UNIFIED COUNCIL

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Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD

CLASS - 9

Question Paper Code : UM9246

KEY

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C	D	C	A	C	D	B	C	A	B

EXPLANATIONS

MATHEMATICS - 1

1: (D) $ab(a+b)^2 + 6ab(a+b) = ab(a+b)(a+b+6)$

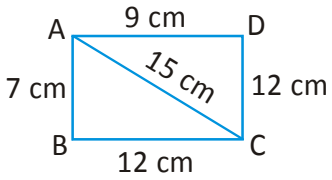
2: (B) Given $y : z = 3 : 7 = 3a : 7a$

$\therefore\ y = 3a$ & $z = 7a$

$\therefore\ x : y = z : y = 7a : 3a = 7 : 3$

[$\angle x = \angle z$ alternative angles]

3: (B) Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$



where $s = \dfrac{a+b+c}{2} = \dfrac{7cm+12cm+15cm}{2}$

$= \dfrac{34cm}{2} = 17$ cm

Area (ΔABC) = $\sqrt{17\times10\times5\times2}$ cm^2

= $10\sqrt{17}$ cm^2

In ΔACD, $15^2 = 12^2 + 9^2 \Rightarrow \angle D = 90°$

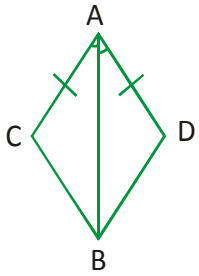
$\therefore$ Area of $\Delta ACD = \frac{1}{2}\times AD \times DC$

$= \frac{1}{\cancel{2}_1}\times 9 \times \cancel{12}^{6}$ cm^2 $= 54$ cm^2

$\therefore$ Area of the quadrilateral ABCD

= $(10\sqrt{7} + 54)$ cm^2

4: (D) In ΔACB & ΔADB



AC = AD (side)

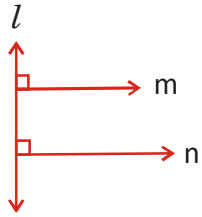
$\angle CAB = \angle DAB$ (Angle)

$\overline{AB} = \overline{AB}$ (side & common)

$\therefore \Delta ACB \cong \Delta ADB$ [$\therefore$ SAS congruency]

5: (A) m||n

[$\therefore$ their corresponding angles are equal]



6: (A) $y = 10$ is line is parallel to $y = 0$ line

i.e X-axis

7: (D) Given $2\cancel{\pi}rh : \cancel{\pi}rl = 8 : 5$

$2h \times 5 = l8$

$5h = 4\left(\sqrt{h^2 + r^2}\right)$

Squaring on both sides

$25h^2 = 16(h^2 + r^2)$

$25h^2 - 16h^2 = 16r^2$

$9h^2 = 16r^2$

$\frac{9}{16} = \frac{r^2}{h^2}$

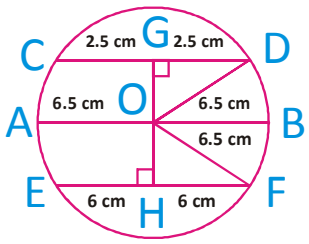
$\left(\frac{3}{4}\right)^{\cancel{2}} = \left(\frac{r}{h}\right)^{\cancel{2}}$

$\therefore$ r : h = 3 : 4

8: (A) $\Delta PSX \cong \Delta QRY$ [$\therefore$ AAS congruency]

$\therefore$ PS = QR

9: (A) Consti : OG $\perp$ CD and OH $\perp$ EF



$\therefore$ CG = GD = $\frac{CD}{2}$ = 2.5 cm

EH = HF = $\frac{EF}{2} = \frac{12cm}{2}$ = 6cm

Radius = $\frac{AB}{2}$ = 6.5 cm

In ΔGOD, $\angle G = 90° \Rightarrow OD^2 = GB^2 + OG^2$

$6.5^2 = 2.5^2 + OG^2$

$42.25 = 6.25 + OG^2$

OG = $\sqrt{42.25 - 6.25}$ = $\sqrt{36}$ = 6cm

Similarly OH =

$\sqrt{(6.5)^2 - 6^2} = \sqrt{42.25 - 36} = \sqrt{6.25}$

= 2.5cm

$\therefore$ GH = OG + OH = 6cm + 2.5 cm = 8.5 cm

10: (D) Given f(1) = g(1)

$\Rightarrow 2(1)^3 + a(1)^2 + 3(1) - 5 = 1^3 + 1^2 - 6(1) - a$

$\Rightarrow \cancel{5} + a - \cancel{5} = 2 - 6 - a$

$2a = -4$

$a = \frac{-\cancel{4}^{2}}{\cancel{2}_1} = -2$

11: (C) Given lb : bh : bl = 15 : 10 : 12 = 15x : 10x : 12x

$\therefore$ lb × bh × bl = 15x × 10x × 12x

($l$bh)^2 = 1800x^3

(960cm^3)^2 = 1800x^3

$96\cancel{0} \times 96\cancel{0} = 18\cancel{0}\,\cancel{0}x^3$

$$x^3 = \frac{\cancel{96}^{16} \times \cancel{96}^{32}}{\cancel{18}_{1}} = 2^9$$

$$x^{\cancel{3}} = \left(2^3\right)^{\cancel{3}}$$

x = 8

$\therefore$ lb = 15x = 15 × 8 = 120 cm^2

$$\therefore h = \frac{960\text{cm}^3}{120\text{cm}^2} = 8\text{cm}$$

$$b = \frac{lbh}{lh} = \frac{\cancel{960}^{10}\text{cm}^3}{\cancel{12\times 8}\ \text{cm}^2} = 10\text{ cm}$$

8 cm × 10 cm × l = 960 cm^3

$$l = \frac{\cancel{960}^{12}\text{cm}^3}{\cancel{80}_{1}\text{cm}} = 12\text{ cm}$$

12: (D) Let the number of chickens be x and number of goats be y

Given $x + y = 35 \rightarrow (1)$

Given $2(x) + 4(y) = 100$

$$\frac{2x}{2} + \frac{4y}{2} = \frac{100}{2}$$

$x + 2y = 50$

$x + y + y = 50$ [$\therefore x + y = 35$]

$y = 50 - 35$

$y = 15$

$x + 15 = 35 \rightarrow (1)$

$x = 20$

$\therefore$ No. of chickens : No. of goats =

$\cancel{20}^{4} : \cancel{15}^{3}$

= 4 : 3

13: (A) $4x^2 + 9y^2 + 25z^2 - 12xy + 30yz - 20zx$ =

$= (2x)^2+(-3y)^2+(-5z)^2+2(2x)(-3y)$

$+2(-3y)(-5z)+2(-5z)(2x)$

$= (2x - 3y - 5z)^2$

$$\sqrt{4x^2 + 9y^2 + 25z^2 - 12xy + 30yz - 20zx}$$

$$= \sqrt{(2x - 3y - 5z)^2}$$

$= (2x - 3y - 5z)$

14: (C) Given $xy = 1 \Rightarrow y = \frac{1}{x}$

$$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = 2 - \sqrt{3}$$

$$\frac{x}{2-x} + \frac{y}{2-y} = \frac{2+\sqrt{3}}{2-\left(2+\sqrt{3}\right)} + \frac{2-\sqrt{3}}{2-\left(2-\sqrt{3}\right)}$$

$$= \frac{2+\sqrt{3}}{\cancel{2}-\cancel{2}-\sqrt{3}} + \frac{2-\sqrt{3}}{\cancel{2}-\cancel{2}+\sqrt{3}}$$

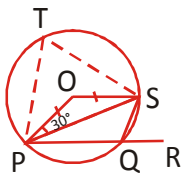
$$= \frac{-\left(2+\sqrt{3}\right)}{\sqrt{3}} + \frac{2-\sqrt{3}}{\sqrt{3}}$$

$$= \frac{-\cancel{2} - \sqrt{3} + \cancel{2} - \sqrt{3}}{\sqrt{3}}$$

$$= \frac{-2\sqrt{3}}{\sqrt{3}}$$

$= -2$

15: (C) In ΔPOS, OP = OS $\Rightarrow \angle$OSP = $\angle$OPS = 30°



$\therefore$ 30° + 30° + $\angle$POS = 180°

$\angle$POS = 180° – 60° = 120°

$$\therefore \angle PTS = \frac{\angle POS}{2} = \frac{120°}{2} = 60°$$

PQST is a cyclic quadrilateral

$\therefore$ $\angle PTS + \angle PQS = 180°$

$60° + \angle PQS = 180° \Rightarrow \angle PQS = 120°$

But $\angle PQS + \angle SQR = 180°$

$120° + \angle SQR = 180 \Rightarrow \angle SQR = 180° - 120° = 60°$

16: (A) Midpoint a rectangle join in order we get a rhombus

Area of rhombus = $\frac{1}{2}$ area of rectangle

17: (A) Option 'A' is true

18: (A) π is an irrational number

19: (D) $s = \frac{a+b+c}{2} = \frac{9cm+40cm+41cm}{2} = 45cm$

Area of triangle

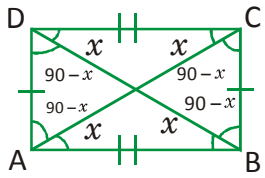
$= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{4\times36\times5\times4}\ cm^2$

$= \sqrt{9\times5\times9\times4\times5\times4}\ cm^2$

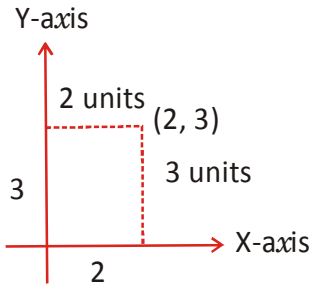
$= 9 \times 5 \times 4\ cm^2$

$= 180\ cm^2$

20: (B) $\Delta ABC \cong \Delta BAD$



21: (A)



22: (B) Given AB > AD $\Rightarrow$ CD > BC

$\Rightarrow \angle CBD > \angle BDC$

$\Rightarrow x > y$

23: (C) $\angle BCE = \angle BAE = 25°$

[$\because$ Angles in the same segment]

$\angle BAE = 25° \Rightarrow \angle EAC = 25°$

$\therefore \angle BAC = 25° + 25° = 50°$

24: (A) In a square interior and exterior angles are equal.

25: (C) Area of $\Delta ABC = \frac{1}{2} \times BC \times 6.5cm = \frac{1}{2} \times 14 \times 6.5\ cm^2 = 45.5\ cm^2$

26: (B) In ΔABC, BC > AC > AB $\Rightarrow \angle A > \angle B > \angle C$

27: (D) In ΔORM, $OR^2 = OM^2 + RM^2$

$(17cm)^2 = (15cm)^2 + RM^2$

$(289 - 225)cm^2 = RM^2$

$RM^2 = 64\ cm^2$

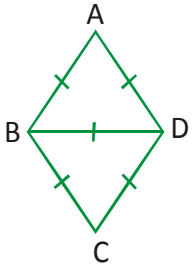
$RM = \sqrt{64cm^2} = 8\ cm$

$\therefore QR = 2RM = 2 \times 8cm = 16\ cm$

28: (B) ABD & BCD are equilataral triangle

$\therefore \angle A = 60°$ & $\angle ABD = 60°$ & $\angle DBC = 60°$

$\therefore$ The angles of rhombus are 60°, 120°, 60° & 120°



29: (B) Given $2x + 3x = 180°$

$5x = 180°$

$x = \frac{180°}{5} = 36°$

$\therefore 3x = 36° \times 3 = 108°$

30: (B) ABCD is a parallelogram

$\therefore$ AC = BD

[$\because$ In a parallelogram opposite sides are equal]

MATHEMATICS - 2

31: (A, B, C)

Given $f(x) = x(y^2 - z^2) + y(z^2 - x^2) + z(x^2 - y^2)$

$f(y) = y(y^2 - z^2) + y(z^2 - y^2) + z(y^2 - y^2)$

$= y^3 - yz^2 + yz^2 - y^3$

$f(y) = 0 \Rightarrow (x - y)$ is a factor of $f(x)$ similarly we can prove.

$(y - z)$ & $(z - x)$ are also factors of $f(x)$

32: (A, D)

$(a^2 - b^2)(c^2 - d^2) - 4abcd =$

$= a^2c^2 - a^2d^2 - b^2c^2 + b^2d^2 - 2abcd - 2abcd$

$= (a^2c^2 + b^2d^2 - 2abcd) - (a^2d^2 + b^2c^2 + 2abcd)$

$= (ac - bd)^2 - (ad + bc)^2$

$= [(ac - bd) + (ad + bc)][(ac - bd) - (ad + bc)]$

$= (ac - bd + ad + bc)\ (ac - bd - ad - bc)$

33: (A, C)

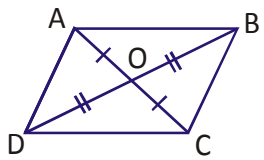
In ΔABC Given $\angle A = \angle B = 70° \Rightarrow 70° + 70° + \angle C = 180°$

$\Rightarrow \angle C = 180° - 140° = 40°$

$\therefore$ BC & AC are bigger sides

34: (A, B, C)

Given in a quadrilateral ABCD diagonals bisect each other



$\therefore$ ABCD is a parallelogram

$\therefore$ BC || AD & BC = AD

AB || CD

35: (A, B, D)

Options A, B & D are true option 'C' is false because in a trapezium both pair of opposite sides are not parallel.

REASONING

36: (B) From bottom left arrow all the arrow turn 90° anticlock wise in each step.

37: (D) (A) $8 - 43 \div 6 \times 1 = 4$ (✗)

(B) $7 \times 6 \div 2 + 3 - 6 = 5$ (✗)

(C) $10 + 7 - 3 \div 2 = 4$ (✗)

(D) $2 \times 5 - 6 + 2 = 6$ (✓)

38: (B) 1 = (d), 2 = (c)

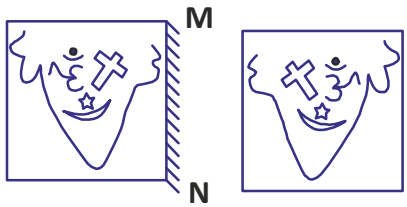
39: (D) Except geography rest of all subjects related to science.

40: (D) In given figure, the dot is placed in the region which is common to the circle and square. Now, we have to search similar common region in four options. Only in figure (D), we find such a region which is common to the circle and square.

41: (C) Image 2 is the textured portion of Image 1 rotated by 90 degrees clockwise.

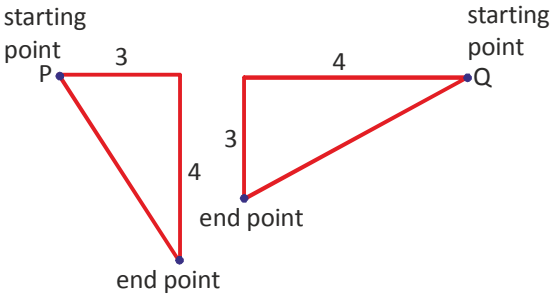
Similarly between image 3 and image 4, textured portion of image 3, when rotated by 90 degrees clockwise, the horizontal lines in the textured portion will become vertical lines and the orientation would also change to the one similar to image in option 3.

42: (D)



43: (C) 14

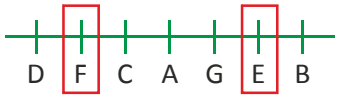
44: (A) $\sqrt{16+9} = \sqrt{25} = 5\,\text{km}$



$p = \sqrt{(3)^2 + (4)^2} \Rightarrow 5\text{ km}$ Q

$= \sqrt{(4)^2 + (3)^2} = \sqrt{25} = 5\text{ km}$

45: (C)



3 boys are there in between E and F.

CRITICAL THINKING

46: (D) P's travel denoted by green line and Q's travel by orange line. R's travel by Purple line.

P met Q at the green dot as shown while Q was supposed to travel along the dotted line. After they met at the dot, they walk and reach R. From R's home, the three travel to T's home.

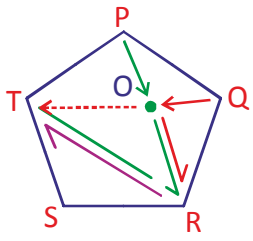
Assuming that all the friends walk at same pace (speed), let's check the option and see which are correct.

A) Actually the distance travelled by P and Q to reach the green dot (O) is the same, i.e., PO = QO. So, the distance travelled by P and Q are actually same.

B) The above discussion applies here. Note that T is quite opposite to T, just like R is quite opposite to P, they meet at the same distance.

C) Since PO = QO, QO + OR = PO + OR, which is equal to RT. R walked a distance of RT while Q walked a distance of QO + OR + RT = 2RT

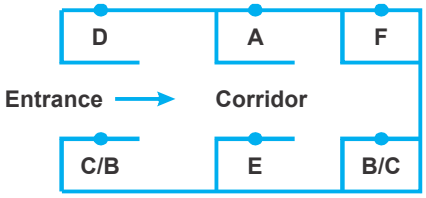
D) As is clear from the drawing, P has to travel just by distance TP, which is the least.



47: (B) Population→unemployment→Poverty →Disease→ Death

48: (C) Option (D) is more unrealistic (troublesome) legs position. Two legs front while he leaning back, that too with the heavy weight, will make him fall behind. A is more like the person was not able to move and was about to collapse or fall down. Discussion for B is similar to that of D.

49: (A) After analysing the given information, we can draw the following diagram A is the only neighbour of F.



50: (B) Bonus and dividend are different from each other. But both these are parts of profit.