UNIFIED COUNCIL

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Unified International Mathematics Olympiad

## UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)

**CLASS - 10**

**Question Paper Code : UM9246**

### KEY

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| B | B | A | C | D | C | D | A | C | D |
| **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
| D | A | C | A | B | D | A | D | C | A |
| **21** | **22** | **23** | **24** | **25** | **26** | **27** | **28** | **29** | **30** |
| C | C | B | A | B | A | D | B | C | B |
| **31** | **32** | **33** | **34** | **35** | **36** | **37** | **38** | **39** | **40** |
| A,C | A,D | A,B,C | A,B | C,D | A | A | D | D | B |
| **41** | **42** | **43** | **44** | **45** | **46** | **47** | **48** | **49** | **50** |
| C | A | C | B | A | D | B | D | B | B |

### EXPLANATIONS

**MATHEMATICS - 1**

1: (B) Given $\tan x = \frac{4}{3}$

$\Rightarrow \frac{RT}{RS} = \frac{4}{3}$

$\Rightarrow \frac{RT}{1\cancel{3}m} = \frac{4}{\cancel{3}}$

$\Rightarrow$ RT = 4m

$\therefore$ PR = 2RT = 8m

In $\Delta PQR$ $\angle 90^\circ$ $\Rightarrow$ $PQ^2 = PR^2 + RQ^2 = (8m)^2 + (6m)^2$

$= 100m^2$

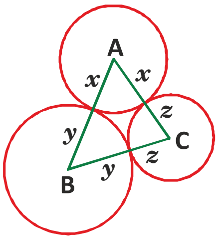
$PQ^2 = (10m)^2$

PQ = 10 m

2: (B) Given $x + y$ = 12cm, $y + z$ = 9cm & $z + x$ = 7cm

$\therefore x + y + y + z + z + x$ = (12 + 9 + 7)cm = 28cm

$2(x + y + z)$ = 28cm




$$x + y + z = \frac{28\text{cm}}{2} = 14\text{cm}$$

$\therefore$ 12cm + $z$ = 14cm $\Rightarrow z$ = 2cm

$x$ + 9cm = 14cm $\Rightarrow x$ = 14cm – 9cm = 5cm

7 + $y$ = 14cm $\Rightarrow y$ = 14cm – 7cm = 7cm

3: (A) Given $(5x + 4)^2 = 9(2x - 1)^2$

$25x^2 + 40x + 16 = 9(4x^2 - 4x + 1)$

$0 = 36x^2 - 25x^2 - 36x - 40x + 9 - 16$

$11x^2 - 76x - 7 = 0$

$11x^2 - 77x + x - 7 = 0$

$11x(x - 7) + 1(x - 7) = 0$

$\therefore x = 7$ (or) $x = \frac{-1}{11}$ is rejected because length is never negative

$\therefore 2x - 1 = 2(7) - 1 = 14 - 1 = 13$ m

4: (C) Area of uncut portion = $\pi(20\text{ cm})^2 - 4 \times \pi(5\text{ cm})^2$

= $400\pi\text{ cm}^2 - 100\pi\text{ cm}^2$

= $300\pi\text{ cm}^2$

Cut portion area = $4 \times \pi(5\text{ cm})^2 = 100\pi\text{ cm}^2$

Ratio of uncut to the cut portions = $300\pi\text{ cm}^2 : 100\pi\text{ cm}^2$

= 3 + 1

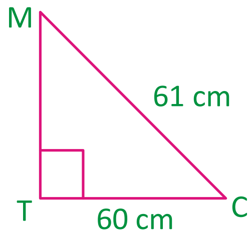
5: (D) In $\Delta MCT$, $\angle T = 90^\circ \Rightarrow MC^2 = MT^2 + TC^2$

$\Rightarrow (61\text{ cm})^2 = MT^2 + (60\text{ cm})^2$

$3721\text{ cm}^2 - 3600\text{ cm}^2 = MT^2$

$\therefore MT = \sqrt{121\text{cm}^2} = 11\text{cm}$

$$\therefore \sin C = \frac{MT}{MC} = \frac{11\text{cm}}{61\text{cm}} = \frac{11}{61}$$




6: (C) Area of

$$\Delta ABC = \frac{1}{2}|3(5-4) + 4(4-0) + (-1)(0-5)|$$

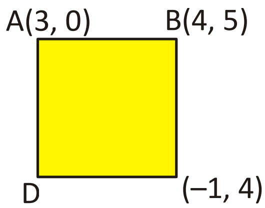
$$= \frac{1}{2}|3 + 16 + 5|$$

$$= \frac{1}{2} \times 24 \text{ units}^2$$

$\therefore$ Area of the rhombus ABCD = 2 Area of

$$\Delta ABC = \cancel{2}^{1} \times \frac{1}{\cancel{2}} \times 24\text{units}^2$$

= 24 units$^2$




7: (D) Sum of three prime numbers = 100

All prime numbers are odd except 2

$\therefore$ '2' is the one prime number.

$2 + x + x + 36 = 100$

$2x = 100 - 38 = 62$

$x = 31$

$\therefore x + 36 = 31 + 36 = 67$ which is a prime

8: (A) Given $\sqrt{l^2 + b^2 + h^2} = 27\text{cm}$

$$\Rightarrow \sqrt{l^2 + (10\text{cm})^2 + (10\text{cm})^2} = 27\text{cm}$$

squaring on both sides

$l^2 + 100\text{cm}^2 + 100\text{cm}^2 = 729\text{cm}^2$

$\therefore l^2 = 529\text{cm}^2 = (23\text{cm})^2$

$\therefore l = 23\text{cm}$

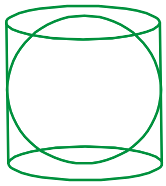
9: (C) Given radius of cylinder and sphere are same and height of cylinder = diameter of sphere

$\therefore$ Volumes ratio of cylinder & sphere

$= \pi r^2(2r) : \frac{4}{3}\pi r^3$

$= 2r^3 : \frac{4^2 r^3}{3}$

= 3 : 2



10: (D) Given a + 3d = 8 $\rightarrow$ eq. (1)

$\frac{12^6}{2}$ [2a + 11d] = 156

2a + 11d = $\frac{156}{6} = 26 \rightarrow eq.(2)$

2a + 11 d = 26 $\longrightarrow$ eq. (2)

2a + 6 d = 16 $\longrightarrow$ eq. (1) × 2

(–) (–) (–)

5d = 10

d = 2

a + 3(2) = 8

a = 8 – 6 = 2

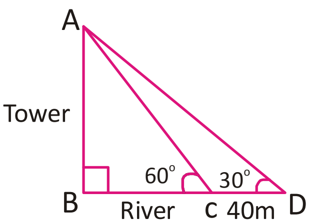
Given $a_n$ = 200

2 + (n – 1) (2) = 200

$2[1 + n - 1]$ = 200

n = $\frac{200}{2} = 100$

11: (D) Given In $\Delta ABC, \angle B = 90° \ \& \ \angle ABC = 60°$




$\therefore$ tan 60° = $\frac{AB}{BC}$

$\sqrt{3} = \frac{AB}{BC} \Rightarrow AB = \sqrt{3}\,BC \rightarrow 1$

In $\Delta ABD$, $\angle D$ = 30° $\Rightarrow$ tan 30° = $\frac{AB}{BC + 40mts}$

$\frac{1}{\sqrt{3}} = \frac{AB}{BC + 40mts}$

$\therefore\ AB = \frac{BC + 40mts}{\sqrt{3}} \rightarrow 2$

from eq. 1 & eq. 2 $\Rightarrow \sqrt{3}BC = \frac{BC + 40mts}{\sqrt{3}}$

$\therefore$ 3BC = BC + 40mts

$\therefore$ 2BC = 40mts

$BC = \frac{40mts}{2} = 20mts$

$\therefore$ Height of tower = AB = $\sqrt{3}BC = 20\sqrt{3}mts$

12: (A) Given $\pi r^2$ = $2\pi r$

$\therefore$ r = 2

$\therefore$ d = 2r = 4 units

13: (C) Two roots of $x^2 - px + q = 0$ be a & a + 1

Given a + a + 1 = $\frac{-(-p)}{1} = p$

p = 2a + 1

a (a + 1) = $\frac{q}{1}$

$a^2 + a = q$

$\therefore\ p^2 - 4q = (2a + 1)^2 - 4(a^2 + a)$

$= 4a^2 + 4a + 1 - 4a^2 - 4a$

= 1

14: (A) Given

$\frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right| = 0$

$\Rightarrow \left|1(2-5) + 5(5-(-1)) + x(-1-2)\right| = 0 \times 2$

$|-3+30-3x| = 0\times 2$

$\therefore$ $27 - 3x = 0$

$3x = 27$

$x = 9$ (OR) verify from options

15: (B) Given $\alpha + \beta = 4$ & $\alpha\beta = -5$

Required quadratic polynomial $= x^2 - x(\alpha + \beta) + \alpha\beta$

$= x^2 - 4x - 5$

16: (D) $\frac{145}{2^3\times 5^2\times 7^2}$ is non terminating but repeating decimal

[$\because$ Denominator is a prime other than 2 & 5]

17: (A) p = –1 & q = 1 satisfies 2p + 3q = 1

18: (D) Given $a_p = a + (p-1)d = q$

$\Rightarrow a + pd - d = q \rightarrow 1$

Given $a_q = a + qd - d = p \rightarrow 2$

$eq1 - eq2 \Rightarrow (a+pd-d)-(a+qd-d) = q-p$

$\Rightarrow \cancel{a} + pd - \cancel{d} - \cancel{a} - qd + \cancel{d} = -p+q$

$d(p-q) = -(p-q)$

$d = -1$

$a + p(-1) - (-1) = q \rightarrow 1$

$a - p + 1 = q$

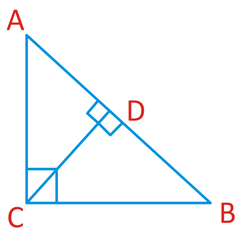
$\therefore$ $a = (p + q - 1)$

$\therefore$ $a_{pq} = a + (pq-1)d = (p+q-1) + (pq-1)(-1)$

$= p + q - \cancel{1} - pq + \cancel{1}$

$= p + q - pq$

19: (C) $\Delta ADC \sim \Delta CDB \sim \Delta ACB$




[A. A similarity]

$\therefore \Delta ADC \sim \Delta ACB \Rightarrow \frac{AD}{AC} = \frac{AC}{AB} \Rightarrow AC^2 = AB\times AD \rightarrow 1$

$\therefore \Delta CDB \sim \Delta ACB \Rightarrow \frac{CD}{AC} = \frac{BD}{BC} = \frac{BC}{AB} \Rightarrow BC^2 = AB\times BD \rightarrow 2$

$\therefore \frac{BC^2}{AC^2} = \frac{\cancel{AB}\times DB}{\cancel{AB}\times AD} = \frac{DB}{AD}$

20: (A) Given $\frac{4}{3}\pi(R^3 - r^3) = \frac{1}{3}\pi\times r^2\times h$

$\Rightarrow \frac{4}{\cancel{3}}\cancel{\pi}(4^3 - 2^3) = \frac{1}{\cancel{3}}\times\cancel{\pi}r_1^2\times 14cm^3$

$\cancel{4}^2\times\frac{(64-8)}{\cancel{14}_7} = r_1^2$

$2\times\frac{\cancel{56}^8}{\cancel{7}_1} = r_1^2$

$r_1 = \sqrt{16cm} = 4cm$

$\therefore$ Diameter $= 2r_1 = 8cm$

21: (C) $\Delta ADC \sim \Delta AGH$ [$\because$ A. A similarity]

$\therefore \frac{AD}{AG} = \frac{DC}{GH}$

$\Rightarrow \frac{2cm}{6cm} = \frac{1cm}{GH} \Rightarrow GH = \frac{6cm^2}{2cm} = 3cm$

$\therefore$ Area of $\Delta AGH$

$= \frac{1}{2}\times AG\times GH = \frac{1}{\cancel{2}}\times\cancel{6}^3\times 3cm^2 = 9cm^2$

Area of shaded region = Area of square + Area of rectangle – Area of $\Delta AGH$ = $2cm^2 + 16cm^2 - 9cm^2 = 9cm^2$

22: (C) Given Diameter of sphere = side of cube = 7cm

$\therefore$ Radius = $\frac{7}{2}cm$

Volume of sphere

$= \frac{4}{3}\pi r^3 = \frac{\cancel{4}^2}{3}\times\frac{\cancel{22}^{11}}{\cancel{7}}\times\frac{\cancel{7}}{\cancel{2}}\times\frac{7}{\cancel{2}_1}\times\frac{7}{\cancel{2}}cm^3$

$= 179.67\ cm^3$

23: (B) Given $x = 2.4488....$

$\therefore 10x = 24.488....$

$x = 2.4488....$

$\therefore 9x = 22.0400$

$$\therefore x = \frac{22.04}{9} \times \frac{100}{100} = \frac{2204}{900} = \frac{551}{225} = 2.44\overline{8}$$

24: (A) Given $p(x) = (k^2x^2 - kx - 2)$ and $(x - 3)$ is a factor of $p(x)$

$\therefore\ p(3) = 0 \Rightarrow p(3) = 9k^2 - 3k - 2 = 0$

$\Rightarrow 9k^2 - 6k + 3k - 2 = 0$

$3k(3k - 2) + 1(3k - 2) = 0$

$(3k - 2)(3k + 1) = 0$

$$k = \frac{2}{3} \text{(or)} -\frac{1}{3}$$

25: (B) $\frac{a_1}{a_2} = \frac{2}{3}$ & $\frac{b_1}{b_2} = \frac{3}{-1} \Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\therefore$ Given lines are intersecting lines.

26: (A) Given $\frac{(x+1)(x+2)+(x-2)(x-1)}{(x-1)(x+2)} = 3$

$x^2 + 3x + 2 + x^2 - 3x + 2 = 3(x^2 + x - 2)$

$2x^2 + 4 = 3x^2 + 3x - 6$

$0 = x^2 + 3x - 10$

$x^2 + 5x - 2x - 10 = 0$

$x = -5$ (or) $2$

27: (D) Given $a_n = a_{41} + 72$

$a + (n-1)(6) = a + 40 \times 6 + 72$

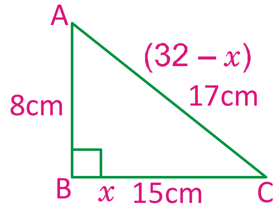
$(n - 1)\ (6) = 6\ (40 + 12)$

$n = 52 + 1 = 53$

$\therefore\ a_{53} = a_{41} + 72$

28: (B) Let BC be $x$ cm

$\Rightarrow AC = (32 - x)$ cm




Given $\angle B = 90° \Rightarrow AC^2 = AB^2 + BC^2$

$\Rightarrow (32 - x)^2 = 8^2 + x^2$

$\Rightarrow 1024 + x^2 - 64x = 64 + x^2$

$1024 + x^2 - 64x = 64 + x^2$

$1024 - 64 = 64x$

$64x = 960$

$$x = \frac{960}{64} = 15 \text{ cm}$$

$\therefore$ AC = 32cm – 15cm = 17cm

$$\therefore \sin A + \cos C = \frac{BC}{AC} + \frac{BC}{AC} = \frac{2BC}{AC} = \frac{2 \times 15\text{cm}}{17\text{cm}} = \frac{30}{17}$$

29: (C) Let the point on $x$ – axis be $p(x, 0)$

Given PA = PB

$$\therefore\ \sqrt{(x+2)^2 + 5^2} = \sqrt{(x-2)^2 + 3^2}$$

Squaring on both sides

$x^2 + 4x + 4 + 25 = x^2 - 4x + 4 + 9$

$4x + 4x = 13 - 29 = -16$

$8x = -16$

$$x = -\frac{16}{8} = -2$$

30: (B) Given $\Delta ABC \sim \Delta DEC$

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EC} = \frac{AC}{DC}$$

$$\Rightarrow \frac{9\text{cm}}{DE} = \frac{10\text{cm} + 2\text{cm}}{8\text{cm}}$$

$$DE = \frac{9 \times 8\text{cm}^2}{12\text{cm}} = 6\text{cm}$$

## MATHEMATICS - 2

31: (A,C)

Given $f(x) = 3x^4 - 21x^3 + 32x^2 + 28x - 48$

$\therefore$ $f(3) = 3(3)^4 - 21(3)^3 + 32(3)^2 + 28(3) - 48$

$= 243 - 567 + 288 + 84 - 48 = 0$

$(x - 3)$ is a factor of $f(x)$

$f(12) = 3(12)^4 - 21(12)^3 + 32(12)^2 + 28(12) - 48$

$= 62\,208 - 36\,288 + 4608 + 336 - 48$

$f(12) = 30\,816$

$\therefore$ $(x - 12)$ is not a factor

$f(4) = 3(4)^4 - 21(4)^3 + 32(4)^2 + 28(4) - 48$

$= 768 - 1344 + 512 + 112 - 48$

$f(4) = 0$

$\therefore$ $(x - 4)$ is a factor of $f(x)$

$f(6) = 3(6)^4 - 21(6)^3 + 32(6)^2 + 28(6) - 48$

$= 3888 - 4536 + 1152 + 168 - 48$

$f(6) = 624 \Rightarrow (x - 6)$ is not a factor.

32: (A, D)

$a_1 = 4, b_1 = -3, c_1 = 3$

$a_2 = 8, b_2 = -6, c_2 = 8$

$\therefore$ $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow$ Given lines are parallel lines

They are inconsistent

33: (A,B,C)

Given $\angle A + \angle C = 180°$ & $\angle B + \angle D = 180°$

$3x + 3° + 2y + 19° = 180°$

$3x + 2y = 158° \rightarrow 1$

$2y + 32° + 2x + 18° = 180°$

$2x + 2y = 180° - 50° = 130°$

eq 1 – eq2 $\Rightarrow 3x + 2y - 2x - 2y = 158° - 130° = 28°$

$x = 28°$ & $3(28°) + 2y = 158° \rightarrow 1$

$84° + 2y = 158°$

$2y = 158° - 84° = 74°$

$y = \frac{74°}{2} = 37°$

$\therefore \angle A = 3x + 3° = 3(28°) + 3° = 84° + 3° = 87°$

$\angle B = 2y + 32° = 2(37°) + 32° = 74° + 32° = 106°$

$\angle C = 2y + 19° = 74° + 19° = 93°$

$\angle D = 2x + 18° = 56° + 18° = 74°$

$\angle B + \angle C = 106° + 93° = 199°$, $\angle C + \angle D = 93° + 74° = 167°$

$\angle A + \angle D = 87° + 74° = 161°$

34: (A,B)

Option A,B angles are in AP and those angle sum is 180°

But option C & D angles are in AP but their sum is not 180°

35: (C,D) Given $2^{4(x^2 + 3x - 1)} = 2^{3(x^2 + 3x + 2)}$

$\therefore 4x^2 + 12x - 4 = 3x^2 + 9x + 6$

$4x^2 - 3x^2 + 12x - 9x - 4 - 6 = 0$

$x^2 + 3x - 10 = 0$

$x = -5$ (or) 2

## REASONING

36: (A) 11, 15, 26, 13, 22

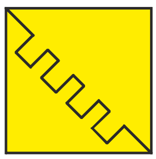
P L A N E is the meaningful word among the options.

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
| 6 | 5 | 4 | 3 | 2 | 1 | 0 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 | | | | | | | | | |

37: (A) The two numbers in the first box are multiplied to get the number in the second. Hence 2 * 8 = 16 will be the answer.

38: (D) As only sons of John have daughters and he has total 8 granddaughters, this means John must have 4 sons as each of his sons has 2 daughters. Therefore he has 8 grandsons and 8 granddaughters from 8 sons. For, the remaining 4 grandsons, he must have 2 daughters as each of his daughters has only 2 sons.

39: (D)



40: (B) The shapes in figures A, C and D have been duplicated and reflected. Although the main shape in B follows this rule, there are two lines in the figure which do not reflect – they appear on the left in both parts of the figure.

41: (C) S, V, Y, B and E are 3 alphabets between them. Numbers 97, 100, 105, 112, 123 are 3, 5, 7, 11, 13 added respectively.

42: (A) If he is facing east direction and his shadow follows him means the shadow is in the west. Therefore sun must be in the east as shadow forms in a straight line with the sun. So it is morning and time could be 10:00 AM.

43: (C) Today is January 31 and next month is Feb. There can be 28 or 29 days in the month of Feb.

A month after today would be March 1st, so option A is incorrect.

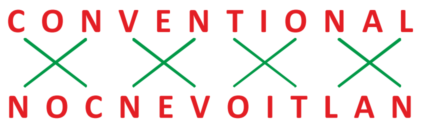
If it is a leap year, then there will be 29 days before the tournament starts to start the tournament and if it is not a leap year, there will be 28 days. Thus we cannot be sure about the option B or option D. Whether it is a leap year or not, there will always be at least 28 days to start the tournament. Hence the conclusion can be drawn

44: (B) Two changes happen alternatively:

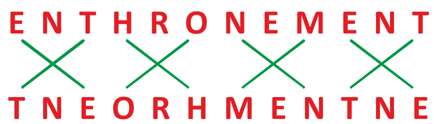
(1) The green shape moves to the back and front.

(2) The orange shapes takes half left turn.

45: (A)




then in the same way




## CRITICAL THINKING

46: (D) None of these

47: (B) In (A) and (B) options, the reflection of the distance shown at the point where the kid was seated must not be the same as in the real situation given the angle from which she was observed by the viewer (us). (B) and (D) suits the reflection of her sitting position. Now, the eyes of the kid in option (D) makes us believe that she was looking straight, but she should be looking slightly above the paper boat.

48: (D) 1-R, 2-Q, 3-P

49: (B) Three person lives between I and M.

| Floor | Person |
|---|---|
| 7 | I |
| 6 | L |
| 5 | N |
| 4 | K |
| 3 | M |
| 2 | J |
| 1 | O |

50: (B)