1} x$ be $\theta \Rightarrow x = \tan\theta$

   Since $x \geq 0$, $0 \leq \theta < \frac{\pi}{2}$

   $\Rightarrow 2(0) \leq 2\theta < \frac{2\pi}{2} \Rightarrow 0 \leq 2\theta < \pi$

   Also, $\frac{1-x^2}{1+x^2} = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta$

   $\Rightarrow \quad \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\theta = 2\tan^{-1}x,\ x \geq 0.$

3. **(B)** f (5) = f (5 + 0) = f (5). f (0) $\Rightarrow$ f (0) = 1.

$$f'(5) = \lim_{h\to 0} = \frac{f(5+h)-f(5)}{h}$$

$$= \lim_{h\to 0} = \frac{f(5).f(h).-f(5)}{h}$$

$$= f(5).\lim_{h\to 0} = \frac{f(h)-1}{h} = f(5).\ \lim_{h\to 0}\frac{f(h)-f(0)}{h}$$

= f (5) × f' (0) = (2 × 3) = 6

$\therefore$ The correct answer is (B)

4. **(B)** Since A is not singular, (Given)

A^{-1} exists

Now, A(adj A) = |A|I = (adj A) A

$\Rightarrow$ |A||adj A| = $|A|^n$ = |adj A||A|

$\therefore$ |adj A| = $|A|^{n-1}$

5. **(D)** Let $\Delta = \begin{vmatrix} a & -b & -c \\ -a & b & -c \\ -a & -b & c \end{vmatrix} = abc\begin{vmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{vmatrix}$

$\begin{bmatrix}\text{Taking a, b, c} \\ \text{common from} \\ C_1, C_2, C_3 \text{ resp.}\end{bmatrix}$

$$= abc\begin{vmatrix} 1 & 0 & 0 \\ -1 & 0 & -2 \\ 1 & -2 & 0 \end{vmatrix} = -4\ abc$$

[$C_2 \to C_1 + C_2$, $C_3 \to C_1 + C_3$]

Now, $\Delta + \lambda$ abc = 0

$\Rightarrow$ – 4abc + λabc = 0

$\Rightarrow \lambda = 4$

6. **(A)** At any instant of time 't' let the length of the rectangle be 'x', breadth be 'y' and the corresponding area be A

Differentiating A = xy w.r.t. 't', we get

$$\frac{dA}{dt} = \frac{d}{dt}(xy) = x\frac{dy}{dt} + y\frac{dx}{dt}$$

= $(-3x + 3.5y)$ cm/sec

$\left(\because \frac{dx}{dt} = 3.5 \text{ cm/sec and } \frac{dy}{dt} = -3 \text{ cm/sec}\right)$

When x = 12 cm and y = 8 cm,

$$\frac{dA}{dt} = (-3\times 12 + 3.5\times 8)\ \text{cm}^2/\text{sec}$$

= – 8 cm^2 / sec

Hence the area of the rectangle decreases at the rate of 8 cm^2 / sec

7. **(B)** Let $\Delta = \begin{bmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{bmatrix} = xyz\begin{bmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{bmatrix}$

$\begin{bmatrix}\text{Taking } x, y, z \\ \text{common from} \\ R_1, R_2, R_3 \text{ resp.}\end{bmatrix}$

$$= xyz\begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & y^2-x^2 \end{vmatrix}$$

$\begin{bmatrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{bmatrix}$

$$= xyz(y-x)(z-x)\begin{bmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+x \end{bmatrix}$$

$\begin{bmatrix}\text{Taking } (y-x) \\ (z-x) \text{ common from} \\ R_2, \text{ and } R_3 \text{ respectively.}\end{bmatrix}$

$= xyz(y-x)(z-x)\ (z-y)$

Now, Δ = 0 $xyz(y-x)(z-x)\ (z-y) = 0$

$\Delta\ xyz = 0\ [\because y - x \neq 0, z - x \neq 0, z - y \neq 0$ since $x \neq y \neq z]$

8. **(A)** As $[x + \pi]$ is always an integer and sine of an integer multiple of π is zero, numerator of f(x) is 0 for all $x \in$ R and denominator is always positive.

($\because$ $[x]^2 \geq 0$ for all $x \in$ R)

$\Rightarrow$ $f(x) = 0$ for all $x \in$ R

So, f is a constant function with D_f = R

Hence, f is continuous and derivable at all $x \in$ R.

9. **(B)** Putting p = $\frac{dy}{dx}$ the given equation can be written as $y = px + 1/p$. Differentating w.r.t. x, we have

$$p = \frac{dy}{dx} = p + x\frac{dp}{dx} - \frac{1}{p^2}\frac{dp}{dx}$$

$$\Rightarrow \quad (x-(1/p^2))\frac{dp}{dx} = 0 \Rightarrow p^2 = \frac{1}{x} \text{ or } \frac{dp}{dx} = 0$$

If $\frac{dp}{dx} = 0$ then p = constant = c putting this value in given equation, we get $y = cx + 1/c$ which represents a straight line. If $p^2 = 1/x$ then $y^2 = (px + 1/p)^2 = p^2x^2 + 1/p^2 + 2x = \frac{1}{x}x^2 + x + 2x = 4x$, which represents a parabola

10. **(D)** Required area

$$= \int_{-1}^{1}|x^3|\,dx = \int_{-1}^{0}|x^3|\,dx + \int_{0}^{1}|x^3|\,dx$$

$$= \int_{-1}^{0}|-x^3|\,dx + \int_{0}^{1}x^3 dx$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

($\because$ $x^3 \leq 0$ for $x \leq 0$ and $x^3 \geq 0$ for $x \geq 0$)

11. **(B)** $\frac{2(3\vec{a}-2\vec{b})+3(2\vec{a}-3\vec{b})}{2+3}$

$$= \frac{6\vec{a}-4\vec{b}+6\vec{a}-9\vec{b})}{5}$$

$$= \frac{12\vec{a}}{5} - \frac{13\vec{b}}{5}$$

12. **(A)** AB = $\begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi \\ \sin\theta\cos\phi & \sin\theta\sin\phi \end{bmatrix}\cos(\theta-\phi)$

Since, $\cos(\theta-\phi) = \cos\frac{\pi}{2} = 0$,

$\therefore$ AB = 0

13. **(B)** Let S be the area and r be the radius of the plate. Then, S = πr^2

$$\Rightarrow \frac{dS}{dt} = 2\pi r.\frac{dr}{dt}$$

When r = 30 cms, we have:

$$\frac{dS}{dt} = \{(2\pi) \times 30 \times 0.025\}\ \text{cm}^2/\text{sec}$$

$$\left[\therefore \frac{dr}{dt} = 0.025\ \text{cm/sec}\right]$$

$$= \frac{3\pi}{2}\ \text{cm}^2/\text{sec}$$

14. **(A)** $\underset{x\to1}{\text{Lt}} \int_4^{f(x)} \frac{2t}{x-1}dt = \underset{x\to1}{\text{Lt}}\left\{\frac{1}{x-1}\int_4^{f(x)} 2t\,dt\right\}$

$$= \underset{x\to1}{\text{Lt}}\left\{\frac{1}{x-1}[t^2]_4^{f(x)}\right\} = \underset{x\to1}{\text{Lt}} \frac{(f(x))^2 - 4^2}{x-1}$$

$$= \underset{x\to1}{\text{Lt}} \frac{(f(x)+4)(f(x)-4)}{x-1}$$

$$= \underset{x\to1}{\text{Lt}}\{f(x)+4\}.\underset{x\to1}{\text{Lt}}\frac{f(x)-f(1)}{x-1}$$

($\because$ $f(x) = 4$)

$$=(4+4).\underset{h\to0}{\text{Lt}}\frac{f(1+h)-f(1)}{h} = 8f'(1)$$

15. **(B)** We know that if G is the centroid of the ΔABC, then $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = \vec{0}$

$$\overrightarrow{GA} - \overrightarrow{BG} + \overrightarrow{GC} = 0$$

$$\overrightarrow{GA} + \overrightarrow{GC} = \overrightarrow{BG}$$

$$\overrightarrow{GA} + \overrightarrow{BG} + \overrightarrow{GC}$$

$$= \overrightarrow{BG} + \overrightarrow{BG} = 2\overrightarrow{BG}$$

16. **(C)** Given differential equation can be written as

$$(1+y^2)\frac{dx}{dy} + x - e^{\tan^{-1}}y = 0$$

or $\frac{dx}{dy}+\left(\frac{1}{1+y^2}\right)x=\frac{e^{\tan^{-1}y}}{1+y^2}$

which is linear in x

$\therefore$ I. F. = $e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$

Hence, the solution of the given equation is given by

$$xe^{\tan^{-1}y}=\int\frac{\left(e^{\tan^{-1}y}\right)^2}{1+y^2}dy+c$$

For integration, put $\tan^{-1}y$ = t

$\Rightarrow$ $xe^{\tan^{-1}y}=\frac{e^{2t}}{2}+x$

i.e., $2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$

17. **(C)** $\int_{-\pi/2}^{\pi/2}\sqrt{\frac{1-\cos 2x}{2}}dx=\int_{-\pi/2}^{\pi/2}\sqrt{\frac{2\sin^2 x}{2}}dx$

$=\int_{-\pi/2}^{\pi/2}|\sin x|\,dx$

$|\sin x|$ is an even function

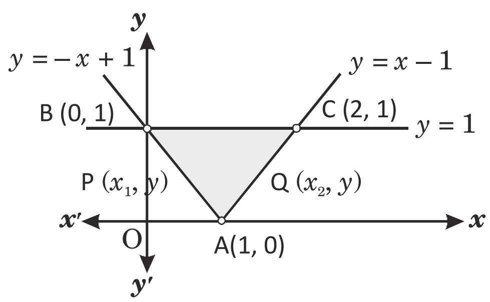
$\therefore$ $\int_{-\pi/2}^{\pi/2}|\sin x|\,dx=2\int_{0}^{\pi/2}|\sin x|\,dx$

$=2\int_{0}^{\pi/2}|\sin x|\,dx=2$

18. **(D)** $\underset{x\to 1}{\text{Lt}}\frac{f(x)-f(1)}{x-1}=f'(1)=-\frac{1}{2}.\frac{-2\times 1}{\sqrt{25-1^2}}=\frac{1}{\sqrt{24}}$

$\left(\because\ f'(x)=-\frac{1}{2}.\frac{-2x}{\sqrt{25-x^2}}\right)$

19. **(A)**



ar(ΔABC) = $\frac{1}{2}$(base) × (altitude)

$\Rightarrow$ ar(ΔABC) = $\frac{1}{2}$(2)(1) = 1

20. **(C)** I =

$\int\frac{1-\sin x}{1-\sin^2 x}dx=\int(\sec^2 x-\sec x\tan x)dx$

= tan x – sec x + C

21. **(B)** $|\hat{a}-\hat{b}|^2=(\hat{a}-\hat{b}).(\hat{a}-\hat{b})$

$=\hat{a}.\hat{a}-2\hat{a}.\hat{b}+\hat{b}.\hat{b}$

$=|\hat{a}|^2-2|\hat{a}||\hat{b}|\cos\theta+|\hat{b}|^2$

= 1 – 2cosθ + 1 = 2(1 – cosθ)

$=2\times 2\sin^2\left(\frac{\theta}{2}\right)$

$\Rightarrow$ $|\hat{a}-\hat{b}|=\left|2\sin\left(\frac{\theta}{2}\right)\right|=2\sin\left(\frac{\theta}{2}\right)$

[$\because$ $0\le\theta\le\pi$, $0\le\theta/2\le\pi/2$]

22. **(B)** Let the length of side of cube be a

Then, coordinates of corner (P) opposite to origin are (a, a, a)

$\therefore$ Direction ratios of diagonal OP are a – 0, a – 0, a – 0 i.e. a, a, a. i.e. 1, 1, 1

23. **(D)** Given f(x) = cos x, $0\le x\le 2\pi$

Differentiating w.r.t 'x', we get

f′(x) = – sin x, $0\le x\le 2\pi$

Now f′(x) < 0 when – sin x < 0,

i.e., when sin x > 0

i.e., when 0 < x < π

Therefore, then given function decreases in [0, π]

24. **(B)** To get the sum as odd number one number must be even & other must be odd

Total changes to select. 2 number sout of 40 nos. = ${}^{40}C_2$

Favourable cases to select 1 even out of

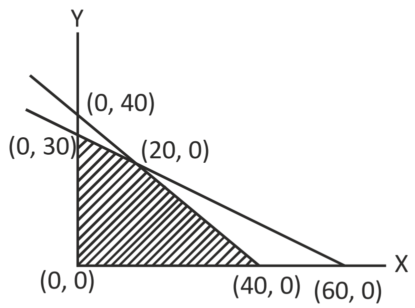
20 even = ${}^{20}C_1$ and favourable cases to select 1 odd out of 20 odd = ${}^{20}C_1$

∴ P(odd sum) = P (1 number even & 1 number odd)

$$= \frac{{}^{20}C_1 \times {}^{20}C_1}{{}^{40}C_2} = \frac{20 \times 20}{(40 \times 39)/2} = \frac{20}{39}$$

25. **(A)** A × B has 'mn' ordered pairs. Each subset of A × B is a relation. The number of subsets of a set consisting of 'mn' elements is 2^{mn}

26. **(B)** On drawing the graphs of given inequations, we have



Now Z = $3x + 4y$

$(Z)_{(0,0)} = 0$

$(Z)_{(40,0)} = 120$

$(Z)_{(0,30)} = 120$

$(Z)_{(20,20)} = 140$

∴ Maximum value = 140

which is obtained at $x = 20$ and $y = 20$

27. **(D)** $\int_0^{\pi/2} \sin^8 x \cos^2 x\, dx$

$$= \frac{7}{10}\cdot\frac{5}{8}\cdot\frac{3}{6}\cdot\frac{1}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2} = \frac{7\pi}{512}$$

28. **(B)** $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$

$$= \tan^{-1} 1 + \left(\frac{\pi}{2} - \cot^{-1} 2\right) + \left(\frac{\pi}{2} - \cot^{-1} 3\right),$$

$$\left(\because \tan^{-1} x = \frac{\pi}{2} - \cot^{-1} x, \forall x \in R\right)$$

$= \tan^{-1} 1 + \pi - (\cot^{-1} 2 + \cot^{-1} 3)$

$$= \frac{\pi}{4} + \pi - \left(\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)\right)$$

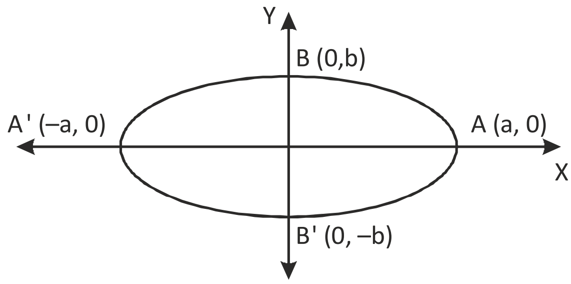
$$= \frac{\pi}{4} + \pi - \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1 - \frac{1}{2}\times\frac{1}{3}}\right)$$

$\left(\because \tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right.$ for $\left. xy < 1\right)$

$$= \frac{\pi}{4} + \pi - \frac{\pi}{4} = \pi$$

29. **(A)** $A = 4\int_0^a y\, dx$

The parametric equations of ellipse are $x = a \cos t$ and $y = b \sin t$



∴ $A = 4\int_0^a (b \sin t)(-a \sin t)\, dt$

⇒ $A = 4ab\int_0^{\pi/2} \sin^2 t\, dt$

∴ $A = 4ab.\frac{1}{2}.\frac{\pi}{2} = \pi ab$

30. **(D)** Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

Then $\vec{a}.\hat{i} = a_1, \vec{a}.\hat{j} = a_2$ and $\vec{a}.\hat{k} = a_3$

∴ $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$$= (\vec{a}.\hat{i})\hat{i} + (\vec{a}.\hat{j})\hat{j} + (\vec{a}.\hat{k})\hat{k}$$

MATHEMATICS - 2 (MAQ)

31. **(A, D)** Solving after putting z = 0 we get

$x = -2, y = 5, z = 0$

Let D.R.'s of the line be $\langle a \quad b \quad c \rangle$

3a + 2b – c = 0 and 4a + b – 2c = 0

$$\Rightarrow \frac{a}{-4+1} = \frac{b}{-4+6} = \frac{c}{3-8}$$

$$\Rightarrow \frac{a}{-3} = \frac{b}{2}\frac{c}{5}$$

$\therefore$ symmetrical form of the line is

$$\boxed{\frac{x+2}{-3} = \frac{y-5}{2} = \frac{z-0}{-5}}$$

or $\dfrac{x+2}{3} = \dfrac{y-5}{-2} = \dfrac{z}{5}$

32. **(A, B, C)** Since $P(A \cup B) \le P(A) + P(B)$

$$\Rightarrow P(A \cup B) \le \frac{1}{8} + \frac{5}{8}$$

$$\Rightarrow \boxed{P(A \cup B) \le \frac{3}{4}}$$

Hence Option (A) is correct.

Again,

$P(A \cap B) \le P(A)$

$$\Rightarrow P(A \cap B) \le \frac{1}{8}$$

Also, $P(A \cap B) \le P(B)$

$$\Rightarrow P(A \cap B) \le \frac{5}{8}$$

From (1) and (2)

$$\boxed{P(A \cap B) \le \frac{1}{8}}$$

Hence, Option (B) is also correct.

Now, $P(A \cap B) \ge 0$

$$\Rightarrow P(B) - P(\bar{A} \cap B) \ge 0$$

$$\Rightarrow P(\bar{A} \cap B) \le P(B)$$

$$\Rightarrow \boxed{P(\bar{A} \cap B) \le \frac{5}{8}}$$

Hence, Option (C) is also correct.

33. **(A, D)** Let $I = \int x^{51}(\tan^{-1} x + \cot^{-1} x)dx$

Since, $\tan^{-1} + \cot^{-1} x = \dfrac{\pi}{2}$

$$\Rightarrow I = \int x^{51}\frac{\pi}{2}dx = \frac{\pi}{2}\frac{x^{52}}{52} + c$$

$$\therefore \boxed{I = \frac{x^{52}}{52}(\tan^{-1} x + \cot^{-1} x) + c}$$

34. **(A, C)** Let $\Delta = \begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$, we have

$$\Delta = \begin{vmatrix} 3a-x & a-x & a-x \\ 3a-x & a+x & a-x \\ 3a-x & a-x & a+x \end{vmatrix} = 0$$

$$\Rightarrow \Delta = (3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 1 & a+x & a-x \\ 1 & a-x & a+x \end{vmatrix} = 0$$

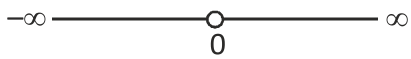
Applying $R_2 \to R_2 + R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow \Delta = (3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 0 & 2x & 0 \\ 0 & 0 & 2x \end{vmatrix} = 0$$

$\Rightarrow$ $\Delta = (3a - x)4x^2 = 0$

either $x = 0$ or $x = 3a$

35. **(A, C, D)** f(x) is discontinuous at x = 0



$$f'(x) = \frac{-12x^2}{\left(e^{x^3} - e^{-x^3}\right)^2}, \; x \ne 0$$

$\therefore$ f(x) is a decreasing function in (-∞, 0) and also in (0,) i.e., in $\mathbb{R}$ = {0} and hence also a one-one function.

Also, $\lim\limits_{x \to -\infty} f(x) = -1$, $\lim\limits_{x \to \infty} f(x) = 1$

$\lim\limits_{x \to 0-0} f(x) = -\infty$, $\lim\limits_{x \to 0+0} f(x) = \infty$

Range of

$f = (-\infty, -1) \cup (1, \infty) = \mathbb{R} - [-1,1]$

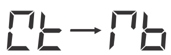
Since, Range equals Codomain

Hence, f is onto

REASONING

36. **(B)** In the first stage, the bottom horizontal line is removed from figure one and the bottom right vertical line in second figure is added to obtain the figure of second stage.

In the similar way, in third stage, the bottom horizontal line of figure one is removed and the bottom right vertical line in second figure is added to obtain the figure of stage 4.



Hence, option (b) is correct.

37. **(B)** The mast is too far forward.

38. **(A)** 85

In the first row, the numbers outside the brackets are divided by 16 and the results placed inside the brackets. In the second row they are divided by 17.

Therefore, in the third row they are divided by 18.

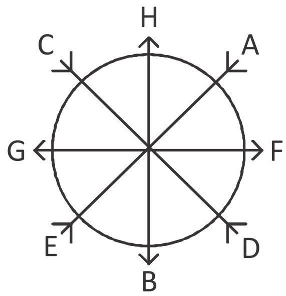
39. **(B)** JX, G = central shape divided into three, H = divided into four, J = divided into six, K = divided into two. L = curved brackets, P = square brackets, X = hexagonal brackets.

40. **(B)** The words formed are AT and UNDER,

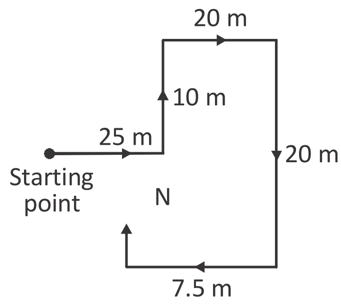
or AS and UNDER

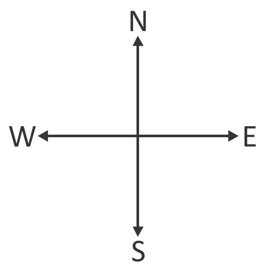
or AT and SOUND

41. **(C)**



42. **(C)** Let us draw the passage followed by Ragini.



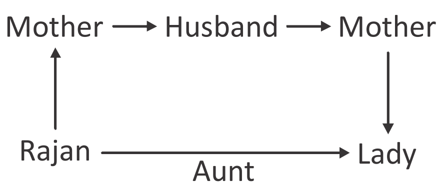


So, the direction in which Ragini is walking, is North.

Hence, option (C) is correct.

43. **(C)** The pattern in the given picture and the fact that the faces are rectangular and option (B) is eliminated as dot on the upper face is on left side instead of right. Option (A) and option (D) also get eliminated as the smaller rectangular faces should be blank instead of having a dot on them. So, option (C) is the only possible fig as it is having three blank faces adjacent to each other.

44. **(A)** Pictorial representation of the relation is



In other words, Rajan's mother's husband is Rajan's father and Rajan's father's

Mother is Rajan's grandmother.

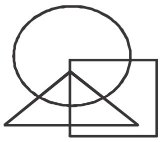
Now, the daughter of Rajan's grandmother will be Rajan's aunt.

45. **(C)**

CRITICAL THINKING

46. **(A)** From statement I alone, we conclude that Manoj is the heaviest. Statement II alone is not sufficient.

47. **(D)**



In the given figure one of the dots lies in the region common to the circle, square and triangle. The other dot lies in the region common to the square and triangle only. In options (A), (B) there is no region common to the triangle and square. In the same way in option (C) there is no region common to the circle, square and triangle. Only option (D) consists of both type of regions.

48. **(A)** When pieces of ice which are several cubic kilometers in size break they will cause earthquakes. So, (A) is the cause and (B) is its effect.

49. **(A)** M is the father of N and N is the son of V. Hence, V is the mother of N. From (1), P is the brother of V Therefore, M is the brother-in-law of P because V is the wife of M. From (2), the daughter of N, is the granddaughter of V. From this we do not get any relation of M to P.

50. **(D)** The ministry suggested yoga army personnel to keep them mentally and physically fit. Statement 'D' indicates that the only way to gain mental and physical fitness is through yoga. This could be the possible reason for the minister to suggest yoga. Hence 'D' is a possible reason.

The End