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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION

Solutions for Class : 12 PCM

MATHEMATICS

1. (C) Let T be the thickness of the sides, then that of the top will be $\frac{5}{4}T$.

$$\therefore S = (2\pi rh)T + \pi r^2 \cdot \frac{5}{4}T = 2\pi T.r \cdot \frac{V}{\pi r^2} + \frac{5\pi}{4} \cdot T.r^2$$

$$= 2TV \cdot \frac{1}{r} + \frac{5\pi}{4} \cdot T.r^2 = f(r)$$

$$f'(r) = \frac{-2TV}{r^2} + \frac{5\pi}{2}T.r = 0 \Rightarrow r^3 = \frac{4V}{5\pi}$$

$$\Rightarrow 5\pi r^3 = 4V = 4\pi r^2h$$

$$\Rightarrow \frac{r}{h} = \frac{4}{5}$$

2. (A) $x\mathbf{A} + \mathbf{B} = \begin{vmatrix} x^3 + x & x + 1 & x - 2 \\ 2x^3 + 3x - 1 & 3x & 3x - 3 \\ x^3 + 2x + 3 & 2x - 1 & 2x - 1 \end{vmatrix}$

$$R_2 - R_1 - R_3 = \begin{vmatrix} x^3 + x & x + 1 & x - 2 \\ -4 & 0 & 0 \\ x^3 + 2x + 3 & 2x - 1 & 2x - 1 \end{vmatrix}$$

$$R_1 + \frac{1}{4}x^3R_2 \quad R_3 + \frac{1}{4}x^3R_2 = \begin{vmatrix} x & x + 1 & x - 2 \\ -4 & 0 & 0 \\ 2x + 2x + 3 & 2x - 1 & 2x - 1 \end{vmatrix}$$

$$R_3 - 2R_1 = \begin{vmatrix} x & x + 1 & x - 2 \\ -4 & 0 & 0 \\ 3 & -3 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} x & x & x \\ -4 & 0 & 0 \\ 3 & -3 & 3 \end{vmatrix} + \begin{vmatrix} 0 & 1 & -2 \\ -4 & 0 & 0 \\ 3 & -3 & 3 \end{vmatrix}$$

$$= x \begin{vmatrix} 1 & 1 & 1 \\ -4 & 0 & 0 \\ 3 & -3 & 3 \end{vmatrix} + \begin{vmatrix} 0 & 1 & -2 \\ -4 & 0 & 0 \\ 3 & -3 & 3 \end{vmatrix}$$

3. (C) Given, $\begin{vmatrix} 0 & x - a & x - b \\ x + a & 0 & x - c \\ x + b & x + c & 0 \end{vmatrix} = 0$

Expanding the given determinant, we have

$$2x^3 + 2x(ac - ab - bc) = 0$$

$$\therefore x = 0$$

4. (C) $A^2 = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 \times \frac{1}{2} & 1 \end{bmatrix}$

$$= A^2 \cdot A = \begin{bmatrix} 1 & 0 \\ 2 \times \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 \times \frac{1}{2} & 1 \end{bmatrix}$$

Continuing in this way, we get

$$A^{100} = \begin{bmatrix} 1 & 0 \\ 100 \times \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}$$

5. (B) The required determinant is obtained by the successive operations

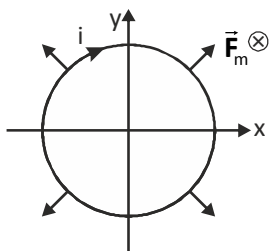
$$C_1 \rightarrow 2C_1 \text{ and } C_1 \rightarrow C_1 + 3C_2 + 4C_3$$

\therefore The value of the determinant is multiplied by 2 (since of the first operation), second operation does not affect the value of the determinant.

PHYSICS

6. (B) Net force on a current carrying loop in uniform magnetic field is zero. Hence, the loop cannot translate. From Fleming's left hand rule we can see that if

magnetic field is perpendicular to paper inwards and current in the loop is clockwise (as shown) the magnetic force \vec{F}_m on each element of the loop is radially outwards, or the loops will have a tendency to expand.



7. (D) According to the given information substance 'X' is ferromagnetic in nature which is gadolinium.

A paramagnetic substance is feebly attracted by a magnet. When a rod of paramagnetic substance is suspended in a magnetic field, it slowly sets itself parallel to the direction of the magnetic field. It also moves from a weaker part of the magnetic field to its stronger part, but it is feebly attracted by the magnet.

When a rod of diamagnetic substance is suspended in a magnetic field, it slowly sets itself at right angles to the direction of field. It moves from stronger part of the magnetic field to its weaker part i.e., it is feebly repelled by the magnet.

Aluminium and oxygen are paramagnetic. Gold is diamagnetic. Gadolinium is ferromagnetic. So, substance X is gadolinium.

8. (A) Given $I = 20 \text{ A}$, $n = 9 \times 10^{30} \text{ m}^{-3}$; $A = 10^{-4} \text{ m}^2$ and $e = 1.6 \times 10^{-19} \text{ C}$

$$V_d = \frac{I}{neA} = \frac{20}{9 \times 10^{30} \times 1.6 \times 10^{-19} \times 10^{-4}}$$

$$= 0.138 \times 10^{-6} \text{ m s}^{-1}$$

9. (B) As the image formed is erect and hence virtual, the magnification produced by the lens is positive i.e. $m = +4$.

Also, $f = +20 \text{ cm}$

$$\text{Now, } m = \frac{f}{u+f}$$

$$\therefore 4 = \frac{20}{u+20} \text{ or } u+20 = 5$$

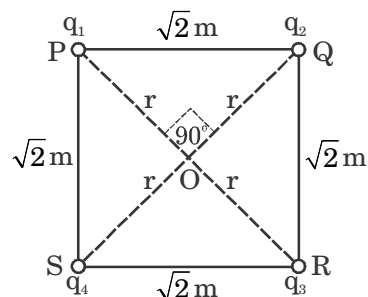
or $u = -15 \text{ cm}$

$$\text{Again, } m = \frac{f}{v} \quad 4 = \frac{20}{v}$$

$$\text{or } v = 20 - 80 = -60 \text{ cm}$$

10. (A) Four charges q_1, q_2, q_3 and q_4 are placed at the four corners of the square PQRS as shown below.

Here,



$$q_1 = 2 \mu\text{C} = 2 \times 10^{-6} \text{ C};$$

$$q_2 = -2 \mu\text{C} = -2 \times 10^{-6} \text{ C};$$

$$q_3 = -3 \mu\text{C} = -3 \times 10^{-6} \text{ C};$$

$$q_4 = 6 \mu\text{C} = 6 \times 10^{-6} \text{ C};$$

$$\text{and } PQ = QR = RS = PS = \sqrt{2} \text{ m}$$

Let r be the distance of each charge from the centre O of the square.

$$\text{Then, } \sqrt{r^2 + r^2} = \sqrt{2} \quad \text{or } r = 1 \text{ m}$$

Potential at point O due to charges at the four corners,

$$V = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r} \right)$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{1}{r} (q_1 + q_2 + q_3 + q_4)$$

$$\frac{9 \times 10^9}{1} (2 \times 10^{-6} + (-2 \times 10^{-6}) + (-3 \times 10^{-6}) + 6 \times 10^{-6})$$

$$= 2.7 \times 10^4 \text{ V}$$

CHEMISTRY

11. (A) Addition of HCl to acetylene in the presence of mercuric salt leads to formation of vinyl chloride which in the monomer for poly vinyl chloride.
12. (A) The ideal conditions for the manufacture of H_2SO_4 by contact process are low temperature, high pressure and high concentration of reactants.
13. (B) The process of heating the quenched steel to a temperature much below redness and cooling it slowly.

14. (C) Amorphous solids do not melt. They simply soften on heating, and gradually begin to flow on further heating. These solids are, therefore considered as super cooled liquids.
15. (D) $[\text{Fe}(\text{CN})_6]^{3-}$ is an octahedral complex ion and is paramagnetic in nature. Secondly it is an inner orbital complex ion with the presence of only one unpaired electron in it.

CRITICAL THINKING

16. (D)
17. (D)
18. (C)
19. (D)
20. (B)